

# Computer algebra independent integration tests

4-Trig-functions/4.5-Secant/4.5.11-e-x-<sup>m</sup>-a+b-sec-c+d-x<sup>n</sup>-<sup>p</sup>

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# Chapter 1

## Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from <https://rulebasedintegration.org>

The number of integrals in this report is [ 83 ]. This is test number [ 117 ].

### 1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

## 1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric2F1 functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 ( 83 )	% 0.00 ( 0 )
Mathematica	% 95.18 ( 79 )	% 4.82 ( 4 )
Maple	% 61.45 ( 51 )	% 38.55 ( 32 )
Maxima	% 33.73 ( 28 )	% 66.27 ( 55 )
Fricas	% 75.90 ( 63 )	% 24.10 ( 20 )
Sympy	% 44.58 ( 37 )	% 55.42 ( 46 )
Giac	% 51.81 ( 43 )	% 48.19 ( 40 )
Mupad	% 56.63 ( 47 )	% 43.37 ( 36 )

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.



grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> <li>1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.</li> <li>2. antiderivative contains a special function and the optimal antiderivative does not.</li> <li>3. antiderivative contains the imaginary unit and the optimal antiderivative does not.</li> </ol>
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

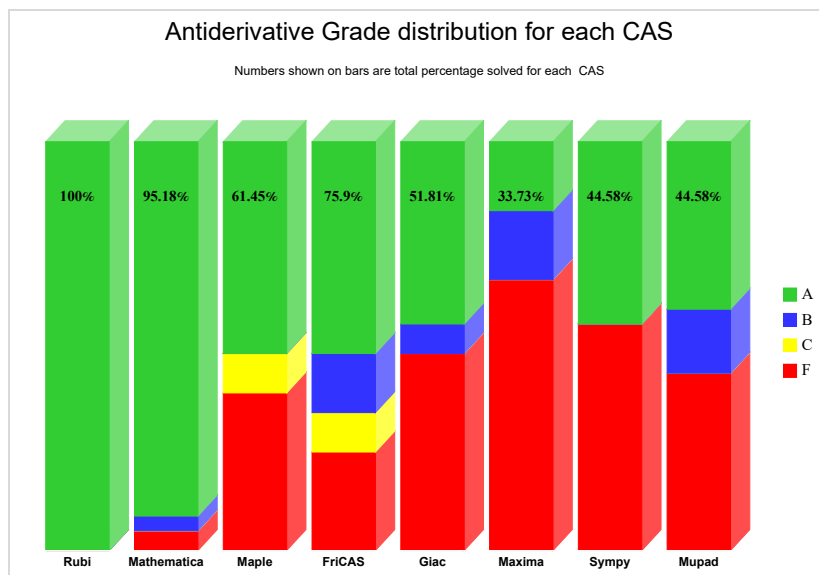
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

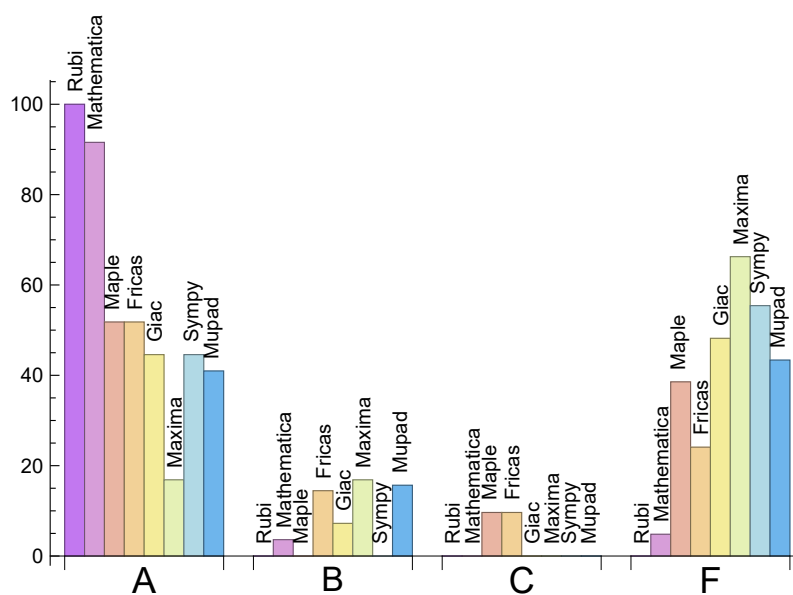
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	91.57	3.61	0.00	4.82
Maple	51.81	0.00	9.64	38.55
Maxima	16.87	16.87	0.00	66.27
Fricas	51.81	14.46	9.64	24.10
Sympy	44.58	0.00	0.00	55.42
Giac	44.58	7.23	0.00	48.19
Mupad	40.96	15.66	0.00	43.37

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This is the typical normal failure **F**.

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	4	100.00 %	0.00 %	0.00 %
Maple	32	100.00 %	0.00 %	0.00 %
Maxima	55	16.36 %	58.18 %	25.45 %
Fricas	20	100.00 %	0.00 %	0.00 %
Sympy	46	97.83 %	2.17 %	0.00 %
Giac	40	97.50 %	0.00 %	2.50 %
Mupad	36	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

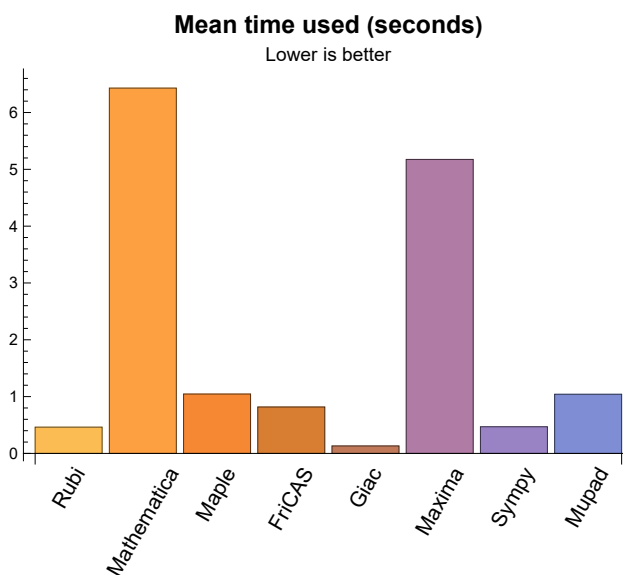
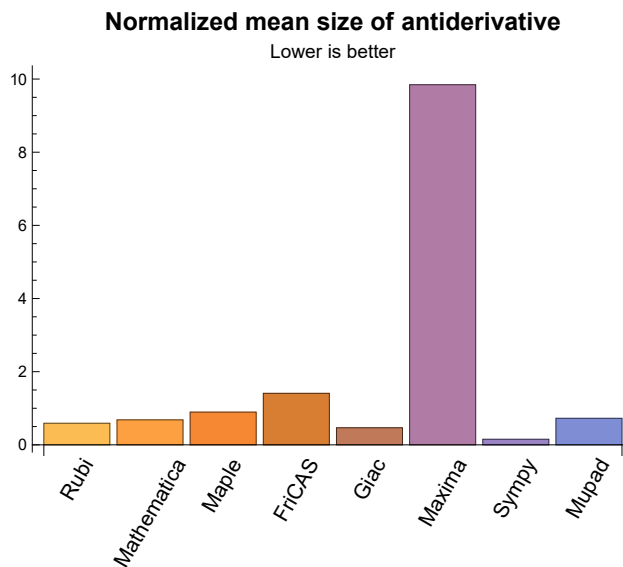
## 1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.46	302.76	0.59	79.00	1.00
Mathematica	6.43	355.39	0.68	54.00	0.92
Maple	1.05	164.71	0.89	0.00	0.00
Maxima	5.17	1443.71	9.85	73.00	1.69
Fricas	0.82	396.29	1.41	0.00	0.00
Sympy	0.47	5.03	0.15	0.00	0.00
Giac	0.13	30.42	0.47	0.00	0.00
Mupad	1.04	58.66	0.73	-1.00	-0.04

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.



## 1.4 list of integrals that has no closed form antiderivative

{2, 4, 6, 7, 9, 11, 13, 14, 17, 19, 21, 22, 24, 26, 28, 29, 30, 34, 35, 39, 40, 44, 45, 49, 50, 54, 55, 59, 60, 64, 65, 69, 70, 71}

## 1.5 list of integrals solved by CAS but has no known antiderivative

**Rubi** {}

**Mathematica** {}

**Maple** {}

**Maxima** {}

**Fricas** {}

**Sympy** {}

**Giac** {}

**Mupad** {}

## 1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

**Rubi** {}

**Mathematica** {23, 25, 46, 47, 48, 66, 67, 76, 82}

**Maple** Verification phase not implemented yet.

**Maxima** Verification phase not implemented yet.

**Fricas** Verification phase not implemented yet.

**Sympy** Verification phase not implemented yet.

**Giac** Verification phase not implemented yet.

**Mupad** Verification phase not implemented yet.

## 1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

## 1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

## 1.9 Important notes about some of the results

### 1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using `sagemath` (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

## 1.9.2 Important note about FriCAS and Giac/X-CAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

## 1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the builtin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at <https://>



[ask.sagemath.org/question/57123/could-we-have-a-leaf\\_count-function-in-base-sagemath/](http://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/)

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount = 1
```

## 1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

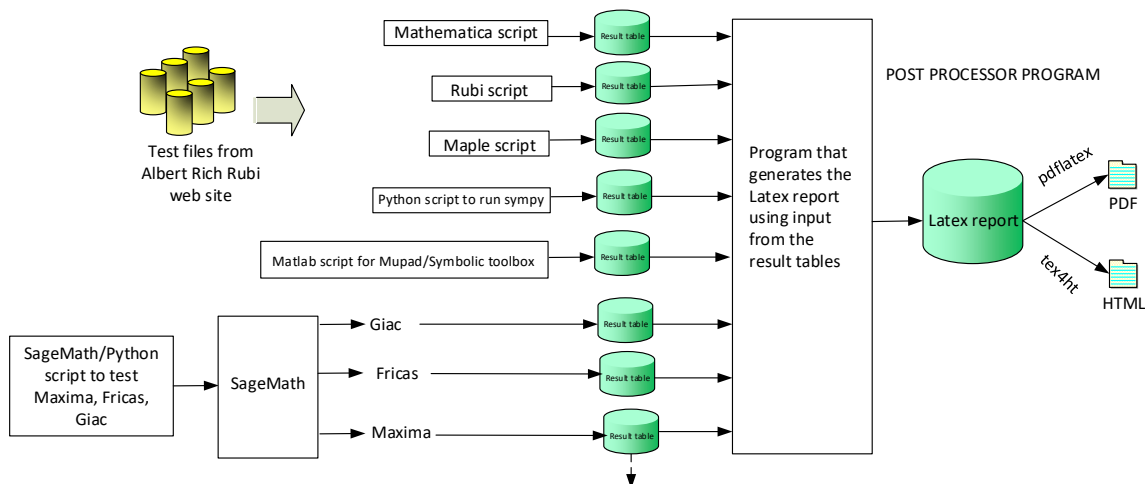
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine, 'cos(x)*sin(x)')
the_variable = evalin(symengine, 'x')
anti = int(integrand, the_variable)
```

Which gives  $\sin(x)^2/2$

## 1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



**One record (line) per one integral result. The line is CSV comma separated. This is description of each record**

1. integer, the problem number.
  2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
  3. integer. Leaf size of result.
  4. integer. Leaf size of the optimal antiderivative.
  5. number. CPU time used to solve this integral. 0 if failed.
  6. string. The integral in Latex format
  7. string. The input used in CAS own syntax.
  8. string. The result (antiderivative) produced by CAS in Latex format
  9. string. The optimal antiderivative in Latex format.
  10. integer. 0 or 1. Indicates if problem has known antiderivative or not
  11. String. The result (antiderivative) in CAS own syntax.
  12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
- The following field present only in Rubi and Mathematica Tables*
13. integer. 1 if result was verified or 0 if not verified.
- The following fields present only in Rubi Tables*
14. integer. Number of rules used.
  15. integer. Integrand leaf size.
  16. real number. Ratio of field 14 over field 15
  17. integer. 1 if result was verified or 0 if not verified.
  18. String of form "{n,n,...}" which is list of the rules used by Rubi

**High level overview of the CAS independent integration test build system**

# Chapter 2

## detailed summary tables of results

### 2.1 List of integrals sorted by grade for each CAS

#### 2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83 }  
}

B grade: { }

C grade: { }

F grade: { }

#### 2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 75, 76, 78, 81 }  
}

B grade: { 18, 79, 82 }  
}

C grade: { }

F grade: { 74, 77, 80, 83 }  
}

#### 2.1.3 Maple

A grade: { 2, 4, 5, 6, 7, 9, 11, 12, 13, 14, 15, 17, 19, 20, 21, 22, 24, 26, 27, 28, 29, 30, 34, 35, 39, 40, 44, 45, 49, 50, 53, 54, 55, 58, 59, 60, 63, 64, 65, 68, 69, 70, 71 }  
}

B grade: { }

C grade: { 72, 73, 75, 76, 78, 79, 81, 82 }

F grade: { 1, 3, 8, 10, 16, 18, 23, 25, 31, 32, 33, 36, 37, 38, 41, 42, 43, 46, 47, 48, 51, 52, 56, 57, 61, 62, 66, 67, 74, 77, 80, 83 }

## 2.1.4 Maxima

A grade: { 2, 4, 5, 6, 7, 13, 22, 34, 39, 40, 53, 58, 59, 71 }

B grade: { 12, 15, 20, 27, 31, 32, 33, 36, 37, 38, 51, 52, 56, 57 }

C grade: { }

F grade: { 1, 3, 8, 9, 10, 11, 14, 16, 17, 18, 19, 21, 23, 24, 25, 26, 28, 29, 30, 35, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 54, 55, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83 }

## 2.1.5 FriCAS

A grade: { 2, 4, 5, 6, 7, 9, 11, 13, 14, 15, 17, 19, 20, 21, 22, 24, 26, 28, 29, 30, 34, 35, 39, 40, 44, 45, 49, 50, 53, 54, 55, 59, 60, 63, 64, 65, 69, 70, 71, 72, 75, 78, 81 }

B grade: { 3, 10, 12, 18, 25, 27, 58, 68, 73, 76, 79, 82 }

C grade: { 1, 8, 16, 23, 74, 77, 80, 83 }

F grade: { 31, 32, 33, 36, 37, 38, 41, 42, 43, 46, 47, 48, 51, 52, 56, 57, 61, 62, 66, 67 }

## 2.1.6 Sympy

A grade: { 2, 4, 5, 6, 7, 9, 11, 13, 14, 17, 19, 21, 22, 24, 26, 28, 29, 30, 34, 35, 39, 40, 44, 45, 49, 50, 53, 54, 55, 58, 59, 60, 64, 65, 69, 70, 71 }

B grade: { }

C grade: { }

F grade: { 1, 3, 8, 10, 12, 15, 16, 18, 20, 23, 25, 27, 31, 32, 33, 36, 37, 38, 41, 42, 43, 46, 47, 48, 51, 52, 56, 57, 61, 62, 63, 66, 67, 68, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83 }

## 2.1.7 Giac

A grade: { 2, 4, 6, 7, 9, 11, 13, 14, 15, 17, 19, 21, 22, 24, 26, 27, 28, 29, 30, 34, 35, 39, 40, 44, 45, 49, 50, 54, 55, 59, 60, 64, 65, 68, 69, 70, 71 }

B grade: { 5, 12, 20, 53, 58, 63 }

C grade: { }

F grade: { 1, 3, 8, 10, 16, 18, 23, 25, 31, 32, 33, 36, 37, 38, 41, 42, 43, 46, 47, 48, 51, 52, 56, 57, 61, 62, 66, 67, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83 }

## 2.1.8 Mupad

A grade: { 2, 4, 6, 7, 9, 11, 13, 14, 17, 19, 21, 22, 24, 26, 28, 29, 30, 34, 35, 39, 40, 44, 45, 49, 50, 54, 55, 59, 60, 64, 65, 69, 70, 71 }

B grade: { 5, 12, 15, 20, 27, 53, 58, 63, 68, 72, 75, 78, 81 }

C grade: { }

F grade: { 1, 3, 8, 10, 16, 18, 23, 25, 31, 32, 33, 36, 37, 38, 41, 42, 43, 46, 47, 48, 51, 52, 56, 57, 61, 62, 66, 67, 73, 74, 76, 77, 79, 80, 82, 83 }

## 2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column **normalized size** is defined as  $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	146	0	0	495	0	0	-1
normalized size	1	1.00	1.02	0.00	0.00	3.46	0.00	0.00	-0.01
time (sec)	N/A	0.162	0.048	0.744	0.000	1.189	0.000	0.000	0.000
Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.015	0.958	0.742	0.000	0.644	0.000	0.000	0.000
Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	95	0	0	346	0	0	-1
normalized size	1	1.00	1.03	0.00	0.00	3.76	0.00	0.00	-0.01
time (sec)	N/A	0.088	0.025	0.742	0.000	0.787	0.000	0.000	0.000

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.015	0.771	0.752	0.000	0.792	0.000	0.000	0.000

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	39	31	42	42	50	67
normalized size	1	1.00	1.00	1.50	1.19	1.62	1.62	1.92	2.58
time (sec)	N/A	0.023	0.014	0.201	0.311	0.746	2.869	0.546	0.581

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.016	0.766	0.758	0.000	0.697	0.000	0.000	0.000

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.016	0.739	0.704	0.000	0.769	0.000	0.000	0.000

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	229	0	0	795	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	3.29	0.00	0.00	-0.00
time (sec)	N/A	0.370	0.610	1.100	0.000	0.751	0.000	0.000	0.000

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.023	9.611	1.134	0.000	0.572	0.000	0.000	0.000

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-1)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	123	0	0	525	0	0	-1
normalized size	1	1.00	0.92	0.00	0.00	3.95	0.00	0.00	-0.01
time (sec)	N/A	0.164	0.465	1.288	0.000	0.870	0.000	0.000	0.000

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.024	7.907	0.979	0.000	0.617	0.000	0.000	0.000

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	41	59	96	91	0	88	100
normalized size	1	1.00	0.93	1.34	2.18	2.07	0.00	2.00	2.27
time (sec)	N/A	0.052	0.213	0.524	0.820	0.664	0.000	0.859	1.497

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.022	17.858	1.155	0.000	0.532	0.000	0.000	0.000



Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.022	9.318	1.122	0.000	1.367	0.000	0.000	0.000

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	62	92	2838	100	0	85	496
normalized size	1	1.00	0.69	1.02	31.53	1.11	0.00	0.94	5.51
time (sec)	N/A	0.073	0.210	0.553	1.421	0.656	0.000	0.336	12.531

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-1)	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	382	382	472	0	0	1457	0	0	-1
normalized size	1	1.00	1.24	0.00	0.00	3.81	0.00	0.00	-0.00
time (sec)	N/A	0.878	1.119	0.846	0.000	0.948	0.000	0.000	0.000

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.026	1.404	0.798	0.000	0.576	0.000	0.000	0.000

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	845	0	0	1063	0	0	-1
normalized size	1	1.00	3.24	0.00	0.00	4.07	0.00	0.00	-0.00
time (sec)	N/A	0.540	1.397	0.825	0.000	1.097	0.000	0.000	0.000

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.025	1.169	0.831	0.000	0.668	0.000	0.000	0.000

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	67	70	7945	251	0	278	157
normalized size	1	1.00	1.02	1.06	120.38	3.80	0.00	4.21	2.38
time (sec)	N/A	0.109	0.179	0.543	63.895	0.754	0.000	0.500	2.239

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.025	0.975	0.701	0.000	0.606	0.000	0.000	0.000

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.014	0.151	0.030	0.000	0.586	0.000	0.000	0.000

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1092	1092	895	0	0	3048	0	0	-1
normalized size	1	1.00	0.82	0.00	0.00	2.79	0.00	0.00	-0.00
time (sec)	N/A	2.288	6.578	1.673	0.000	1.069	0.000	0.000	0.000

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.024	6.880	1.413	0.000	0.623	0.000	0.000	0.000

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	596	596	1069	0	0	1928	0	0	-1
normalized size	1	1.00	1.79	0.00	0.00	3.23	0.00	0.00	-0.00
time (sec)	N/A	1.204	9.033	1.535	0.000	1.011	0.000	0.000	0.000

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.024	5.802	1.128	0.000	0.609	0.000	0.000	0.000

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	153	214	8871	525	0	195	340
normalized size	1	1.00	1.24	1.74	72.12	4.27	0.00	1.59	2.76
time (sec)	N/A	0.254	0.689	0.529	66.778	2.060	0.000	0.541	5.013

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.024	11.296	1.311	0.000	0.672	0.000	0.000	0.000

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.024	7.373	1.465	0.000	0.994	0.000	0.000	0.000

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.024	7.815	1.635	0.000	0.841	0.000	0.000	0.000

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	476	476	479	0	1512	0	0	0	-1
normalized size	1	1.00	1.01	0.00	3.18	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.457	0.147	1.289	1.070	2.106	0.000	0.000	0.000

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	348	348	351	0	966	0	0	0	-1
normalized size	1	1.00	1.01	0.00	2.78	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.311	0.084	1.165	1.006	0.591	0.000	0.000	0.000

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	223	0	540	0	0	0	-1
normalized size	1	1.00	1.01	0.00	2.45	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.186	0.063	1.151	0.895	0.820	0.000	0.000	0.000

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.017	2.063	1.349	0.000	0.750	0.000	0.000	0.000

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.018	2.199	1.337	0.000	0.477	0.000	0.000	0.000

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	749	749	739	0	6373	0	0	0	-1
normalized size	1	1.00	0.99	0.00	8.51	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.860	2.198	1.385	1.623	0.583	0.000	0.000	0.000

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	551	551	543	0	3898	0	0	0	-1
normalized size	1	1.00	0.99	0.00	7.07	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.634	1.257	1.384	1.427	0.618	0.000	0.000	0.000

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	355	355	347	0	2006	0	0	0	-1
normalized size	1	1.00	0.98	0.00	5.65	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.451	0.708	1.378	0.920	0.903	0.000	0.000	0.000

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.023	40.078	1.402	0.000	0.754	0.000	0.000	0.000

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.023	21.034	1.398	0.000	0.739	0.000	0.000	0.000

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1041	1041	1122	0	0	0	0	0	-1
normalized size	1	1.00	1.08	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.476	2.667	1.134	0.000	1.038	0.000	0.000	0.000

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	781	781	858	0	0	0	0	0	-1
normalized size	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.180	2.129	1.128	0.000	0.847	0.000	0.000	0.000

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	521	521	632	0	0	0	0	0	-1
normalized size	1	1.00	1.21	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.966	20.442	1.358	0.000	0.587	0.000	0.000	0.000

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.025	2.013	1.234	0.000	0.723	0.000	0.000	0.000

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.015	0.390	0.026	0.000	0.539	0.000	0.000	0.000

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	3123	3123	3702	0	0	0	0	0	-1
normalized size	1	1.00	1.19	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	4.118	14.878	1.374	0.000	0.838	0.000	0.000	0.000

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	2323	2323	2777	0	0	0	0	0	-1
normalized size	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.228	13.262	1.296	0.000	0.755	0.000	0.000	0.000

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1523	1523	1767	0	0	0	0	0	-1
normalized size	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.504	15.938	1.306	0.000	0.626	0.000	0.000	0.000

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.026	44.417	1.303	0.000	1.387	0.000	0.000	0.000

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.025	31.565	1.281	0.000	0.821	0.000	0.000	0.000

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	281	0	738	0	0	0	-1
normalized size	1	1.00	0.99	0.00	2.60	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.248	0.254	1.204	0.972	0.595	0.000	0.000	0.000

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	155	0	374	0	0	0	-1
normalized size	1	1.00	0.98	0.00	2.37	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.132	0.135	1.181	0.981	0.712	0.000	0.000	0.000

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	32	31	41	56	50	71
normalized size	1	1.00	1.00	1.23	1.19	1.58	2.15	1.92	2.73
time (sec)	N/A	0.021	0.083	0.264	0.372	0.663	4.742	0.848	2.943



Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.014	2.899	1.381	0.000	0.590	0.000	0.000	0.000

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.014	2.833	1.456	0.000	0.553	0.000	0.000	0.000

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	451	451	443	0	2876	0	0	0	-1
normalized size	1	1.00	0.98	0.00	6.38	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.537	1.583	1.287	0.866	0.530	0.000	0.000	0.000

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	247	0	1279	0	0	0	-1
normalized size	1	1.00	0.97	0.00	5.02	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.325	0.823	1.289	0.998	0.736	0.000	0.000	0.000

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	45	60	50	91	88	88	109
normalized size	1	1.00	0.96	1.28	1.06	1.94	1.87	1.87	2.32
time (sec)	N/A	0.052	0.279	0.758	0.529	0.786	9.752	0.747	2.683

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.022	23.126	1.290	0.000	0.518	0.000	0.000	0.000

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.023	23.811	1.315	0.000	0.702	0.000	0.000	0.000

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	653	653	725	0	0	0	0	0	-1
normalized size	1	1.00	1.11	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.049	2.492	1.071	0.000	0.535	0.000	0.000	0.000

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	393	393	486	0	0	0	0	0	-1
normalized size	1	1.00	1.24	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.826	6.700	1.011	0.000	0.617	0.000	0.000	0.000

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	69	71	0	274	0	278	153
normalized size	1	1.00	1.01	1.04	0.00	4.03	0.00	4.09	2.25
time (sec)	N/A	0.082	0.263	0.706	0.000	0.548	0.000	0.537	2.746

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.025	4.399	1.000	0.000	0.662	0.000	0.000	0.000

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.024	3.955	0.987	0.000	0.494	0.000	0.000	0.000

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1925	1925	2254	0	0	0	0	0	-1
normalized size	1	1.00	1.17	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.858	13.833	1.332	0.000	0.616	0.000	0.000	0.000

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1125	1125	1210	0	0	0	0	0	-1
normalized size	1	1.00	1.08	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.106	9.439	1.159	0.000	0.551	0.000	0.000	0.000

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	163	216	0	574	0	196	330
normalized size	1	1.00	1.28	1.70	0.00	4.52	0.00	1.54	2.60
time (sec)	N/A	0.199	0.793	0.852	0.000	0.581	0.000	0.736	5.251

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.023	29.076	1.308	0.000	0.513	0.000	0.000	0.000

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.023	29.211	1.237	0.000	0.575	0.000	0.000	0.000

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.048	3.087	1.509	0.000	0.745	0.000	0.000	0.000

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	38	159	0	60	0	0	104
normalized size	1	1.00	0.86	3.61	0.00	1.36	0.00	0.00	2.36
time (sec)	N/A	0.047	0.116	1.584	0.000	0.754	0.000	0.000	2.656

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	188	873	0	470	0	0	-1
normalized size	1	1.00	1.26	5.86	0.00	3.15	0.00	0.00	-0.01
time (sec)	N/A	0.114	0.646	1.867	0.000	1.484	0.000	0.000	0.000

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	C	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	0	0	0	655	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	2.79	0.00	0.00	-0.00
time (sec)	N/A	0.190	1.324	2.273	0.000	0.826	0.000	0.000	0.000

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	54	276	0	113	0	0	180
normalized size	1	1.00	0.68	3.49	0.00	1.43	0.00	0.00	2.28
time (sec)	N/A	0.090	0.417	1.191	0.000	0.581	0.000	0.000	2.710

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	347	1096	0	656	0	0	-1
normalized size	1	1.00	1.57	4.96	0.00	2.97	0.00	0.00	-0.00
time (sec)	N/A	0.200	5.162	1.202	0.000	0.936	0.000	0.000	0.000

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	C	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	390	390	0	0	0	1028	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	2.64	0.00	0.00	-0.00
time (sec)	N/A	0.396	11.662	3.020	0.000	1.277	0.000	0.000	0.000

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	80	314	0	300	0	0	223
normalized size	1	1.00	0.92	3.61	0.00	3.45	0.00	0.00	2.56
time (sec)	N/A	0.147	0.288	1.071	0.000	0.638	0.000	0.000	2.753

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F(-1)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	328	328	861	1308	0	1291	0	0	-1
normalized size	1	1.00	2.62	3.99	0.00	3.94	0.00	0.00	-0.00
time (sec)	N/A	0.595	1.947	1.522	0.000	0.707	0.000	0.000	0.000

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F(-1)	C	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	485	485	0	0	0	1733	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	3.57	0.00	0.00	-0.00
time (sec)	N/A	0.923	1.837	1.951	0.000	2.327	0.000	0.000	0.000

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	191	704	0	628	0	0	461
normalized size	1	1.00	1.22	4.48	0.00	4.00	0.00	0.00	2.94
time (sec)	N/A	0.287	0.932	1.330	0.000	0.663	0.000	0.000	5.331

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F(-1)	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	757	757	2450	2817	0	2531	0	0	-1
normalized size	1	1.00	3.24	3.72	0.00	3.34	0.00	0.00	-0.00
time (sec)	N/A	1.273	10.473	1.142	0.000	1.096	0.000	0.000	0.000

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F(-1)	C	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1384	1384	0	0	0	3855	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	2.79	0.00	0.00	-0.00
time (sec)	N/A	2.367	11.748	2.933	0.000	1.314	0.000	0.000	0.000

## 2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio  $\frac{\text{number of rules}}{\text{integrand size}}$  is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [23] had the largest ratio of [.6667]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	10	6	1.00	16	0.375
2	A	0	0	0.00	0	0.000
3	A	8	5	1.00	16	0.312
4	A	0	0	0.00	0	0.000
5	A	4	3	1.00	14	0.214
6	A	0	0	0.00	0	0.000
7	A	0	0	0.00	0	0.000
8	A	15	11	1.00	18	0.611
9	A	0	0	0.00	0	0.000
10	A	10	7	1.00	18	0.389
11	A	0	0	0.00	0	0.000
12	A	5	5	1.00	16	0.312
13	A	0	0	0.00	0	0.000
14	A	0	0	0.00	0	0.000
15	A	5	3	1.00	12	0.250
Continued on next page						

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
16	A	13	8	1.00	18	0.444
17	A	0	0	0.00	0	0.000
18	A	11	7	1.00	18	0.389
19	A	0	0	0.00	0	0.000
20	A	4	4	1.00	16	0.250
21	A	0	0	0.00	0	0.000
22	A	0	0	0.00	0	0.000
23	A	31	12	1.00	18	0.667
24	A	0	0	0.00	0	0.000
25	A	22	10	1.00	18	0.556
26	A	0	0	0.00	0	0.000
27	A	6	6	1.00	16	0.375
28	A	0	0	0.00	0	0.000
29	A	0	0	0.00	0	0.000
30	A	0	0	0.00	0	0.000
31	A	20	7	1.00	18	0.389
32	A	16	7	1.00	18	0.389
33	A	12	7	1.00	16	0.438
34	A	0	0	0.00	0	0.000
35	A	0	0	0.00	0	0.000
36	A	30	10	1.00	20	0.500
37	A	24	10	1.00	20	0.500
38	A	18	10	1.00	18	0.556
39	A	0	0	0.00	0	0.000
40	A	0	0	0.00	0	0.000
41	A	23	9	1.00	20	0.450

Continued on next page



Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
42	A	19	9	1.00	20	0.450
43	A	15	9	1.00	18	0.500
44	A	0	0	0.00	0	0.000
45	A	0	0	0.00	0	0.000
46	A	61	11	1.00	20	0.550
47	A	49	11	1.00	20	0.550
48	A	37	11	1.00	18	0.611
49	A	0	0	0.00	0	0.000
50	A	0	0	0.00	0	0.000
51	A	14	7	1.00	20	0.350
52	A	10	6	1.00	20	0.300
53	A	4	3	1.00	20	0.150
54	A	0	0	0.00	0	0.000
55	A	0	0	0.00	0	0.000
56	A	21	10	1.00	22	0.454
57	A	15	11	1.00	22	0.500
58	A	5	5	1.00	22	0.227
59	A	0	0	0.00	0	0.000
60	A	0	0	0.00	0	0.000
61	A	17	9	1.00	22	0.409
62	A	13	8	1.00	22	0.364
63	A	4	4	1.00	22	0.182
64	A	0	0	0.00	0	0.000
65	A	0	0	0.00	0	0.000
66	A	43	11	1.00	22	0.500
67	A	31	12	1.00	22	0.546

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
68	A	6	6	1.00	22	0.273
69	A	0	0	0.00	0	0.000
70	A	0	0	0.00	0	0.000
71	A	0	0	0.00	0	0.000
72	A	5	4	1.00	20	0.200
73	A	9	6	1.00	22	0.273
74	A	11	7	1.00	22	0.318
75	A	6	6	1.00	22	0.273
76	A	11	8	1.00	24	0.333
77	A	16	12	1.00	24	0.500
78	A	5	5	1.00	22	0.227
79	A	12	8	1.00	24	0.333
80	A	14	9	1.00	24	0.375
81	A	7	7	1.00	22	0.318
82	A	23	11	1.00	24	0.458
83	A	32	13	1.00	24	0.542

# Chapter 3

## Listing of integrals

### 3.1 $\int x^5 (a + b \sec(c + dx^2)) dx$

**Optimal.** Leaf size=143

$$\frac{ax^6}{6} - \frac{b\text{Li}_3(-ie^{i(dx^2+c)})}{d^3} + \frac{b\text{Li}_3(ie^{i(dx^2+c)})}{d^3} + \frac{ibx^2\text{Li}_2(-ie^{i(dx^2+c)})}{d^2} - \frac{ibx^2\text{Li}_2(ie^{i(dx^2+c)})}{d^2} - \frac{ibx^4 \tan^{-1}(e^{i(c+dx^2)})}{d}$$

[Out]  $1/6*a*x^6 - I*b*x^4*\arctan(\exp(I*(d*x^2+c)))/d + I*b*x^2*\text{polylog}(2, -I*\exp(I*(d*x^2+c)))/d^2 - I*b*x^2*\text{polylog}(2, I*\exp(I*(d*x^2+c)))/d^2 - b*\text{polylog}(3, -I*\exp(I*(d*x^2+c)))/d^3 + b*\text{polylog}(3, I*\exp(I*(d*x^2+c)))/d^3$

**Rubi [A]** time = 0.16, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {14, 4204, 4181, 2531, 2282, 6589}

$$\frac{ibx^2\text{PolyLog}(2, -ie^{i(c+dx^2)})}{d^2} - \frac{ibx^2\text{PolyLog}(2, ie^{i(c+dx^2)})}{d^2} - \frac{b\text{PolyLog}(3, -ie^{i(c+dx^2)})}{d^3} + \frac{b\text{PolyLog}(3, ie^{i(c+dx^2)})}{d^3} + \dots$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^5*(a + b*\text{Sec}[c + d*x^2]), x]$

[Out]  $(a*x^6)/6 - (I*b*x^4*\text{ArcTan}[E^{I*(c + d*x^2)}])/d + (I*b*x^2*\text{PolyLog}[2, (-I)*E^{I*(c + d*x^2)}])/d^2 - (I*b*x^2*\text{PolyLog}[2, I*E^{I*(c + d*x^2)}])/d^2 - (b*\text{PolyLog}[3, (-I)*E^{I*(c + d*x^2)}])/d^3 + (b*\text{PolyLog}[3, I*E^{I*(c + d*x^2)}])/d^3$

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_) + (b_.)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]
```

### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)*(x_))^(m_), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

### Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f, x] + (-Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))], x], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

### Rule 4204

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps

$$\begin{aligned}
\int x^5 (a + b \sec(c + dx^2)) dx &= \int (ax^5 + bx^5 \sec(c + dx^2)) dx \\
&= \frac{ax^6}{6} + b \int x^5 \sec(c + dx^2) dx \\
&= \frac{ax^6}{6} + \frac{1}{2}b \text{Subst} \left( \int x^2 \sec(c + dx) dx, x, x^2 \right) \\
&= \frac{ax^6}{6} - \frac{ibx^4 \tan^{-1} \left( e^{i(c+dx^2)} \right)}{d} - \frac{b \text{Subst} \left( \int x \log(1 - ie^{i(c+dx)}) dx, x, x^2 \right)}{d} + \frac{b \text{Subst} \left( \int x \log(1 + ie^{i(c+dx)}) dx, x, x^2 \right)}{d} \\
&= \frac{ax^6}{6} - \frac{ibx^4 \tan^{-1} \left( e^{i(c+dx^2)} \right)}{d} + \frac{ibx^2 \text{Li}_2 \left( -ie^{i(c+dx^2)} \right)}{d^2} - \frac{ibx^2 \text{Li}_2 \left( ie^{i(c+dx^2)} \right)}{d^2} - \frac{b \text{Li}_3 \left( -ie^{i(c+dx^2)} \right)}{d^3} + \frac{b \text{Li}_3 \left( ie^{i(c+dx^2)} \right)}{d^3} \\
&= \frac{ax^6}{6} - \frac{ibx^4 \tan^{-1} \left( e^{i(c+dx^2)} \right)}{d} + \frac{ibx^2 \text{Li}_2 \left( -ie^{i(c+dx^2)} \right)}{d^2} - \frac{ibx^2 \text{Li}_2 \left( ie^{i(c+dx^2)} \right)}{d^2} - \frac{b \text{Li}_3 \left( -ie^{i(c+dx^2)} \right)}{d^3} + \frac{b \text{Li}_3 \left( ie^{i(c+dx^2)} \right)}{d^3} \\
&= \frac{ax^6}{6} - \frac{ibx^4 \tan^{-1} \left( e^{i(c+dx^2)} \right)}{d} + \frac{ibx^2 \text{Li}_2 \left( -ie^{i(c+dx^2)} \right)}{d^2} - \frac{ibx^2 \text{Li}_2 \left( ie^{i(c+dx^2)} \right)}{d^2} - \frac{b \text{Li}_3 \left( -ie^{i(c+dx^2)} \right)}{d^3} + \frac{b \text{Li}_3 \left( ie^{i(c+dx^2)} \right)}{d^3}
\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 146, normalized size = 1.02

$$\frac{ax^6}{6} - \frac{b \text{Li}_3 \left( -ie^{i(dx^2+c)} \right)}{d^3} + \frac{b \text{Li}_3 \left( ie^{i(dx^2+c)} \right)}{d^3} + \frac{ibx^2 \text{Li}_2 \left( -ie^{i(dx^2+c)} \right)}{d^2} - \frac{ibx^2 \text{Li}_2 \left( ie^{i(dx^2+c)} \right)}{d^2} - \frac{ibx^4 \tan^{-1} \left( e^{ic+idx^2} \right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[x^5\*(a + b\*Sec[c + d\*x^2]),x]

[Out] (a\*x^6)/6 - (I\*b\*x^4\*ArcTan[E^(I\*c + I\*d\*x^2)])/d + (I\*b\*x^2\*PolyLog[2, (-I)\*E^(I\*(c + d\*x^2))])/d^2 - (I\*b\*x^2\*PolyLog[2, I\*E^(I\*(c + d\*x^2))])/d^2 - (b\*PolyLog[3, (-I)\*E^(I\*(c + d\*x^2))])/d^3 + (b\*PolyLog[3, I\*E^(I\*(c + d\*x^2))])/d^3

**fricas [C]** time = 1.19, size = 495, normalized size = 3.46

$$\frac{2ad^3x^6 - 6ibdx^2\text{Li}_2(i \cos(dx^2 + c) + \sin(dx^2 + c)) - 6ibdx^2\text{Li}_2(i \cos(dx^2 + c) - \sin(dx^2 + c)) + 6ibdx^2\text{Li}_2(i \cos(dx^2 + c) + \sin(dx^2 + c)) - 6ibdx^2\text{Li}_2(i \cos(dx^2 + c) - \sin(dx^2 + c))}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*sec(d\*x^2+c)),x, algorithm="fricas")

```
[Out] 1/12*(2*a*d^3*x^6 - 6*I*b*d*x^2*dilog(I*cos(d*x^2 + c) + sin(d*x^2 + c)) -
6*I*b*d*x^2*dilog(I*cos(d*x^2 + c) - sin(d*x^2 + c)) + 6*I*b*d*x^2*dilog(-I
*cos(d*x^2 + c) + sin(d*x^2 + c)) + 6*I*b*d*x^2*dilog(-I*cos(d*x^2 + c) - s
in(d*x^2 + c)) + 3*b*c^2*log(cos(d*x^2 + c) + I*sin(d*x^2 + c) + I) - 3*b*c
^2*log(cos(d*x^2 + c) - I*sin(d*x^2 + c) + I) + 3*b*c^2*log(-cos(d*x^2 + c)
+ I*sin(d*x^2 + c) + I) - 3*b*c^2*log(-cos(d*x^2 + c) - I*sin(d*x^2 + c) +
I) + 3*(b*d^2*x^4 - b*c^2)*log(I*cos(d*x^2 + c) + sin(d*x^2 + c) + 1) - 3*
(b*d^2*x^4 - b*c^2)*log(I*cos(d*x^2 + c) - sin(d*x^2 + c) + 1) + 3*(b*d^2*x
^4 - b*c^2)*log(-I*cos(d*x^2 + c) + sin(d*x^2 + c) + 1) - 3*(b*d^2*x^4 - b*
c^2)*log(-I*cos(d*x^2 + c) - sin(d*x^2 + c) + 1) - 6*b*polylog(3, I*cos(d*x
^2 + c) + sin(d*x^2 + c)) + 6*b*polylog(3, I*cos(d*x^2 + c) - sin(d*x^2 + c
)) - 6*b*polylog(3, -I*cos(d*x^2 + c) + sin(d*x^2 + c)) + 6*b*polylog(3, -I
*cos(d*x^2 + c) - sin(d*x^2 + c)))/d^3
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sec(dx^2 + c) + a)x^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*sec(d*x^2+c)),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x^2 + c) + a)*x^5, x)
```

**maple** [F] time = 0.74, size = 0, normalized size = 0.00

$$\int x^5 (a + b \sec(dx^2 + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*sec(d*x^2+c)),x)
```

```
[Out] int(x^5*(a+b*sec(d*x^2+c)),x)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{6}ax^6 + 2b \int \frac{x^5 \cos(2dx^2 + 2c) \cos(dx^2 + c) + x^5 \sin(2dx^2 + 2c) \sin(dx^2 + c) + x^5 \cos(dx^2 + c)}{\cos(2dx^2 + 2c)^2 + \sin(2dx^2 + 2c)^2 + 2 \cos(2dx^2 + 2c) + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*sec(d*x^2+c)),x, algorithm="maxima")
```

```
[Out] 1/6*a*x^6 + 2*b*integrate((x^5*cos(2*d*x^2 + 2*c)*cos(d*x^2 + c) + x^5*sin(
2*d*x^2 + 2*c)*sin(d*x^2 + c) + x^5*cos(d*x^2 + c))/(cos(2*d*x^2 + 2*c)^2 +
sin(2*d*x^2 + 2*c)^2 + 2*cos(2*d*x^2 + 2*c) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^5 \left( a + \frac{b}{\cos(dx^2 + c)} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5*(a + b/cos(c + d*x^2)),x)`

[Out] `int(x^5*(a + b/cos(c + d*x^2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5 (a + b \sec(c + dx^2)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5*(a+b*sec(d*x**2+c)),x)`

[Out] `Integral(x**5*(a + b*sec(c + d*x**2)), x)`

### 3.2 $\int x^4 (a + b \sec(c + dx^2)) dx$

**Optimal.** Leaf size=26

$$b \operatorname{Int}(x^4 \sec(c + dx^2), x) + \frac{ax^5}{5}$$

[Out]  $1/5*a*x^5+b*\operatorname{Unintegrable}(x^4*\sec(d*x^2+c), x)$

**Rubi [A]** time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int x^4 (a + b \sec(c + dx^2)) dx$$

Verification is Not applicable to the result.

[In]  $\operatorname{Int}[x^4*(a + b*\operatorname{Sec}[c + d*x^2]), x]$

[Out]  $(a*x^5)/5 + b*\operatorname{Defer}[\operatorname{Int}[x^4*\operatorname{Sec}[c + d*x^2], x]$

Rubi steps

$$\begin{aligned} \int x^4 (a + b \sec(c + dx^2)) dx &= \int (ax^4 + bx^4 \sec(c + dx^2)) dx \\ &= \frac{ax^5}{5} + b \int x^4 \sec(c + dx^2) dx \end{aligned}$$

**Mathematica [A]** time = 0.96, size = 0, normalized size = 0.00

$$\int x^4 (a + b \sec(c + dx^2)) dx$$

Verification is Not applicable to the result.

[In]  $\operatorname{Integrate}[x^4*(a + b*\operatorname{Sec}[c + d*x^2]), x]$

[Out]  $\operatorname{Integrate}[x^4*(a + b*\operatorname{Sec}[c + d*x^2]), x]$

**fricas [A]** time = 0.64, size = 0, normalized size = 0.00

$$\operatorname{integral}(bx^4 \sec(dx^2 + c) + ax^4, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{integrate}(x^4*(a+b*\sec(d*x^2+c)), x, \text{algorithm}="fricas")$



[Out] `integral(b*x^4*sec(d*x^2 + c) + a*x^4, x)`

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sec(dx^2 + c) + a)x^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*(a+b*sec(d*x^2+c)),x, algorithm="giac")`

[Out] `integrate((b*sec(d*x^2 + c) + a)*x^4, x)`

**maple** [A] time = 0.74, size = 0, normalized size = 0.00

$$\int x^4 (a + b \sec(dx^2 + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4*(a+b*sec(d*x^2+c)),x)`

[Out] `int(x^4*(a+b*sec(d*x^2+c)),x)`

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{5} ax^5 + 2b \int \frac{x^4 \cos(2dx^2 + 2c) \cos(dx^2 + c) + x^4 \sin(2dx^2 + 2c) \sin(dx^2 + c) + x^4 \cos(dx^2 + c)}{\cos(2dx^2 + 2c)^2 + \sin(2dx^2 + 2c)^2 + 2 \cos(2dx^2 + 2c) + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*(a+b*sec(d*x^2+c)),x, algorithm="maxima")`

[Out] `1/5*a*x^5 + 2*b*integrate((x^4*cos(2*d*x^2 + 2*c)*cos(d*x^2 + c) + x^4*sin(2*d*x^2 + 2*c)*sin(d*x^2 + c) + x^4*cos(d*x^2 + c))/(cos(2*d*x^2 + 2*c)^2 + sin(2*d*x^2 + 2*c)^2 + 2*cos(2*d*x^2 + 2*c) + 1), x)`

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int x^4 \left( a + \frac{b}{\cos(dx^2 + c)} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4*(a + b/cos(c + d*x^2)),x)`

[Out] `int(x^4*(a + b/cos(c + d*x^2)), x)`

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 (a + b \sec(c + dx^2)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4\*(a+b\*sec(d\*x\*\*2+c)),x)

[Out] Integral(x\*\*4\*(a + b\*sec(c + d\*x\*\*2)), x)

### 3.3 $\int x^3 (a + b \sec(c + dx^2)) dx$

**Optimal.** Leaf size=92

$$\frac{ax^4}{4} + \frac{ib\text{Li}_2\left(-ie^{i(dx^2+c)}\right)}{2d^2} - \frac{ib\text{Li}_2\left(ie^{i(dx^2+c)}\right)}{2d^2} - \frac{ibx^2 \tan^{-1}\left(e^{i(c+dx^2)}\right)}{d}$$

[Out]  $1/4*a*x^4 - I*b*x^2*\arctan(\exp(I*(d*x^2+c)))/d + 1/2*I*b*\text{polylog}(2, -I*\exp(I*(d*x^2+c)))/d^2 - 1/2*I*b*\text{polylog}(2, I*\exp(I*(d*x^2+c)))/d^2$

**Rubi [A]** time = 0.09, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$ , Rules used = {14, 4204, 4181, 2279, 2391}

$$\frac{ib\text{PolyLog}\left(2, -ie^{i(c+dx^2)}\right)}{2d^2} - \frac{ib\text{PolyLog}\left(2, ie^{i(c+dx^2)}\right)}{2d^2} + \frac{ax^4}{4} - \frac{ibx^2 \tan^{-1}\left(e^{i(c+dx^2)}\right)}{d}$$

Antiderivative was successfully verified.

[In] Int[x^3\*(a + b\*Sec[c + d\*x^2]), x]

[Out]  $(a*x^4)/4 - (I*b*x^2*\text{ArcTan}[E^{I*(c + d*x^2)}])/d + ((I/2)*b*\text{PolyLog}[2, (-I)*E^{I*(c + d*x^2)}])/d^2 - ((I/2)*b*\text{PolyLog}[2, I*E^{I*(c + d*x^2)}])/d^2$

#### Rule 14

Int[(u\_)\*((c\_)\*(x\_))^(m\_), x\_Symbol] :> Int[ExpandIntegrand[(c\*x)^m\*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a\_ + (b\_)\*(v\_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

#### Rule 2279

Int[Log[(a\_) + (b\_)\*((F\_)^((e\_)\*((c\_) + (d\_)\*(x\_)))^(n\_))], x\_Symbol] :> Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

#### Rule 2391

Int[Log[(c\_)\*((d\_) + (e\_)\*(x\_)^(n\_))]/(x\_), x\_Symbol] :> -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

#### Rule 4181

Int[csc[(e\_) + Pi\*(k\_) + (f\_)\*(x\_)]\*((c\_) + (d\_)\*(x\_))^(m\_), x\_Symbol] :> Simp[(-2\*(c + d\*x)^m\*ArcTanh[E^(I\*k\*Pi)\*E^(I\*(e + f\*x))])/f, x] + (-Di

```
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))], x], x) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

### Rule 4204

```
Int[(x_)^(m_)*((a_) + (b_)*Sec[(c_) + (d_)*(x_)^(n_)])^(p_), x_Symbol
] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

### Rubi steps

$$\begin{aligned}
\int x^3 (a + b \sec(c + dx^2)) dx &= \int (ax^3 + bx^3 \sec(c + dx^2)) dx \\
&= \frac{ax^4}{4} + b \int x^3 \sec(c + dx^2) dx \\
&= \frac{ax^4}{4} + \frac{1}{2} b \text{Subst}\left(\int x \sec(c + dx) dx, x, x^2\right) \\
&= \frac{ax^4}{4} - \frac{ibx^2 \tan^{-1}\left(e^{i(c+dx^2)}\right)}{d} - \frac{b \text{Subst}\left(\int \log(1 - ie^{i(c+dx)}) dx, x, x^2\right)}{2d} + \frac{b \text{Subst}\left(\int \log(1 + ie^{i(c+dx)}) dx, x, x^2\right)}{2d} \\
&= \frac{ax^4}{4} - \frac{ibx^2 \tan^{-1}\left(e^{i(c+dx^2)}\right)}{d} + \frac{(ib) \text{Subst}\left(\int \frac{\log(1-ix)}{x} dx, x, e^{i(c+dx^2)}\right)}{2d^2} - \frac{(ib) \text{Subst}\left(\int \frac{\log(1+ix)}{x} dx, x, e^{i(c+dx^2)}\right)}{2d^2} \\
&= \frac{ax^4}{4} - \frac{ibx^2 \tan^{-1}\left(e^{i(c+dx^2)}\right)}{d} + \frac{ib \text{Li}_2\left(-ie^{i(c+dx^2)}\right)}{2d^2} - \frac{ib \text{Li}_2\left(ie^{i(c+dx^2)}\right)}{2d^2}
\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 95, normalized size = 1.03

$$\frac{ax^4}{4} + \frac{ib \text{Li}_2\left(-ie^{i(dx^2+c)}\right)}{2d^2} - \frac{ib \text{Li}_2\left(ie^{i(dx^2+c)}\right)}{2d^2} - \frac{ibx^2 \tan^{-1}\left(e^{ic+idx^2}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Sec[c + d*x^2]),x]
```

```
[Out] (a*x^4)/4 - (I*b*x^2*ArcTan[E^(I*c + I*d*x^2)])/d + ((I/2)*b*PolyLog[2, (-I)*E^(I*(c + d*x^2))])/d^2 - ((I/2)*b*PolyLog[2, I*E^(I*(c + d*x^2))])/d^2
```

**fricas** [B] time = 0.79, size = 346, normalized size = 3.76

$$\frac{ad^2x^4 - bc \log(\cos(dx^2 + c) + i \sin(dx^2 + c) + i) + bc \log(\cos(dx^2 + c) - i \sin(dx^2 + c) + i) - bc \log(-\cos(dx^2 + c) - i \sin(dx^2 + c) + i)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*sec(d\*x^2+c)),x, algorithm="fricas")

[Out]  $\frac{1}{4} * (a * d^2 * x^4 - b * c * \log(\cos(d * x^2 + c) + I * \sin(d * x^2 + c) + I) + b * c * \log(\cos(d * x^2 + c) - I * \sin(d * x^2 + c) + I) - b * c * \log(-\cos(d * x^2 + c) + I * \sin(d * x^2 + c) + I) + b * c * \log(-\cos(d * x^2 + c) - I * \sin(d * x^2 + c) + I) - I * b * \operatorname{dilog}(I * \cos(d * x^2 + c) + \sin(d * x^2 + c)) - I * b * \operatorname{dilog}(I * \cos(d * x^2 + c) - \sin(d * x^2 + c)) + I * b * \operatorname{dilog}(-I * \cos(d * x^2 + c) + \sin(d * x^2 + c)) + I * b * \operatorname{dilog}(-I * \cos(d * x^2 + c) - \sin(d * x^2 + c)) + (b * d * x^2 + b * c) * \log(I * \cos(d * x^2 + c) + \sin(d * x^2 + c) + 1) - (b * d * x^2 + b * c) * \log(I * \cos(d * x^2 + c) - \sin(d * x^2 + c) + 1) + (b * d * x^2 + b * c) * \log(-I * \cos(d * x^2 + c) + \sin(d * x^2 + c) + 1) - (b * d * x^2 + b * c) * \log(-I * \cos(d * x^2 + c) - \sin(d * x^2 + c) + 1)) / d^2$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sec(dx^2 + c) + a)x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*sec(d\*x^2+c)),x, algorithm="giac")

[Out] integrate((b\*sec(d\*x^2 + c) + a)\*x^3, x)

**maple** [F] time = 0.74, size = 0, normalized size = 0.00

$$\int x^3 (a + b \sec(dx^2 + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(a+b\*sec(d\*x^2+c)),x)

[Out] int(x^3\*(a+b\*sec(d\*x^2+c)),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4} ax^4 + 2b \int \frac{x^3 \cos(2dx^2 + 2c) \cos(dx^2 + c) + x^3 \sin(2dx^2 + 2c) \sin(dx^2 + c) + x^3 \cos(dx^2 + c)}{\cos(2dx^2 + 2c)^2 + \sin(2dx^2 + 2c)^2 + 2 \cos(2dx^2 + 2c) + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*sec(d\*x^2+c)),x, algorithm="maxima")

[Out]  $\frac{1}{4}ax^4 + 2b \int (x^3 \cos(2dx^2 + 2c) \cos(dx^2 + c) + x^3 \sin(2dx^2 + 2c) \sin(dx^2 + c) + x^3 \cos(dx^2 + c)) / (\cos(2dx^2 + 2c)^2 + \sin(2dx^2 + 2c)^2 + 2\cos(2dx^2 + 2c) + 1), x$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \left( a + \frac{b}{\cos(dx^2 + c)} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(a + b/cos(c + d\*x^2)),x)

[Out] int(x^3\*(a + b/cos(c + d\*x^2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 (a + b \sec(c + dx^2)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(a+b\*sec(d\*x\*\*2+c)),x)

[Out] Integral(x\*\*3\*(a + b\*sec(c + d\*x\*\*2)), x)

### 3.4 $\int x^2 (a + b \sec(c + dx^2)) dx$

Optimal. Leaf size=26

$$b \operatorname{Int}(x^2 \sec(c + dx^2), x) + \frac{ax^3}{3}$$

[Out]  $1/3*a*x^3+b*\operatorname{Unintegrable}(x^2*\sec(d*x^2+c), x)$

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int x^2 (a + b \sec(c + dx^2)) dx$$

Verification is Not applicable to the result.

[In]  $\operatorname{Int}[x^2*(a + b*\operatorname{Sec}[c + d*x^2]), x]$

[Out]  $(a*x^3)/3 + b*\operatorname{Defer}[\operatorname{Int}[x^2*\operatorname{Sec}[c + d*x^2], x]$

Rubi steps

$$\begin{aligned} \int x^2 (a + b \sec(c + dx^2)) dx &= \int (ax^2 + bx^2 \sec(c + dx^2)) dx \\ &= \frac{ax^3}{3} + b \int x^2 \sec(c + dx^2) dx \end{aligned}$$

Mathematica [A] time = 0.77, size = 0, normalized size = 0.00

$$\int x^2 (a + b \sec(c + dx^2)) dx$$

Verification is Not applicable to the result.

[In]  $\operatorname{Integrate}[x^2*(a + b*\operatorname{Sec}[c + d*x^2]), x]$

[Out]  $\operatorname{Integrate}[x^2*(a + b*\operatorname{Sec}[c + d*x^2]), x]$

fricas [A] time = 0.79, size = 0, normalized size = 0.00

$$\operatorname{integral}(bx^2 \sec(dx^2 + c) + ax^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{integrate}(x^2*(a+b*\sec(d*x^2+c)), x, \operatorname{algorithm}="fricas")$

[Out] integral(b\*x^2\*sec(d\*x^2 + c) + a\*x^2, x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sec(dx^2 + c) + a)x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*sec(d\*x^2+c)),x, algorithm="giac")

[Out] integrate((b\*sec(d\*x^2 + c) + a)\*x^2, x)

**maple** [A] time = 0.75, size = 0, normalized size = 0.00

$$\int x^2 (a + b \sec(dx^2 + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a+b\*sec(d\*x^2+c)),x)

[Out] int(x^2\*(a+b\*sec(d\*x^2+c)),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}ax^3 + 2b \int \frac{x^2 \cos(2dx^2 + 2c) \cos(dx^2 + c) + x^2 \sin(2dx^2 + 2c) \sin(dx^2 + c) + x^2 \cos(dx^2 + c)}{\cos(2dx^2 + 2c)^2 + \sin(2dx^2 + 2c)^2 + 2 \cos(2dx^2 + 2c) + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*sec(d\*x^2+c)),x, algorithm="maxima")

[Out] 1/3\*a\*x^3 + 2\*b\*integrate((x^2\*cos(2\*d\*x^2 + 2\*c)\*cos(d\*x^2 + c) + x^2\*sin(2\*d\*x^2 + 2\*c)\*sin(d\*x^2 + c) + x^2\*cos(d\*x^2 + c))/(cos(2\*d\*x^2 + 2\*c)^2 + sin(2\*d\*x^2 + 2\*c)^2 + 2\*cos(2\*d\*x^2 + 2\*c) + 1), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int x^2 \left( a + \frac{b}{\cos(dx^2 + c)} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a + b/cos(c + d\*x^2)),x)

[Out] int(x^2\*(a + b/cos(c + d\*x^2)), x)



sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 (a + b \sec(c + dx^2)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*sec(d*x**2+c)),x)
```

```
[Out] Integral(x**2*(a + b*sec(c + d*x**2)), x)
```

### 3.5 $\int x \left( a + b \sec \left( c + dx^2 \right) \right) dx$

Optimal. Leaf size=26

$$\frac{ax^2}{2} + \frac{b \tanh^{-1}(\sin(c + dx^2))}{2d}$$

[Out] 1/2\*a\*x^2+1/2\*b\*arctanh(sin(d\*x^2+c))/d

**Rubi [A]** time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {14, 4204, 3770}

$$\frac{ax^2}{2} + \frac{b \tanh^{-1}(\sin(c + dx^2))}{2d}$$

Antiderivative was successfully verified.

[In] Int[x\*(a + b\*Sec[c + d\*x^2]),x]

[Out] (a\*x^2)/2 + (b\*ArcTanh[Sin[c + d\*x^2]])/(2\*d)

#### Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

#### Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]
```

#### Rule 4204

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]
```

#### Rubi steps

$$\begin{aligned}
\int x(a + b \sec(c + dx^2)) dx &= \int (ax + bx \sec(c + dx^2)) dx \\
&= \frac{ax^2}{2} + b \int x \sec(c + dx^2) dx \\
&= \frac{ax^2}{2} + \frac{1}{2}b \text{Subst} \left( \int \sec(c + dx) dx, x, x^2 \right) \\
&= \frac{ax^2}{2} + \frac{b \tanh^{-1}(\sin(c + dx^2))}{2d}
\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 26, normalized size = 1.00

$$\frac{ax^2}{2} + \frac{b \tanh^{-1}(\sin(c + dx^2))}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[x\*(a + b\*Sec[c + d\*x^2]),x]

[Out] (a\*x^2)/2 + (b\*ArcTanh[Sin[c + d\*x^2]])/(2\*d)

**fricas [A]** time = 0.75, size = 42, normalized size = 1.62

$$\frac{2adx^2 + b \log(\sin(dx^2 + c) + 1) - b \log(-\sin(dx^2 + c) + 1)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*sec(d\*x^2+c)),x, algorithm="fricas")

[Out] 1/4\*(2\*a\*d\*x^2 + b\*log(sin(d\*x^2 + c) + 1) - b\*log(-sin(d\*x^2 + c) + 1))/d

**giac [B]** time = 0.55, size = 50, normalized size = 1.92

$$\frac{(dx^2 + c)a + b \log\left(\left|\tan\left(\frac{1}{2}dx^2 + \frac{1}{2}c\right) + 1\right|\right) - b \log\left(\left|\tan\left(\frac{1}{2}dx^2 + \frac{1}{2}c\right) - 1\right|\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*sec(d\*x^2+c)),x, algorithm="giac")

[Out] 1/2\*((d\*x^2 + c)\*a + b\*log(abs(tan(1/2\*d\*x^2 + 1/2\*c) + 1)) - b\*log(abs(tan(1/2\*d\*x^2 + 1/2\*c) - 1)))/d

**maple** [A] time = 0.20, size = 39, normalized size = 1.50

$$\frac{ax^2}{2} + \frac{b \ln(\sec(dx^2 + c) + \tan(dx^2 + c))}{2d} + \frac{ca}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(a+b*sec(d*x^2+c)),x)`

[Out] `1/2*a*x^2+1/2/d*b*ln(sec(d*x^2+c)+tan(d*x^2+c))+1/2/d*c*a`

**maxima** [A] time = 0.31, size = 31, normalized size = 1.19

$$\frac{1}{2}ax^2 + \frac{b \log(\sec(dx^2 + c) + \tan(dx^2 + c))}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*sec(d*x^2+c)),x, algorithm="maxima")`

[Out] `1/2*a*x^2 + 1/2*b*log(sec(d*x^2 + c) + tan(d*x^2 + c))/d`

**mupad** [B] time = 0.58, size = 67, normalized size = 2.58

$$\frac{ax^2}{2} + \frac{b \ln(-bx^{2i} - 2bx e^{dx^{2i}} e^{c1i})}{2d} - \frac{b \ln(bx^{2i} - 2bx e^{dx^{2i}} e^{c1i})}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(a + b/cos(c + d*x^2)),x)`

[Out] `(a*x^2)/2 + (b*log(- b*x*2i - 2*b*x*exp(d*x^2*1i)*exp(c*1i)))/(2*d) - (b*log(b*x*2i - 2*b*x*exp(d*x^2*1i)*exp(c*1i)))/(2*d)`

**sympy** [A] time = 2.87, size = 42, normalized size = 1.62

$$\begin{cases} \frac{a(c+dx^2)+b \log(\tan(c+dx^2)+\sec(c+dx^2))}{2d} & \text{for } d \neq 0 \\ \frac{x^2(a+b \sec(c))}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*sec(d*x**2+c)),x)`

[Out] `Piecewise(((a*(c + d*x**2) + b*log(tan(c + d*x**2) + sec(c + d*x**2)))/(2*d), Ne(d, 0)), (x**2*(a + b*sec(c))/2, True))`

$$3.6 \quad \int \frac{a+b \sec(c+dx^2)}{x} dx$$

Optimal. Leaf size=22

$$b \operatorname{Int} \left( \frac{\sec(c+dx^2)}{x}, x \right) + a \log(x)$$

[Out] a\*ln(x)+b\*Unintegrable(sec(d\*x^2+c)/x,x)

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{a+b \sec(c+dx^2)}{x} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*Sec[c + d\*x^2])/x,x]

[Out] a\*Log[x] + b\*Defer[Int][Sec[c + d\*x^2]/x, x]

Rubi steps

$$\begin{aligned} \int \frac{a+b \sec(c+dx^2)}{x} dx &= \int \left( \frac{a}{x} + \frac{b \sec(c+dx^2)}{x} \right) dx \\ &= a \log(x) + b \int \frac{\sec(c+dx^2)}{x} dx \end{aligned}$$

Mathematica [A] time = 0.77, size = 0, normalized size = 0.00

$$\int \frac{a+b \sec(c+dx^2)}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*Sec[c + d\*x^2])/x,x]

[Out] Integrate[(a + b\*Sec[c + d\*x^2])/x, x]

fricas [A] time = 0.70, size = 0, normalized size = 0.00

$$\operatorname{integral} \left( \frac{b \sec(dx^2+c) + a}{x}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sec(d\*x^2+c))/x,x, algorithm="fricas")

[Out] integral((b\*sec(d\*x^2 + c) + a)/x, x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \sec(dx^2 + c) + a}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sec(d\*x^2+c))/x,x, algorithm="giac")

[Out] integrate((b\*sec(d\*x^2 + c) + a)/x, x)

maple [A] time = 0.76, size = 0, normalized size = 0.00

$$\int \frac{a + b \sec(dx^2 + c)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*sec(d\*x^2+c))/x,x)

[Out] int((a+b\*sec(d\*x^2+c))/x,x)

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$2b \int \frac{\cos(2dx^2 + 2c) \cos(dx^2 + c) + \sin(2dx^2 + 2c) \sin(dx^2 + c) + \cos(dx^2 + c)}{x \cos(2dx^2 + 2c)^2 + x \sin(2dx^2 + 2c)^2 + 2x \cos(2dx^2 + 2c) + x} dx + a \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sec(d\*x^2+c))/x,x, algorithm="maxima")

[Out] 2\*b\*integrate((cos(2\*d\*x^2 + 2\*c)\*cos(d\*x^2 + c) + sin(2\*d\*x^2 + 2\*c)\*sin(d\*x^2 + c) + cos(d\*x^2 + c))/(x\*cos(2\*d\*x^2 + 2\*c)^2 + x\*sin(2\*d\*x^2 + 2\*c)^2 + 2\*x\*cos(2\*d\*x^2 + 2\*c) + x), x) + a\*log(x)

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{a + \frac{b}{\cos(dx^2+c)}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b/cos(c + d*x^2))/x,x)`

[Out] `int((a + b/cos(c + d*x^2))/x, x)`

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \sec(c + dx^2)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sec(d*x**2+c))/x,x)`

[Out] `Integral((a + b*sec(c + d*x**2))/x, x)`

$$3.7 \quad \int \frac{a+b \sec(c+dx^2)}{x^2} dx$$

Optimal. Leaf size=24

$$b \operatorname{Int} \left( \frac{\sec(c+dx^2)}{x^2}, x \right) - \frac{a}{x}$$

[Out]  $-a/x + b \operatorname{Unintegrable}(\sec(dx^2+c)/x^2, x)$

**Rubi** [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{a + b \sec(c + dx^2)}{x^2} dx$$

Verification is Not applicable to the result.

[In]  $\operatorname{Int}[(a + b \operatorname{Sec}[c + d x^2])/x^2, x]$

[Out]  $-(a/x) + b \operatorname{Defer}[\operatorname{Int}[\operatorname{Sec}[c + d x^2]/x^2, x]]$

Rubi steps

$$\begin{aligned} \int \frac{a + b \sec(c + dx^2)}{x^2} dx &= \int \left( \frac{a}{x^2} + \frac{b \sec(c + dx^2)}{x^2} \right) dx \\ &= -\frac{a}{x} + b \int \frac{\sec(c + dx^2)}{x^2} dx \end{aligned}$$

**Mathematica** [A] time = 0.74, size = 0, normalized size = 0.00

$$\int \frac{a + b \sec(c + dx^2)}{x^2} dx$$

Verification is Not applicable to the result.

[In]  $\operatorname{Integrate}[(a + b \operatorname{Sec}[c + d x^2])/x^2, x]$

[Out]  $\operatorname{Integrate}[(a + b \operatorname{Sec}[c + d x^2])/x^2, x]$

**fricas** [A] time = 0.77, size = 0, normalized size = 0.00

$$\operatorname{integral} \left( \frac{b \sec(dx^2 + c) + a}{x^2}, x \right)$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sec(d\*x^2+c))/x^2,x, algorithm="fricas")

[Out] integral((b\*sec(d\*x^2 + c) + a)/x^2, x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \sec(dx^2 + c) + a}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sec(d\*x^2+c))/x^2,x, algorithm="giac")

[Out] integrate((b\*sec(d\*x^2 + c) + a)/x^2, x)

maple [A] time = 0.70, size = 0, normalized size = 0.00

$$\int \frac{a + b \sec(dx^2 + c)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*sec(d\*x^2+c))/x^2,x)

[Out] int((a+b\*sec(d\*x^2+c))/x^2,x)

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$2b \int \frac{\cos(2dx^2 + 2c) \cos(dx^2 + c) + \sin(2dx^2 + 2c) \sin(dx^2 + c) + \cos(dx^2 + c)}{x^2 \cos(2dx^2 + 2c)^2 + x^2 \sin(2dx^2 + 2c)^2 + 2x^2 \cos(2dx^2 + 2c) + x^2} dx - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sec(d\*x^2+c))/x^2,x, algorithm="maxima")

[Out] 2\*b\*integrate((cos(2\*d\*x^2 + 2\*c)\*cos(d\*x^2 + c) + sin(2\*d\*x^2 + 2\*c)\*sin(d\*x^2 + c) + cos(d\*x^2 + c))/(x^2\*cos(2\*d\*x^2 + 2\*c)^2 + x^2\*sin(2\*d\*x^2 + 2\*c)^2 + 2\*x^2\*cos(2\*d\*x^2 + 2\*c) + x^2), x) - a/x

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a + \frac{b}{\cos(dx^2+c)}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x^2))/x^2,x)
```

```
[Out] int((a + b/cos(c + d*x^2))/x^2, x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \sec(c + dx^2)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x**2+c))/x**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x**2))/x**2, x)
```

### 3.8 $\int x^5 \left( a + b \sec(c + dx^2) \right)^2 dx$

**Optimal.** Leaf size=242

$$\frac{a^2 x^6}{6} - \frac{2ab \operatorname{Li}_3\left(-ie^{i(dx^2+c)}\right)}{d^3} + \frac{2ab \operatorname{Li}_3\left(ie^{i(dx^2+c)}\right)}{d^3} + \frac{2iabx^2 \operatorname{Li}_2\left(-ie^{i(dx^2+c)}\right)}{d^2} - \frac{2iabx^2 \operatorname{Li}_2\left(ie^{i(dx^2+c)}\right)}{d^2} - \frac{2iabx^4 \tan^{-1}\left(e^{i(dx^2+c)}\right)}{d}$$

```
[Out] -1/2*I*b^2*x^4/d+1/6*a^2*x^6-2*I*a*b*x^4*arctan(exp(I*(d*x^2+c)))/d+b^2*x^2
*ln(1+exp(2*I*(d*x^2+c)))/d^2+2*I*a*b*x^2*polylog(2,-I*exp(I*(d*x^2+c)))/d^
2-2*I*a*b*x^2*polylog(2,I*exp(I*(d*x^2+c)))/d^2-1/2*I*b^2*polylog(2,-exp(2*
I*(d*x^2+c)))/d^3-2*a*b*polylog(3,-I*exp(I*(d*x^2+c)))/d^3+2*a*b*polylog(3,
I*exp(I*(d*x^2+c)))/d^3+1/2*b^2*x^4*tan(d*x^2+c)/d
```

**Rubi [A]** time = 0.37, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$ , Rules used = {4204, 4190, 4181, 2531, 2282, 6589, 4184, 3719, 2190, 2279, 2391}

$$\frac{2iabx^2 \operatorname{PolyLog}\left(2, -ie^{i(c+dx^2)}\right)}{d^2} - \frac{2iabx^2 \operatorname{PolyLog}\left(2, ie^{i(c+dx^2)}\right)}{d^2} - \frac{2ab \operatorname{PolyLog}\left(3, -ie^{i(c+dx^2)}\right)}{d^3} + \frac{2ab \operatorname{PolyLog}\left(3, ie^{i(c+dx^2)}\right)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*Sec[c + d*x^2])^2,x]
```

```
[Out] ((-I/2)*b^2*x^4)/d + (a^2*x^6)/6 - ((2*I)*a*b*x^4*ArcTan[E^(I*(c + d*x^2))
]/d + (b^2*x^2*Log[1 + E^((2*I)*(c + d*x^2))])/d^2 + ((2*I)*a*b*x^2*PolyLog
[2, (-I)*E^(I*(c + d*x^2))])/d^2 - ((2*I)*a*b*x^2*PolyLog[2, I*E^(I*(c + d*
x^2))])/d^2 - ((I/2)*b^2*PolyLog[2, -E^((2*I)*(c + d*x^2))])/d^3 - (2*a*b*P
olyLog[3, (-I)*E^(I*(c + d*x^2))])/d^3 + (2*a*b*PolyLog[3, I*E^(I*(c + d*x^
2))])/d^3 + (b^2*x^4*Tan[c + d*x^2])/(2*d)
```

**Rule 2190**

```
Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_))*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))]^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

**Rule 2279**

```
Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_)))^(n_))], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

### Rule 2282

Int[u\_, x\_Symbol] :=> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_.)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_.)\*((a\_.) + (b\_.)\*x))\*(F\_) [v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] :=> -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rule 2531

Int[Log[1 + (e\_.)\*((F\_)^(c\_.)\*((a\_.) + (b\_.)\*(x\_)))^(n\_.)]\*((f\_.) + (g\_.)\*(x\_))^(m\_.), x\_Symbol] :=> -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x))))^n]]/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m - 1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x))))^n]], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

### Rule 3719

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*tan[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] :=> Simp[(I\*(c + d\*x)^(m + 1))/(d\*(m + 1)), x] - Dist[2\*I, Int[((c + d\*x)^m\*E^(2\*I\*(e + f\*x)))/(1 + E^(2\*I\*(e + f\*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]

### Rule 4181

Int[csc[(e\_.) + Pi\*(k\_.) + (f\_.)\*(x\_)]\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :=> Simp[(-2\*(c + d\*x)^m\*ArcTanh[E^(I\*k\*Pi)\*E^(I\*(e + f\*x))]/f, x] + (-Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Log[1 - E^(I\*k\*Pi)\*E^(I\*(e + f\*x))], x], x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Log[1 + E^(I\*k\*Pi)\*E^(I\*(e + f\*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2\*k] && IGtQ[m, 0]

### Rule 4184

Int[csc[(e\_.) + (f\_.)\*(x\_)]^2\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :=> -Simp[((c + d\*x)^m\*Cot[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cot[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 4190

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(n_.)*((c_.) + (d_.)*(x_.))^(m_.)
, x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 4204

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_.)])^(p_.), x_Symbol
] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_.))^(p_.)]/((d_.) + (e_.)*(x_.)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int x^5 (a + b \sec(c + dx^2))^2 dx &= \frac{1}{2} \text{Subst} \left( \int x^2 (a + b \sec(c + dx))^2 dx, x, x^2 \right) \\
&= \frac{1}{2} \text{Subst} \left( \int (a^2 x^2 + 2abx^2 \sec(c + dx) + b^2 x^2 \sec^2(c + dx)) dx, x, x^2 \right) \\
&= \frac{a^2 x^6}{6} + (ab) \text{Subst} \left( \int x^2 \sec(c + dx) dx, x, x^2 \right) + \frac{1}{2} b^2 \text{Subst} \left( \int x^2 \sec^2(c + dx) dx, x, x^2 \right) \\
&= \frac{a^2 x^6}{6} - \frac{2iabx^4 \tan^{-1} \left( e^{i(c+dx^2)} \right)}{d} + \frac{b^2 x^4 \tan(c + dx^2)}{2d} - \frac{(2ab) \text{Subst} \left( \int x \log(1 + e^{2i(c+dx^2)}) dx, x, x^2 \right)}{d} \\
&= -\frac{ib^2 x^4}{2d} + \frac{a^2 x^6}{6} - \frac{2iabx^4 \tan^{-1} \left( e^{i(c+dx^2)} \right)}{d} + \frac{2iabx^2 \text{Li}_2 \left( -ie^{i(c+dx^2)} \right)}{d^2} - \frac{2iabx^2 \text{Li}_2 \left( e^{2i(c+dx^2)} \right)}{d^2} \\
&= -\frac{ib^2 x^4}{2d} + \frac{a^2 x^6}{6} - \frac{2iabx^4 \tan^{-1} \left( e^{i(c+dx^2)} \right)}{d} + \frac{b^2 x^2 \log \left( 1 + e^{2i(c+dx^2)} \right)}{d^2} + \frac{2iabx^2 \text{Li}_2 \left( e^{2i(c+dx^2)} \right)}{d^2} \\
&= -\frac{ib^2 x^4}{2d} + \frac{a^2 x^6}{6} - \frac{2iabx^4 \tan^{-1} \left( e^{i(c+dx^2)} \right)}{d} + \frac{b^2 x^2 \log \left( 1 + e^{2i(c+dx^2)} \right)}{d^2} + \frac{2iabx^2 \text{Li}_2 \left( e^{2i(c+dx^2)} \right)}{d^2} \\
&= -\frac{ib^2 x^4}{2d} + \frac{a^2 x^6}{6} - \frac{2iabx^4 \tan^{-1} \left( e^{i(c+dx^2)} \right)}{d} + \frac{b^2 x^2 \log \left( 1 + e^{2i(c+dx^2)} \right)}{d^2} + \frac{2iabx^2 \text{Li}_2 \left( e^{2i(c+dx^2)} \right)}{d^2}
\end{aligned}$$

**Mathematica [A]** time = 0.61, size = 229, normalized size = 0.95

$$\frac{a^2 d^3 x^6 - 12 i a b d^2 x^4 \tan^{-1}\left(e^{i(c+dx^2)}\right) + 12 i a b d x^2 \operatorname{Li}_2\left(-ie^{i(dx^2+c)}\right) - 12 i a b d x^2 \operatorname{Li}_2\left(ie^{i(dx^2+c)}\right) - 12 a b \operatorname{Li}_3\left(-ie^{i(dx^2+c)}\right)}{6d^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^5\*(a + b\*Sec[c + d\*x^2])^2,x]

[Out]  $((-3*I)*b^2*d^2*x^4 + a^2*d^3*x^6 - (12*I)*a*b*d^2*x^4*\operatorname{ArcTan}[E^{(I*(c + d*x^2))}] + 6*b^2*d*x^2*\operatorname{Log}[1 + E^{((2*I)*(c + d*x^2))}] + (12*I)*a*b*d*x^2*\operatorname{PolyLog}[2, (-I)*E^{(I*(c + d*x^2))}] - (12*I)*a*b*d*x^2*\operatorname{PolyLog}[2, I*E^{(I*(c + d*x^2))}] - (3*I)*b^2*\operatorname{PolyLog}[2, -E^{((2*I)*(c + d*x^2))}] - 12*a*b*\operatorname{PolyLog}[3, (-I)*E^{(I*(c + d*x^2))}] + 12*a*b*\operatorname{PolyLog}[3, I*E^{(I*(c + d*x^2))}] + 3*b^2*d^2*x^4*\operatorname{Tan}[c + d*x^2])/(6*d^3)$

**fricas [C]** time = 0.75, size = 795, normalized size = 3.29

$$\frac{a^2 d^3 x^6 \cos(dx^2 + c) + 3 b^2 d^2 x^4 \sin(dx^2 + c) - 6 a b \cos(dx^2 + c) \operatorname{polylog}(3, i \cos(dx^2 + c) + \sin(dx^2 + c)) + 6 a b \cos(dx^2 + c) \operatorname{polylog}(3, -i \cos(dx^2 + c) + \sin(dx^2 + c)) + (-6 I a b d x^2 + 3 I b^2) \cos(dx^2 + c) \operatorname{dilog}(I \cos(dx^2 + c) + \sin(dx^2 + c)) + (-6 I a b d x^2 - 3 I b^2) \cos(dx^2 + c) \operatorname{dilog}(I \cos(dx^2 + c) - \sin(dx^2 + c)) + (6 I a b d x^2 - 3 I b^2) \cos(dx^2 + c) \operatorname{dilog}(-I \cos(dx^2 + c) + \sin(dx^2 + c)) + (6 I a b d x^2 + 3 I b^2) \cos(dx^2 + c) \operatorname{dilog}(-I \cos(dx^2 + c) - \sin(dx^2 + c)) + 3*(a*b*c^2 - b^2*c)*\cos(dx^2 + c)*\log(\cos(dx^2 + c) + I*\sin(dx^2 + c) + I) - 3*(a*b*c^2 + b^2*c)*\cos(dx^2 + c)*\log(\cos(dx^2 + c) - I*\sin(dx^2 + c) + I) + 3*(a*b*d^2*x^4 + b^2*d*x^2 - a*b*c^2 + b^2*c)*\cos(dx^2 + c)*\log(I*\cos(dx^2 + c) + \sin(dx^2 + c) + 1) - 3*(a*b*d^2*x^4 - b^2*d*x^2 - a*b*c^2 - b^2*c)*\cos(dx^2 + c)*\log(I*\cos(dx^2 + c) - \sin(dx^2 + c) + 1) + 3*(a*b*d^2*x^4 + b^2*d*x^2 - a*b*c^2 + b^2*c)*\cos(dx^2 + c)*\log(-I*\cos(dx^2 + c) + \sin(dx^2 + c) + 1) - 3*(a*b*d^2*x^4 - b^2*d*x^2 - a*b*c^2 - b^2*c)*\cos(dx^2 + c)*\log(-I*\cos(dx^2 + c) - \sin(dx^2 + c) + 1) + 3*(a*b*c^2 - b^2*c)*\cos(dx^2 + c)*\log(-\cos(dx^2 + c) + I*\sin(dx^2 + c) + I) - 3*(a*b*c^2 + b^2*c)*\cos(dx^2 + c)*\log(-\cos(dx^2 + c) - I*\sin(dx^2 + c) + I))/(d^3*\cos(dx^2 + c))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*sec(d\*x^2+c))^2,x, algorithm="fricas")

[Out]  $1/6*(a^2*d^3*x^6*\cos(d*x^2 + c) + 3*b^2*d^2*x^4*\sin(d*x^2 + c) - 6*a*b*\cos(d*x^2 + c)*\operatorname{polylog}(3, I*\cos(d*x^2 + c) + \sin(d*x^2 + c)) + 6*a*b*\cos(d*x^2 + c)*\operatorname{polylog}(3, I*\cos(d*x^2 + c) - \sin(d*x^2 + c)) - 6*a*b*\cos(d*x^2 + c)*\operatorname{polylog}(3, -I*\cos(d*x^2 + c) + \sin(d*x^2 + c)) + 6*a*b*\cos(d*x^2 + c)*\operatorname{polylog}(3, -I*\cos(d*x^2 + c) - \sin(d*x^2 + c)) + (-6*I*a*b*d*x^2 + 3*I*b^2)*\cos(d*x^2 + c)*\operatorname{dilog}(I*\cos(d*x^2 + c) + \sin(d*x^2 + c)) + (-6*I*a*b*d*x^2 - 3*I*b^2)*\cos(d*x^2 + c)*\operatorname{dilog}(I*\cos(d*x^2 + c) - \sin(d*x^2 + c)) + (6*I*a*b*d*x^2 - 3*I*b^2)*\cos(d*x^2 + c)*\operatorname{dilog}(-I*\cos(d*x^2 + c) + \sin(d*x^2 + c)) + (6*I*a*b*d*x^2 + 3*I*b^2)*\cos(d*x^2 + c)*\operatorname{dilog}(-I*\cos(d*x^2 + c) - \sin(d*x^2 + c)) + 3*(a*b*c^2 - b^2*c)*\cos(d*x^2 + c)*\log(\cos(d*x^2 + c) + I*\sin(d*x^2 + c) + I) - 3*(a*b*c^2 + b^2*c)*\cos(d*x^2 + c)*\log(\cos(d*x^2 + c) - I*\sin(d*x^2 + c) + I) + 3*(a*b*d^2*x^4 + b^2*d*x^2 - a*b*c^2 + b^2*c)*\cos(d*x^2 + c)*\log(I*\cos(d*x^2 + c) + \sin(d*x^2 + c) + 1) - 3*(a*b*d^2*x^4 - b^2*d*x^2 - a*b*c^2 - b^2*c)*\cos(d*x^2 + c)*\log(I*\cos(d*x^2 + c) - \sin(d*x^2 + c) + 1) + 3*(a*b*d^2*x^4 + b^2*d*x^2 - a*b*c^2 + b^2*c)*\cos(d*x^2 + c)*\log(-I*\cos(d*x^2 + c) + \sin(d*x^2 + c) + 1) - 3*(a*b*d^2*x^4 - b^2*d*x^2 - a*b*c^2 - b^2*c)*\cos(d*x^2 + c)*\log(-I*\cos(d*x^2 + c) - \sin(d*x^2 + c) + 1) + 3*(a*b*c^2 - b^2*c)*\cos(d*x^2 + c)*\log(-\cos(d*x^2 + c) + I*\sin(d*x^2 + c) + I) - 3*(a*b*c^2 + b^2*c)*\cos(d*x^2 + c)*\log(-\cos(d*x^2 + c) - I*\sin(d*x^2 + c) + I))/(d^3*\cos(d*x^2 + c))$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sec(dx^2 + c) + a)^2 x^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*sec(d\*x^2+c))^2,x, algorithm="giac")

[Out] integrate((b\*sec(d\*x^2 + c) + a)^2\*x^5, x)

**maple** [F] time = 1.10, size = 0, normalized size = 0.00

$$\int x^5 (a + b \sec(dx^2 + c))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5\*(a+b\*sec(d\*x^2+c))^2,x)

[Out] int(x^5\*(a+b\*sec(d\*x^2+c))^2,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{6} a^2 x^6 + \frac{b^2 x^4 \sin(2 dx^2 + 2 c) + 4 \left( d \cos(2 dx^2 + 2 c)^2 + d \sin(2 dx^2 + 2 c)^2 + 2 d \cos(2 dx^2 + 2 c) + d \right) \int \frac{ab dx^5}{d \cos(2 dx^2 + 2 c)^2 + d \sin(2 dx^2 + 2 c)^2 + 2 d \cos(2 dx^2 + 2 c) + d}}{d \cos(2 dx^2 + 2 c)^2 + d \sin(2 dx^2 + 2 c)^2 + 2 d \cos(2 dx^2 + 2 c) + d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*sec(d\*x^2+c))^2,x, algorithm="maxima")

[Out] 1/6\*a^2\*x^6 + (b^2\*x^4\*sin(2\*d\*x^2 + 2\*c) + (d\*cos(2\*d\*x^2 + 2\*c)^2 + d\*sin(2\*d\*x^2 + 2\*c)^2 + 2\*d\*cos(2\*d\*x^2 + 2\*c) + d)\*integrate(4\*(a\*b\*d\*x^5\*cos(2\*d\*x^2 + 2\*c)\*cos(d\*x^2 + c) + a\*b\*d\*x^5\*cos(d\*x^2 + c) + (a\*b\*d\*x^5\*sin(d\*x^2 + c) - b^2\*x^3)\*sin(2\*d\*x^2 + 2\*c))/(d\*cos(2\*d\*x^2 + 2\*c)^2 + d\*sin(2\*d\*x^2 + 2\*c)^2 + 2\*d\*cos(2\*d\*x^2 + 2\*c) + d), x))/(d\*cos(2\*d\*x^2 + 2\*c)^2 + d\*sin(2\*d\*x^2 + 2\*c)^2 + 2\*d\*cos(2\*d\*x^2 + 2\*c) + d)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^5 \left( a + \frac{b}{\cos(dx^2 + c)} \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5*(a + b/cos(c + d*x^2))^2,x)`

[Out] `int(x^5*(a + b/cos(c + d*x^2))^2, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5 (a + b \sec(c + dx^2))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5*(a+b*sec(d*x**2+c))**2,x)`

[Out] `Integral(x**5*(a + b*sec(c + d*x**2))**2, x)`



$$3.9 \quad \int x^4 (a + b \sec(c + dx^2))^2 dx$$

Optimal. Leaf size=21

$$\text{Int}\left(x^4 (a + b \sec(c + dx^2))^2, x\right)$$

[Out] Unintegrable(x^4\*(a+b\*sec(d\*x^2+c))^2,x)

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int x^4 (a + b \sec(c + dx^2))^2 dx$$

Verification is Not applicable to the result.

[In] Int[x^4\*(a + b\*Sec[c + d\*x^2])^2,x]

[Out] Defer[Int][x^4\*(a + b\*Sec[c + d\*x^2])^2, x]

Rubi steps

$$\int x^4 (a + b \sec(c + dx^2))^2 dx = \int x^4 (a + b \sec(c + dx^2))^2 dx$$

Mathematica [A] time = 9.61, size = 0, normalized size = 0.00

$$\int x^4 (a + b \sec(c + dx^2))^2 dx$$

Verification is Not applicable to the result.

[In] Integrate[x^4\*(a + b\*Sec[c + d\*x^2])^2,x]

[Out] Integrate[x^4\*(a + b\*Sec[c + d\*x^2])^2, x]

fricas [A] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(b^2 x^4 \sec(dx^2 + c)^2 + 2 abx^4 \sec(dx^2 + c) + a^2 x^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(a+b\*sec(d\*x^2+c))^2,x, algorithm="fricas")

[Out] integral(b^2\*x^4\*sec(d\*x^2 + c)^2 + 2\*a\*b\*x^4\*sec(d\*x^2 + c) + a^2\*x^4, x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sec(dx^2 + c) + a)^2 x^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(a+b\*sec(d\*x^2+c))^2,x, algorithm="giac")

[Out] integrate((b\*sec(d\*x^2 + c) + a)^2\*x^4, x)

**maple** [A] time = 1.13, size = 0, normalized size = 0.00

$$\int x^4 (a + b \sec(dx^2 + c))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(a+b\*sec(d\*x^2+c))^2,x)

[Out] int(x^4\*(a+b\*sec(d\*x^2+c))^2,x)

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(a+b\*sec(d\*x^2+c))^2,x, algorithm="maxima")

[Out] Timed out

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int x^4 \left( a + \frac{b}{\cos(dx^2 + c)} \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(a + b/cos(c + d\*x^2))^2,x)

[Out] int(x^4\*(a + b/cos(c + d\*x^2))^2, x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 (a + b \sec(c + dx^2))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4\*(a+b\*sec(d\*x\*\*2+c))\*\*2,x)

[Out] Integral(x\*\*4\*(a + b\*sec(c + d\*x\*\*2))\*\*2, x)

### 3.10 $\int x^3 (a + b \sec(c + dx^2))^2 dx$

**Optimal.** Leaf size=133

$$\frac{a^2 x^4}{4} + \frac{iab \operatorname{Li}_2(-ie^{i(dx^2+c)})}{d^2} - \frac{iab \operatorname{Li}_2(ie^{i(dx^2+c)})}{d^2} - \frac{2iabx^2 \tan^{-1}(e^{i(c+dx^2)})}{d} + \frac{b^2 \log(\cos(c + dx^2))}{2d^2} + \frac{b^2 x^2 \tan(c + dx^2)}{2d}$$

[Out]  $1/4*a^2*x^4-2*I*a*b*x^2*\arctan(\exp(I*(d*x^2+c)))/d+1/2*b^2*\ln(\cos(d*x^2+c))/d^2+I*a*b*\operatorname{polylog}(2,-I*\exp(I*(d*x^2+c)))/d^2-I*a*b*\operatorname{polylog}(2,I*\exp(I*(d*x^2+c)))/d^2+1/2*b^2*x^2*\tan(d*x^2+c)/d$

**Rubi [A]** time = 0.16, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$ , Rules used = {4204, 4190, 4181, 2279, 2391, 4184, 3475}

$$\frac{iab \operatorname{PolyLog}(2, -ie^{i(c+dx^2)})}{d^2} - \frac{iab \operatorname{PolyLog}(2, ie^{i(c+dx^2)})}{d^2} + \frac{a^2 x^4}{4} - \frac{2iabx^2 \tan^{-1}(e^{i(c+dx^2)})}{d} + \frac{b^2 \log(\cos(c + dx^2))}{2d^2} +$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^3*(a + b*\operatorname{Sec}[c + d*x^2])^2, x]$

[Out]  $(a^2*x^4)/4 - ((2*I)*a*b*x^2*\operatorname{ArcTan}[E^{I*(c + d*x^2)}])/d + (b^2*\operatorname{Log}[\operatorname{Cos}[c + d*x^2]])/(2*d^2) + (I*a*b*\operatorname{PolyLog}[2, (-I)*E^{I*(c + d*x^2)}])/d^2 - (I*a*b*\operatorname{PolyLog}[2, I*E^{I*(c + d*x^2)}])/d^2 + (b^2*x^2*\operatorname{Tan}[c + d*x^2])/(2*d)$

Rule 2279

$\operatorname{Int}[\operatorname{Log}[(a_) + (b_)*((F_)^{((e_)*((c_) + (d_)*(x_)))})^{(n_)}], x\_Symbol]$   
 $:\> \operatorname{Dist}[1/(d*e*n*\operatorname{Log}[F]), \operatorname{Subst}[\operatorname{Int}[\operatorname{Log}[a + b*x]/x, x], x, (F^{(e*(c + d*x))})^n], x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d, e, n\}, x \ \&\& \operatorname{GtQ}[a, 0]$

Rule 2391

$\operatorname{Int}[\operatorname{Log}[(c_)*((d_) + (e_)*(x_)^{(n_)})]/(x_), x\_Symbol] :\> -\operatorname{Simp}[\operatorname{PolyLog}[2, -(c*e*x^n)]/n, x] /;$   $\operatorname{FreeQ}\{c, d, e, n\}, x \ \&\& \operatorname{EqQ}[c*d, 1]$

Rule 3475

$\operatorname{Int}[\operatorname{tan}[(c_) + (d_)*(x_)], x\_Symbol] :\> -\operatorname{Simp}[\operatorname{Log}[\operatorname{RemoveContent}[\operatorname{Cos}[c + d*x], x]]/d, x] /;$   $\operatorname{FreeQ}\{c, d\}, x$

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_.)]*((c_.) + (d_.)*(x_.))^(m_.), x_Symbol]
:> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f, x] + (-Dist
[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

#### Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_.)]^2*((c_.) + (d_.)*(x_.))^(m_.), x_Symbol]
:> -Simp[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

#### Rule 4190

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(n_.)*((c_.) + (d_.)*(x_.))^(m_.)
, x_Symbol]
:> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

#### Rule 4204

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_.)])^(p_.), x_Symbol]
:> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

#### Rubi steps

$$\begin{aligned}
\int x^3 (a + b \sec(c + dx^2))^2 dx &= \frac{1}{2} \text{Subst} \left( \int x (a + b \sec(c + dx))^2 dx, x, x^2 \right) \\
&= \frac{1}{2} \text{Subst} \left( \int (a^2 x + 2abx \sec(c + dx) + b^2 x \sec^2(c + dx)) dx, x, x^2 \right) \\
&= \frac{a^2 x^4}{4} + (ab) \text{Subst} \left( \int x \sec(c + dx) dx, x, x^2 \right) + \frac{1}{2} b^2 \text{Subst} \left( \int x \sec^2(c + dx) dx, x, x^2 \right) \\
&= \frac{a^2 x^4}{4} - \frac{2iabx^2 \tan^{-1} \left( e^{i(c+dx^2)} \right)}{d} + \frac{b^2 x^2 \tan(c + dx^2)}{2d} - \frac{(ab) \text{Subst} \left( \int \log(1 - ie^{i(c+dx^2)}) dx, x, x^2 \right)}{d} \\
&= \frac{a^2 x^4}{4} - \frac{2iabx^2 \tan^{-1} \left( e^{i(c+dx^2)} \right)}{d} + \frac{b^2 \log(\cos(c + dx^2))}{2d^2} + \frac{b^2 x^2 \tan(c + dx^2)}{2d} + \frac{iab \text{Li}_2 \left( -ie^{i(c+dx^2)} \right)}{d^2}
\end{aligned}$$

**Mathematica [A]** time = 0.47, size = 123, normalized size = 0.92

$$\frac{a^2 d^2 x^4 + 4iab \operatorname{Li}_2\left(-ie^{i(dx^2+c)}\right) - 4iab \operatorname{Li}_2\left(ie^{i(dx^2+c)}\right) - 8iabdx^2 \tan^{-1}\left(e^{i(c+dx^2)}\right) + 2b^2 dx^2 \tan(c + dx^2) + 2b^2 \log\left(e^{i(c+dx^2)}\right)}{4d^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*(a + b\*Sec[c + d\*x^2])^2,x]

[Out] (a^2\*d^2\*x^4 - (8\*I)\*a\*b\*d\*x^2\*ArcTan[E^(I\*(c + d\*x^2))]) + 2\*b^2\*Log[Cos[c + d\*x^2]] + (4\*I)\*a\*b\*PolyLog[2, (-I)\*E^(I\*(c + d\*x^2))] - (4\*I)\*a\*b\*PolyLog[2, I\*E^(I\*(c + d\*x^2))] + 2\*b^2\*d\*x^2\*Tan[c + d\*x^2])/(4\*d^2)

**fricas [B]** time = 0.87, size = 525, normalized size = 3.95

$$\frac{a^2 d^2 x^4 \cos(dx^2 + c) + 2b^2 dx^2 \sin(dx^2 + c) - 2iab \cos(dx^2 + c) \operatorname{Li}_2(i \cos(dx^2 + c) + \sin(dx^2 + c)) - 2iab \cos(dx^2 + c) \operatorname{Li}_2(-i \cos(dx^2 + c) + \sin(dx^2 + c))}{4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*sec(d\*x^2+c))^2,x, algorithm="fricas")

[Out] 1/4\*(a^2\*d^2\*x^4\*cos(d\*x^2 + c) + 2\*b^2\*d\*x^2\*sin(d\*x^2 + c) - 2\*I\*a\*b\*cos(d\*x^2 + c)\*dilog(I\*cos(d\*x^2 + c) + sin(d\*x^2 + c)) - 2\*I\*a\*b\*cos(d\*x^2 + c)\*dilog(I\*cos(d\*x^2 + c) - sin(d\*x^2 + c)) + 2\*I\*a\*b\*cos(d\*x^2 + c)\*dilog(-I\*cos(d\*x^2 + c) + sin(d\*x^2 + c)) + 2\*I\*a\*b\*cos(d\*x^2 + c)\*dilog(-I\*cos(d\*x^2 + c) - sin(d\*x^2 + c)) - (2\*a\*b\*c - b^2)\*cos(d\*x^2 + c)\*log(cos(d\*x^2 + c) + I\*sin(d\*x^2 + c) + I) + (2\*a\*b\*c + b^2)\*cos(d\*x^2 + c)\*log(cos(d\*x^2 + c) - I\*sin(d\*x^2 + c) + I) + 2\*(a\*b\*d\*x^2 + a\*b\*c)\*cos(d\*x^2 + c)\*log(I\*cos(d\*x^2 + c) + sin(d\*x^2 + c) + 1) - 2\*(a\*b\*d\*x^2 + a\*b\*c)\*cos(d\*x^2 + c)\*log(I\*cos(d\*x^2 + c) - sin(d\*x^2 + c) + 1) + 2\*(a\*b\*d\*x^2 + a\*b\*c)\*cos(d\*x^2 + c)\*log(-I\*cos(d\*x^2 + c) + sin(d\*x^2 + c) + 1) - 2\*(a\*b\*d\*x^2 + a\*b\*c)\*cos(d\*x^2 + c)\*log(-I\*cos(d\*x^2 + c) - sin(d\*x^2 + c) + 1) - (2\*a\*b\*c - b^2)\*cos(d\*x^2 + c)\*log(-cos(d\*x^2 + c) + I\*sin(d\*x^2 + c) + I) + (2\*a\*b\*c + b^2)\*cos(d\*x^2 + c)\*log(-cos(d\*x^2 + c) - I\*sin(d\*x^2 + c) + I))/(d^2\*cos(d\*x^2 + c))

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sec(dx^2 + c) + a)^2 x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*sec(d\*x^2+c))^2,x, algorithm="giac")

[Out] integrate((b\*sec(d\*x^2 + c) + a)^2\*x^3, x)

**maple** [F] time = 1.29, size = 0, normalized size = 0.00

$$\int x^3 (a + b \sec(dx^2 + c))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(a+b\*sec(d\*x^2+c))^2,x)

[Out] int(x^3\*(a+b\*sec(d\*x^2+c))^2,x)

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*sec(d\*x^2+c))^2,x, algorithm="maxima")

[Out] Timed out

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \left( a + \frac{b}{\cos(dx^2 + c)} \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(a + b/cos(c + d\*x^2))^2,x)

[Out] int(x^3\*(a + b/cos(c + d\*x^2))^2, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 (a + b \sec(c + dx^2))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(a+b\*sec(d\*x\*\*2+c))\*\*2,x)

[Out] Integral(x\*\*3\*(a + b\*sec(c + d\*x\*\*2))\*\*2, x)

### 3.11 $\int x^2 (a + b \sec(c + dx^2))^2 dx$

Optimal. Leaf size=21

$$\text{Int}\left(x^2 (a + b \sec(c + dx^2))^2, x\right)$$

[Out] Unintegrable(x^2\*(a+b\*sec(d\*x^2+c))^2,x)

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int x^2 (a + b \sec(c + dx^2))^2 dx$$

Verification is Not applicable to the result.

[In] Int[x^2\*(a + b\*Sec[c + d\*x^2])^2,x]

[Out] Defer[Int][x^2\*(a + b\*Sec[c + d\*x^2])^2, x]

Rubi steps

$$\int x^2 (a + b \sec(c + dx^2))^2 dx = \int x^2 (a + b \sec(c + dx^2))^2 dx$$

Mathematica [A] time = 7.91, size = 0, normalized size = 0.00

$$\int x^2 (a + b \sec(c + dx^2))^2 dx$$

Verification is Not applicable to the result.

[In] Integrate[x^2\*(a + b\*Sec[c + d\*x^2])^2,x]

[Out] Integrate[x^2\*(a + b\*Sec[c + d\*x^2])^2, x]

fricas [A] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(b^2 x^2 \sec(dx^2 + c)^2 + 2 abx^2 \sec(dx^2 + c) + a^2 x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*sec(d\*x^2+c))^2,x, algorithm="fricas")

[Out] integral(b^2\*x^2\*sec(d\*x^2 + c)^2 + 2\*a\*b\*x^2\*sec(d\*x^2 + c) + a^2\*x^2, x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sec(dx^2 + c) + a)^2 x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*sec(d\*x^2+c))^2,x, algorithm="giac")

[Out] integrate((b\*sec(d\*x^2 + c) + a)^2\*x^2, x)

**maple** [A] time = 0.98, size = 0, normalized size = 0.00

$$\int x^2 (a + b \sec(dx^2 + c))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a+b\*sec(d\*x^2+c))^2,x)

[Out] int(x^2\*(a+b\*sec(d\*x^2+c))^2,x)

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*sec(d\*x^2+c))^2,x, algorithm="maxima")

[Out] Timed out

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int x^2 \left( a + \frac{b}{\cos(dx^2 + c)} \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a + b/cos(c + d\*x^2))^2,x)

[Out] int(x^2\*(a + b/cos(c + d\*x^2))^2, x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 (a + b \sec(c + dx^2))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(a+b\*sec(d\*x\*\*2+c))\*\*2,x)

[Out] Integral(x\*\*2\*(a + b\*sec(c + d\*x\*\*2))\*\*2, x)



### 3.12 $\int x \left( a + b \sec \left( c + dx^2 \right) \right)^2 dx$

Optimal. Leaf size=44

$$\frac{a^2 x^2}{2} + \frac{ab \tanh^{-1}(\sin(c + dx^2))}{d} + \frac{b^2 \tan(c + dx^2)}{2d}$$

[Out]  $1/2*a^2*x^2+a*b*\operatorname{arctanh}(\sin(d*x^2+c))/d+1/2*b^2*\tan(d*x^2+c)/d$

**Rubi [A]** time = 0.05, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$ , Rules used = {4204, 3773, 3770, 3767, 8}

$$\frac{a^2 x^2}{2} + \frac{ab \tanh^{-1}(\sin(c + dx^2))}{d} + \frac{b^2 \tan(c + dx^2)}{2d}$$

Antiderivative was successfully verified.

[In] `Int[x*(a + b*Sec[c + d*x^2])^2,x]`

[Out]  $(a^2*x^2)/2 + (a*b*\operatorname{ArcTanh}[\operatorname{Sin}[c + d*x^2]])/d + (b^2*\operatorname{Tan}[c + d*x^2])/(2*d)$

#### Rule 8

`Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

#### Rule 3767

`Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]`

#### Rule 3770

`Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]`

#### Rule 3773

`Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^2, x_Symbol] := Simp[a^2*x, x] + (Dist[2*a*b, Int[Csc[c + d*x], x], x] + Dist[b^2, Int[Csc[c + d*x]^2, x], x]) /; FreeQ[{a, b, c, d}, x]`

#### Rule 4204

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

### Rubi steps

$$\begin{aligned} \int x (a + b \sec(c + dx^2))^2 dx &= \frac{1}{2} \text{Subst} \left( \int (a + b \sec(c + dx))^2 dx, x, x^2 \right) \\ &= \frac{a^2 x^2}{2} + (ab) \text{Subst} \left( \int \sec(c + dx) dx, x, x^2 \right) + \frac{1}{2} b^2 \text{Subst} \left( \int \sec^2(c + dx) dx, x, x^2 \right) \\ &= \frac{a^2 x^2}{2} + \frac{ab \tanh^{-1}(\sin(c + dx^2))}{d} - \frac{b^2 \text{Subst} \left( \int 1 dx, x, -\tan(c + dx^2) \right)}{2d} \\ &= \frac{a^2 x^2}{2} + \frac{ab \tanh^{-1}(\sin(c + dx^2))}{d} + \frac{b^2 \tan(c + dx^2)}{2d} \end{aligned}$$

**Mathematica [A]** time = 0.21, size = 41, normalized size = 0.93

$$\frac{a^2 dx^2 + 2ab \tanh^{-1}(\sin(c + dx^2)) + b^2 \tan(c + dx^2)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Sec[c + d*x^2])^2,x]
```

```
[Out] (a^2*d*x^2 + 2*a*b*ArcTanh[Sin[c + d*x^2]] + b^2*Tan[c + d*x^2])/(2*d)
```

**fricas [B]** time = 0.66, size = 91, normalized size = 2.07

$$\frac{a^2 dx^2 \cos(dx^2 + c) + ab \cos(dx^2 + c) \log(\sin(dx^2 + c) + 1) - ab \cos(dx^2 + c) \log(-\sin(dx^2 + c) + 1) + b^2 \sin(dx^2 + c)}{2d \cos(dx^2 + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*sec(d*x^2+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*d*x^2*cos(d*x^2 + c) + a*b*cos(d*x^2 + c)*log(sin(d*x^2 + c) + 1)
- a*b*cos(d*x^2 + c)*log(-sin(d*x^2 + c) + 1) + b^2*sin(d*x^2 + c))/(d*cos(
d*x^2 + c))
```

**giac [B]** time = 0.86, size = 88, normalized size = 2.00

$$\frac{(dx^2 + c)a^2 + 2ab \log\left(\left|\tan\left(\frac{1}{2}dx^2 + \frac{1}{2}c\right) + 1\right|\right) - 2ab \log\left(\left|\tan\left(\frac{1}{2}dx^2 + \frac{1}{2}c\right) - 1\right|\right) - \frac{2b^2 \tan\left(\frac{1}{2}dx^2 + \frac{1}{2}c\right)}{\tan\left(\frac{1}{2}dx^2 + \frac{1}{2}c\right)^2 - 1}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*sec(d\*x^2+c))^2,x, algorithm="giac")

[Out] 1/2\*((d\*x^2 + c)\*a^2 + 2\*a\*b\*log(abs(tan(1/2\*d\*x^2 + 1/2\*c) + 1)) - 2\*a\*b\*log(abs(tan(1/2\*d\*x^2 + 1/2\*c) - 1)) - 2\*b^2\*tan(1/2\*d\*x^2 + 1/2\*c)/(tan(1/2\*d\*x^2 + 1/2\*c)^2 - 1))/d

**maple [A]** time = 0.52, size = 59, normalized size = 1.34

$$\frac{a^2x^2}{2} + \frac{b^2 \tan(dx^2 + c)}{2d} + \frac{ab \ln(\sec(dx^2 + c) + \tan(dx^2 + c))}{d} + \frac{a^2c}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a+b\*sec(d\*x^2+c))^2,x)

[Out] 1/2\*a^2\*x^2+1/2\*b^2\*tan(d\*x^2+c)/d+1/d\*a\*b\*ln(sec(d\*x^2+c)+tan(d\*x^2+c))+1/2/d\*a^2\*c

**maxima [B]** time = 0.82, size = 96, normalized size = 2.18

$$\frac{1}{2}a^2x^2 + \frac{ab \log(\sec(dx^2 + c) + \tan(dx^2 + c))}{d} + \frac{b^2 \sin(2dx^2 + 2c)}{d \cos(2dx^2 + 2c)^2 + d \sin(2dx^2 + 2c)^2 + 2d \cos(2dx^2 + 2c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*sec(d\*x^2+c))^2,x, algorithm="maxima")

[Out] 1/2\*a^2\*x^2 + a\*b\*log(sec(d\*x^2 + c) + tan(d\*x^2 + c))/d + b^2\*sin(2\*d\*x^2 + 2\*c)/(d\*cos(2\*d\*x^2 + 2\*c)^2 + d\*sin(2\*d\*x^2 + 2\*c)^2 + 2\*d\*cos(2\*d\*x^2 + 2\*c) + d)

**mupad [B]** time = 1.50, size = 100, normalized size = 2.27

$$\frac{a^2x^2}{2} + \frac{b^2 \operatorname{Im}}{d(e^{2i dx^2 + c 2i} + 1)} + \frac{ab \ln(-abx^4i - 4abxe^{dx^2i}e^{c1i})}{d} - \frac{ab \ln(abx^4i - 4abxe^{dx^2i}e^{c1i})}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b/cos(c + d*x^2))^2,x)
```

```
[Out] (a^2*x^2)/2 + (b^2*1i)/(d*(exp(c*2i + d*x^2*2i) + 1)) + (a*b*log(- a*b*x*4i
- 4*a*b*x*exp(d*x^2*1i)*exp(c*1i)))/d - (a*b*log(a*b*x*4i - 4*a*b*x*exp(d*
x^2*1i)*exp(c*1i)))/d
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int x (a + b \sec(c + dx^2))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*sec(d*x**2+c))**2,x)
```

```
[Out] Integral(x*(a + b*sec(c + d*x**2))**2, x)
```

$$3.13 \quad \int \frac{(a+b \sec(c+dx^2))^2}{x} dx$$

Optimal. Leaf size=21

$$\text{Int} \left( \frac{(a+b \sec(c+dx^2))^2}{x}, x \right)$$

[Out] Unintegrable((a+b\*sec(d\*x^2+c))^2/x,x)

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{(a+b \sec(c+dx^2))^2}{x} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*Sec[c + d\*x^2])^2/x,x]

[Out] Defer[Int] [(a + b\*Sec[c + d\*x^2])^2/x, x]

Rubi steps

$$\int \frac{(a+b \sec(c+dx^2))^2}{x} dx = \int \frac{(a+b \sec(c+dx^2))^2}{x} dx$$

Mathematica [A] time = 17.86, size = 0, normalized size = 0.00

$$\int \frac{(a+b \sec(c+dx^2))^2}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*Sec[c + d\*x^2])^2/x,x]

[Out] Integrate[(a + b\*Sec[c + d\*x^2])^2/x, x]

fricas [A] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{b^2 \sec(dx^2+c)^2 + 2ab \sec(dx^2+c) + a^2}{x}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sec(d\*x^2+c))^2/x,x, algorithm="fricas")

[Out] integral((b^2\*sec(d\*x^2 + c)^2 + 2\*a\*b\*sec(d\*x^2 + c) + a^2)/x, x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \sec(dx^2 + c) + a)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sec(d\*x^2+c))^2/x,x, algorithm="giac")

[Out] integrate((b\*sec(d\*x^2 + c) + a)^2/x, x)

maple [A] time = 1.16, size = 0, normalized size = 0.00

$$\int \frac{(a + b \sec(dx^2 + c))^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*sec(d\*x^2+c))^2/x,x)

[Out] int((a+b\*sec(d\*x^2+c))^2/x,x)

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$a^2 \log(x) + \frac{b^2 \sin(2 dx^2 + 2 c) + 2 \left( dx^2 \cos(2 dx^2 + 2 c) \right)^2 + dx^2 \sin(2 dx^2 + 2 c)^2 + 2 dx^2 \cos(2 dx^2 + 2 c) + dx^2}{dx^2 \cos(2 dx^2 + 2 c)^2 + dx^2 \sin(2 dx^2 + 2 c)^2 + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sec(d\*x^2+c))^2/x,x, algorithm="maxima")

[Out] a^2\*log(x) + (b^2\*sin(2\*d\*x^2 + 2\*c) + (d\*x^2\*cos(2\*d\*x^2 + 2\*c))^2 + d\*x^2\*sin(2\*d\*x^2 + 2\*c)^2 + 2\*d\*x^2\*cos(2\*d\*x^2 + 2\*c) + d\*x^2)\*integrate(2\*(2\*a\*b\*d\*x^2\*cos(2\*d\*x^2 + 2\*c)\*cos(d\*x^2 + c) + 2\*a\*b\*d\*x^2\*cos(d\*x^2 + c) + (2\*a\*b\*d\*x^2\*sin(d\*x^2 + c) + b^2)\*sin(2\*d\*x^2 + 2\*c))/(d\*x^3\*cos(2\*d\*x^2 + 2\*c)^2 + d\*x^3\*sin(2\*d\*x^2 + 2\*c)^2 + 2\*d\*x^3\*cos(2\*d\*x^2 + 2\*c) + d\*x^3), x)/(d\*x^2\*cos(2\*d\*x^2 + 2\*c)^2 + d\*x^2\*sin(2\*d\*x^2 + 2\*c)^2 + 2\*d\*x^2\*cos(2\*d\*x^2 + 2\*c) + d\*x^2)

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\left(a + \frac{b}{\cos(dx^2+c)}\right)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cos(c + d\*x^2))^2/x,x)

[Out] int((a + b/cos(c + d\*x^2))^2/x, x)

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \sec(c + dx^2)\right)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sec(d\*x\*\*2+c))\*\*2/x,x)

[Out] Integral((a + b\*sec(c + d\*x\*\*2))\*\*2/x, x)

$$3.14 \quad \int \frac{(a+b \sec(c+dx^2))^2}{x^2} dx$$

Optimal. Leaf size=21

$$\text{Int} \left( \frac{(a + b \sec(c + dx^2))^2}{x^2}, x \right)$$

[Out] Unintegrable((a+b\*sec(d\*x^2+c))^2/x^2,x)

**Rubi [A]** time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{(a + b \sec(c + dx^2))^2}{x^2} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*Sec[c + d\*x^2])^2/x^2,x]

[Out] Defer[Int] [(a + b\*Sec[c + d\*x^2])^2/x^2, x]

Rubi steps

$$\int \frac{(a + b \sec(c + dx^2))^2}{x^2} dx = \int \frac{(a + b \sec(c + dx^2))^2}{x^2} dx$$

**Mathematica [A]** time = 9.32, size = 0, normalized size = 0.00

$$\int \frac{(a + b \sec(c + dx^2))^2}{x^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*Sec[c + d\*x^2])^2/x^2,x]

[Out] Integrate[(a + b\*Sec[c + d\*x^2])^2/x^2, x]

**fricas [A]** time = 1.37, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{b^2 \sec(dx^2 + c)^2 + 2ab \sec(dx^2 + c) + a^2}{x^2}, x \right)$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sec(d\*x^2+c))^2/x^2,x, algorithm="fricas")

[Out] integral((b^2\*sec(d\*x^2 + c)^2 + 2\*a\*b\*sec(d\*x^2 + c) + a^2)/x^2, x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \sec(dx^2 + c) + a)^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sec(d\*x^2+c))^2/x^2,x, algorithm="giac")

[Out] integrate((b\*sec(d\*x^2 + c) + a)^2/x^2, x)

**maple** [A] time = 1.12, size = 0, normalized size = 0.00

$$\int \frac{(a + b \sec(dx^2 + c))^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*sec(d\*x^2+c))^2/x^2,x)

[Out] int((a+b\*sec(d\*x^2+c))^2/x^2,x)

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sec(d\*x^2+c))^2/x^2,x, algorithm="maxima")

[Out] Timed out

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\left(a + \frac{b}{\cos(dx^2+c)}\right)^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cos(c + d\*x^2))^2/x^2,x)

[Out] `int((a + b/cos(c + d*x^2))^2/x^2, x)`

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \sec(c + dx^2))^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sec(d*x**2+c))**2/x**2,x)`

[Out] `Integral((a + b*sec(c + d*x**2))**2/x**2, x)`

### 3.15 $\int x \sec^7(a + bx^2) dx$

**Optimal.** Leaf size=90

$$\frac{5 \tanh^{-1}(\sin(a + bx^2))}{32b} + \frac{\tan(a + bx^2) \sec^5(a + bx^2)}{12b} + \frac{5 \tan(a + bx^2) \sec^3(a + bx^2)}{48b} + \frac{5 \tan(a + bx^2) \sec(a + bx^2)}{32b}$$

[Out] 5/32\*arctanh(sin(b\*x^2+a))/b+5/32\*sec(b\*x^2+a)\*tan(b\*x^2+a)/b+5/48\*sec(b\*x^2+a)^3\*tan(b\*x^2+a)/b+1/12\*sec(b\*x^2+a)^5\*tan(b\*x^2+a)/b

**Rubi [A]** time = 0.07, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {4204, 3768, 3770}

$$\frac{5 \tanh^{-1}(\sin(a + bx^2))}{32b} + \frac{\tan(a + bx^2) \sec^5(a + bx^2)}{12b} + \frac{5 \tan(a + bx^2) \sec^3(a + bx^2)}{48b} + \frac{5 \tan(a + bx^2) \sec(a + bx^2)}{32b}$$

Antiderivative was successfully verified.

[In] Int[x\*Sec[a + b\*x^2]^7,x]

[Out] (5\*ArcTanh[Sin[a + b\*x^2]])/(32\*b) + (5\*Sec[a + b\*x^2]\*Tan[a + b\*x^2])/(32\*b) + (5\*Sec[a + b\*x^2]^3\*Tan[a + b\*x^2])/(48\*b) + (Sec[a + b\*x^2]^5\*Tan[a + b\*x^2])/(12\*b)

#### Rule 3768

Int[(csc[(c\_.) + (d\_.)\*(x\_.)]\*(b\_.))^n], x\_Symbol] := -Simp[(b\*Csc[c + d\*x]\*(b\*Csc[c + d\*x])^(n - 1))/(d\*(n - 1)), x] + Dist[(b^2\*(n - 2))/(n - 1), Int[(b\*Csc[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

#### Rule 3770

Int[csc[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] := -Simp[ArcTanh[Cos[c + d\*x]]/d, x] /; FreeQ[{c, d}, x]

#### Rule 4204

Int[(x\_)^(m\_.)\*((a\_.) + (b\_.)\*Sec[(c\_.) + (d\_.)\*(x\_)^(n\_.)])^(p\_.), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*Sec[c + d\*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]

#### Rubi steps

$$\begin{aligned}
\int x \sec^7(a + bx^2) dx &= \frac{1}{2} \text{Subst} \left( \int \sec^7(a + bx) dx, x, x^2 \right) \\
&= \frac{\sec^5(a + bx^2) \tan(a + bx^2)}{12b} + \frac{5}{12} \text{Subst} \left( \int \sec^5(a + bx) dx, x, x^2 \right) \\
&= \frac{5 \sec^3(a + bx^2) \tan(a + bx^2)}{48b} + \frac{\sec^5(a + bx^2) \tan(a + bx^2)}{12b} + \frac{5}{16} \text{Subst} \left( \int \sec^3(a + bx) dx, x, x^2 \right) \\
&= \frac{5 \sec(a + bx^2) \tan(a + bx^2)}{32b} + \frac{5 \sec^3(a + bx^2) \tan(a + bx^2)}{48b} + \frac{\sec^5(a + bx^2) \tan(a + bx^2)}{12b} \\
&= \frac{5 \tanh^{-1}(\sin(a + bx^2))}{32b} + \frac{5 \sec(a + bx^2) \tan(a + bx^2)}{32b} + \frac{5 \sec^3(a + bx^2) \tan(a + bx^2)}{48b}
\end{aligned}$$

**Mathematica** [A] time = 0.21, size = 62, normalized size = 0.69

$$\frac{15 \tanh^{-1}(\sin(a + bx^2)) + \tan(a + bx^2) \sec(a + bx^2) (8 \sec^4(a + bx^2) + 10 \sec^2(a + bx^2) + 15)}{96b}$$

Antiderivative was successfully verified.

[In] Integrate[x\*Sec[a + b\*x^2]^7,x]

[Out] (15\*ArcTanh[Sin[a + b\*x^2]] + Sec[a + b\*x^2]\*(15 + 10\*Sec[a + b\*x^2]^2 + 8\*Sec[a + b\*x^2]^4)\*Tan[a + b\*x^2])/ (96\*b)

**fricas** [A] time = 0.66, size = 100, normalized size = 1.11

$$\frac{15 \cos(bx^2 + a)^6 \log(\sin(bx^2 + a) + 1) - 15 \cos(bx^2 + a)^6 \log(-\sin(bx^2 + a) + 1) + 2(15 \cos(bx^2 + a)^4 + 10 \cos(bx^2 + a)^2 + 8) \sin(bx^2 + a)}{192 b \cos(bx^2 + a)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*sec(b\*x^2+a)^7,x, algorithm="fricas")

[Out] 1/192\*(15\*cos(b\*x^2 + a)^6\*log(sin(b\*x^2 + a) + 1) - 15\*cos(b\*x^2 + a)^6\*log(-sin(b\*x^2 + a) + 1) + 2\*(15\*cos(b\*x^2 + a)^4 + 10\*cos(b\*x^2 + a)^2 + 8)\*sin(b\*x^2 + a))/(b\*cos(b\*x^2 + a)^6)

**giac** [A] time = 0.34, size = 85, normalized size = 0.94

$$\frac{2(15 \sin(bx^2+a)^5 - 40 \sin(bx^2+a)^3 + 33 \sin(bx^2+a))}{(\sin(bx^2+a)^2 - 1)^3} - 15 \log(\sin(bx^2 + a) + 1) + 15 \log(-\sin(bx^2 + a) + 1)$$

---

192 b

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*sec(b\*x^2+a)^7,x, algorithm="giac")

[Out] 
$$-1/192*(2*(15*\sin(b*x^2 + a)^5 - 40*\sin(b*x^2 + a)^3 + 33*\sin(b*x^2 + a))/(\sin(b*x^2 + a)^2 - 1)^3 - 15*\log(\sin(b*x^2 + a) + 1) + 15*\log(-\sin(b*x^2 + a) + 1))/b$$

**maple [A]** time = 0.55, size = 92, normalized size = 1.02

$$\frac{(\sec^5(bx^2 + a)) \tan(bx^2 + a)}{12b} + \frac{5(\sec^3(bx^2 + a)) \tan(bx^2 + a)}{48b} + \frac{5 \sec(bx^2 + a) \tan(bx^2 + a)}{32b} + \frac{5 \ln(\sec(bx^2 + a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*sec(b\*x^2+a)^7,x)

[Out] 
$$1/12*\sec(b*x^2+a)^5*\tan(b*x^2+a)/b+5/48*\sec(b*x^2+a)^3*\tan(b*x^2+a)/b+5/32*\sec(b*x^2+a)*\tan(b*x^2+a)/b+5/32/b*\ln(\sec(b*x^2+a)+\tan(b*x^2+a))$$

**maxima [B]** time = 1.42, size = 2838, normalized size = 31.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*sec(b\*x^2+a)^7,x, algorithm="maxima")

[Out] 
$$\begin{aligned} &1/192*(4*(15*\sin(11*b*x^2 + 11*a) + 85*\sin(9*b*x^2 + 9*a) + 198*\sin(7*b*x^2 + 7*a) - 198*\sin(5*b*x^2 + 5*a) - 85*\sin(3*b*x^2 + 3*a) - 15*\sin(b*x^2 + a)) * \\ &\cos(12*b*x^2 + 12*a) - 60*(6*\sin(10*b*x^2 + 10*a) + 15*\sin(8*b*x^2 + 8*a) + 20*\sin(6*b*x^2 + 6*a) + 15*\sin(4*b*x^2 + 4*a) + 6*\sin(2*b*x^2 + 2*a)) * \\ &\cos(11*b*x^2 + 11*a) + 24*(85*\sin(9*b*x^2 + 9*a) + 198*\sin(7*b*x^2 + 7*a) - 198*\sin(5*b*x^2 + 5*a) - 85*\sin(3*b*x^2 + 3*a) - 15*\sin(b*x^2 + a)) * \\ &\cos(10*b*x^2 + 10*a) - 340*(15*\sin(8*b*x^2 + 8*a) + 20*\sin(6*b*x^2 + 6*a) + 15*\sin(4*b*x^2 + 4*a) + 6*\sin(2*b*x^2 + 2*a)) * \\ &\cos(9*b*x^2 + 9*a) + 60*(198*\sin(7*b*x^2 + 7*a) - 198*\sin(5*b*x^2 + 5*a) - 85*\sin(3*b*x^2 + 3*a) - 15*\sin(b*x^2 + a)) * \\ &\cos(8*b*x^2 + 8*a) - 792*(20*\sin(6*b*x^2 + 6*a) + 15*\sin(4*b*x^2 + 4*a) + 6*\sin(2*b*x^2 + 2*a)) * \\ &\cos(7*b*x^2 + 7*a) - 80*(198*\sin(5*b*x^2 + 5*a) + 85*\sin(3*b*x^2 + 3*a) + 15*\sin(b*x^2 + a)) * \\ &\cos(6*b*x^2 + 6*a) + 2376*(5*\sin(4*b*x^2 + 4*a) + 2*\sin(2*b*x^2 + 2*a)) * \\ &\cos(5*b*x^2 + 5*a) - 300*(17*\sin(3*b*x^2 + 3*a) + 3*\sin(b*x^2 + a)) * \\ &\cos(4*b*x^2 + 4*a) - 15*(2*(6*\cos(10*b*x^2 + 10*a) + 15*\cos(8*b*x^2 + 8*a) + 20*\cos(6*b*x^2 + 6*a) + 15*\cos(4*b*x^2 + 4*a) + 6*\cos(2*b*x^2 + 2*a) + 1) * \\ &\cos(12*b*x^2 + 12*a) + \cos(12*b*x^2 + 12*a)^2 + 12*(15*\cos(8*b*x^2 + 8*a) + 20*\cos(6*b*x^2 + 6*a) + 15*\cos(4*b*x^2 + 4*a) + 6*\cos(2*b*x^2 + 2*a) + 1) * \\ &\cos(10*b*x^2 + 10*a) + 36*\cos(10*b*x^2 + 10*a)^2 + 30*(20*\cos(6*b*x^2 + 6*a) + 15*\cos(4*b*x^2 + 4*a) + 6*\cos(2*b \end{aligned}$$

$$\begin{aligned}
& *x^2 + 2*a) + 1)*\cos(8*b*x^2 + 8*a) + 225*\cos(8*b*x^2 + 8*a)^2 + 40*(15*\cos \\
& (4*b*x^2 + 4*a) + 6*\cos(2*b*x^2 + 2*a) + 1)*\cos(6*b*x^2 + 6*a) + 400*\cos(6* \\
& b*x^2 + 6*a)^2 + 30*(6*\cos(2*b*x^2 + 2*a) + 1)*\cos(4*b*x^2 + 4*a) + 225*\cos \\
& (4*b*x^2 + 4*a)^2 + 36*\cos(2*b*x^2 + 2*a)^2 + 2*(6*\sin(10*b*x^2 + 10*a) + 1 \\
& 5*\sin(8*b*x^2 + 8*a) + 20*\sin(6*b*x^2 + 6*a) + 15*\sin(4*b*x^2 + 4*a) + 6*si \\
& n(2*b*x^2 + 2*a))*\sin(12*b*x^2 + 12*a) + \sin(12*b*x^2 + 12*a)^2 + 12*(15*si \\
& n(8*b*x^2 + 8*a) + 20*\sin(6*b*x^2 + 6*a) + 15*\sin(4*b*x^2 + 4*a) + 6*\sin(2* \\
& b*x^2 + 2*a))*\sin(10*b*x^2 + 10*a) + 36*\sin(10*b*x^2 + 10*a)^2 + 30*(20*\sin \\
& (6*b*x^2 + 6*a) + 15*\sin(4*b*x^2 + 4*a) + 6*\sin(2*b*x^2 + 2*a))*\sin(8*b*x^2 \\
& + 8*a) + 225*\sin(8*b*x^2 + 8*a)^2 + 120*(5*\sin(4*b*x^2 + 4*a) + 2*\sin(2*b* \\
& x^2 + 2*a))*\sin(6*b*x^2 + 6*a) + 400*\sin(6*b*x^2 + 6*a)^2 + 225*\sin(4*b*x^2 \\
& + 4*a)^2 + 180*\sin(4*b*x^2 + 4*a)*\sin(2*b*x^2 + 2*a) + 36*\sin(2*b*x^2 + 2* \\
& a)^2 + 12*\cos(2*b*x^2 + 2*a) + 1)*\log((\cos(b*x^2 + 2*a)^2 + \cos(a)^2 - 2*\co \\
& s(a)*\sin(b*x^2 + 2*a) + \sin(b*x^2 + 2*a)^2 + 2*\cos(b*x^2 + 2*a)*\sin(a) + si \\
& n(a)^2)/(\cos(b*x^2 + 2*a)^2 + \cos(a)^2 + 2*\cos(a)*\sin(b*x^2 + 2*a) + \sin(b* \\
& x^2 + 2*a)^2 - 2*\cos(b*x^2 + 2*a)*\sin(a) + \sin(a)^2)) - 4*(15*\cos(11*b*x^2 \\
& + 11*a) + 85*\cos(9*b*x^2 + 9*a) + 198*\cos(7*b*x^2 + 7*a) - 198*\cos(5*b*x^2 \\
& + 5*a) - 85*\cos(3*b*x^2 + 3*a) - 15*\cos(b*x^2 + a))*\sin(12*b*x^2 + 12*a) + \\
& 60*(6*\cos(10*b*x^2 + 10*a) + 15*\cos(8*b*x^2 + 8*a) + 20*\cos(6*b*x^2 + 6*a) \\
& + 15*\cos(4*b*x^2 + 4*a) + 6*\cos(2*b*x^2 + 2*a) + 1)*\sin(11*b*x^2 + 11*a) - \\
& 24*(85*\cos(9*b*x^2 + 9*a) + 198*\cos(7*b*x^2 + 7*a) - 198*\cos(5*b*x^2 + 5*a) \\
& - 85*\cos(3*b*x^2 + 3*a) - 15*\cos(b*x^2 + a))*\sin(10*b*x^2 + 10*a) + 340*(1 \\
& 5*\cos(8*b*x^2 + 8*a) + 20*\cos(6*b*x^2 + 6*a) + 15*\cos(4*b*x^2 + 4*a) + 6*\co \\
& s(2*b*x^2 + 2*a) + 1)*\sin(9*b*x^2 + 9*a) - 60*(198*\cos(7*b*x^2 + 7*a) - 198 \\
& *\cos(5*b*x^2 + 5*a) - 85*\cos(3*b*x^2 + 3*a) - 15*\cos(b*x^2 + a))*\sin(8*b*x^ \\
& 2 + 8*a) + 792*(20*\cos(6*b*x^2 + 6*a) + 15*\cos(4*b*x^2 + 4*a) + 6*\cos(2*b*x \\
& ^2 + 2*a) + 1)*\sin(7*b*x^2 + 7*a) + 80*(198*\cos(5*b*x^2 + 5*a) + 85*\cos(3*b \\
& *x^2 + 3*a) + 15*\cos(b*x^2 + a))*\sin(6*b*x^2 + 6*a) - 792*(15*\cos(4*b*x^2 + \\
& 4*a) + 6*\cos(2*b*x^2 + 2*a) + 1)*\sin(5*b*x^2 + 5*a) + 300*(17*\cos(3*b*x^2 \\
& + 3*a) + 3*\cos(b*x^2 + a))*\sin(4*b*x^2 + 4*a) - 340*(6*\cos(2*b*x^2 + 2*a) + \\
& 1)*\sin(3*b*x^2 + 3*a) + 2040*\cos(3*b*x^2 + 3*a)*\sin(2*b*x^2 + 2*a) + 360*c \\
& os(b*x^2 + a)*\sin(2*b*x^2 + 2*a) - 360*\cos(2*b*x^2 + 2*a)*\sin(b*x^2 + a) - \\
& 60*\sin(b*x^2 + a))/(b*\cos(12*b*x^2 + 12*a)^2 + 36*b*\cos(10*b*x^2 + 10*a)^2 \\
& + 225*b*\cos(8*b*x^2 + 8*a)^2 + 400*b*\cos(6*b*x^2 + 6*a)^2 + 225*b*\cos(4*b*x \\
& ^2 + 4*a)^2 + 36*b*\cos(2*b*x^2 + 2*a)^2 + b*\sin(12*b*x^2 + 12*a)^2 + 36*b*s \\
& in(10*b*x^2 + 10*a)^2 + 225*b*\sin(8*b*x^2 + 8*a)^2 + 400*b*\sin(6*b*x^2 + 6* \\
& a)^2 + 225*b*\sin(4*b*x^2 + 4*a)^2 + 180*b*\sin(4*b*x^2 + 4*a)*\sin(2*b*x^2 + \\
& 2*a) + 36*b*\sin(2*b*x^2 + 2*a)^2 + 2*(6*b*\cos(10*b*x^2 + 10*a) + 15*b*\cos(8 \\
& *b*x^2 + 8*a) + 20*b*\cos(6*b*x^2 + 6*a) + 15*b*\cos(4*b*x^2 + 4*a) + 6*b*\cos \\
& (2*b*x^2 + 2*a) + b)*\cos(12*b*x^2 + 12*a) + 12*(15*b*\cos(8*b*x^2 + 8*a) + 2 \\
& 0*b*\cos(6*b*x^2 + 6*a) + 15*b*\cos(4*b*x^2 + 4*a) + 6*b*\cos(2*b*x^2 + 2*a) + \\
& b)*\cos(10*b*x^2 + 10*a) + 30*(20*b*\cos(6*b*x^2 + 6*a) + 15*b*\cos(4*b*x^2 + \\
& 4*a) + 6*b*\cos(2*b*x^2 + 2*a) + b)*\cos(8*b*x^2 + 8*a) + 40*(15*b*\cos(4*b*x \\
& ^2 + 4*a) + 6*b*\cos(2*b*x^2 + 2*a) + b)*\cos(6*b*x^2 + 6*a) + 30*(6*b*\cos(2* \\
& b*x^2 + 2*a) + b)*\cos(4*b*x^2 + 4*a) + 12*b*\cos(2*b*x^2 + 2*a) + 2*(6*b*\sin
\end{aligned}$$

$(10*b*x^2 + 10*a) + 15*b*\sin(8*b*x^2 + 8*a) + 20*b*\sin(6*b*x^2 + 6*a) + 15*b*\sin(4*b*x^2 + 4*a) + 6*b*\sin(2*b*x^2 + 2*a))*\sin(12*b*x^2 + 12*a) + 12*(15*b*\sin(8*b*x^2 + 8*a) + 20*b*\sin(6*b*x^2 + 6*a) + 15*b*\sin(4*b*x^2 + 4*a) + 6*b*\sin(2*b*x^2 + 2*a))*\sin(10*b*x^2 + 10*a) + 30*(20*b*\sin(6*b*x^2 + 6*a) + 15*b*\sin(4*b*x^2 + 4*a) + 6*b*\sin(2*b*x^2 + 2*a))*\sin(8*b*x^2 + 8*a) + 120*(5*b*\sin(4*b*x^2 + 4*a) + 2*b*\sin(2*b*x^2 + 2*a))*\sin(6*b*x^2 + 6*a) + b)$

**mupad [B]** time = 12.53, size = 496, normalized size = 5.51

$$\frac{5 \ln\left(-\frac{x5i}{8} - \frac{5x e^{a1i} e^{bx^2 1i}}{8}\right)}{32b} - \frac{5 \ln\left(\frac{x5i}{8} - \frac{5x e^{a1i} e^{bx^2 1i}}{8}\right)}{32b} + \frac{e^{3i b x^2 + a 3i} 8i}{3b (5 e^{2i b x^2 + a 2i} + 10 e^{4i b x^2 + a 4i} + 10 e^{6i b x^2 + a 6i} + 5 e^{8i b x^2 + a 8i})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/cos(a + b\*x^2)^7,x)

[Out]  $(5*\log(- (x*5i)/8 - (5*x*\exp(a*1i)*\exp(b*x^2*1i))/8))/(32*b) - (5*\log((x*5i)/8 - (5*x*\exp(a*1i)*\exp(b*x^2*1i))/8))/(32*b) + (\exp(a*3i + b*x^2*3i)*8i)/(3*b*(5*\exp(a*2i + b*x^2*2i) + 10*\exp(a*4i + b*x^2*4i) + 10*\exp(a*6i + b*x^2*6i) + 5*\exp(a*8i + b*x^2*8i) + \exp(a*10i + b*x^2*10i) + 1)) - (\exp(a*1i + b*x^2*1i)*1i)/(6*b*(3*\exp(a*2i + b*x^2*2i) + 3*\exp(a*4i + b*x^2*4i) + \exp(a*6i + b*x^2*6i) + 1)) - (\exp(a*1i + b*x^2*1i)*5i)/(16*b*(\exp(a*2i + b*x^2*2i) + 1)) + (\exp(a*5i + b*x^2*5i)*16i)/(3*b*(6*\exp(a*2i + b*x^2*2i) + 15*\exp(a*4i + b*x^2*4i) + 20*\exp(a*6i + b*x^2*6i) + 15*\exp(a*8i + b*x^2*8i) + 6*\exp(a*10i + b*x^2*10i) + \exp(a*12i + b*x^2*12i) + 1)) + (\exp(a*1i + b*x^2*1i)*1i)/(b*(4*\exp(a*2i + b*x^2*2i) + 6*\exp(a*4i + b*x^2*4i) + 4*\exp(a*6i + b*x^2*6i) + \exp(a*8i + b*x^2*8i) + 1)) - (\exp(a*1i + b*x^2*1i)*5i)/(24*b*(2*\exp(a*2i + b*x^2*2i) + \exp(a*4i + b*x^2*4i) + 1))$

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x \sec^7(a + bx^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*sec(b\*x\*\*2+a)\*\*7,x)

[Out] Integral(x\*sec(a + b\*x\*\*2)\*\*7, x)

$$3.16 \quad \int \frac{x^5}{a+b \sec(c+dx^2)} dx$$

**Optimal.** Leaf size=382

$$\frac{ibLi_3\left(-\frac{ae^{i(dx^2+c)}}{b-\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}} - \frac{ibLi_3\left(-\frac{ae^{i(dx^2+c)}}{b+\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}} + \frac{bx^2Li_2\left(-\frac{ae^{i(dx^2+c)}}{b-\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}} - \frac{bx^2Li_2\left(-\frac{ae^{i(dx^2+c)}}{b+\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}} + \frac{ibx^4 \log\left(1 + \frac{ae^{i(c+dx^2)}}{b-\sqrt{b^2-a^2}}\right)}{2ad\sqrt{b^2-a^2}} - \frac{ibx^4 \log\left(1 + \frac{ae^{i(c+dx^2)}}{b+\sqrt{b^2-a^2}}\right)}{2ad\sqrt{b^2-a^2}}$$

[Out]  $\frac{1}{6}x^6/a + \frac{1}{2}I*b*x^4*\ln(1+a*\exp(I*(d*x^2+c)))/(b-(-a^2+b^2)^{(1/2)})/a/d/(-a^2+b^2)^{(1/2)} - \frac{1}{2}I*b*x^4*\ln(1+a*\exp(I*(d*x^2+c)))/(b+(-a^2+b^2)^{(1/2)})/a/d/(-a^2+b^2)^{(1/2)} + b*x^2*polylog(2,-a*\exp(I*(d*x^2+c)))/(b-(-a^2+b^2)^{(1/2)})/a/d^2/(-a^2+b^2)^{(1/2)} - b*x^2*polylog(2,-a*\exp(I*(d*x^2+c)))/(b+(-a^2+b^2)^{(1/2)})/a/d^2/(-a^2+b^2)^{(1/2)} + I*b*polylog(3,-a*\exp(I*(d*x^2+c)))/(b-(-a^2+b^2)^{(1/2)})/a/d^3/(-a^2+b^2)^{(1/2)} - I*b*polylog(3,-a*\exp(I*(d*x^2+c)))/(b+(-a^2+b^2)^{(1/2)})/a/d^3/(-a^2+b^2)^{(1/2)}$

**Rubi [A]** time = 0.88, antiderivative size = 382, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$ , Rules used = {4204, 4191, 3321, 2264, 2190, 2531, 2282, 6589}

$$\frac{bx^2PolyLog\left(2, -\frac{ae^{i(c+dx^2)}}{b-\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}} - \frac{bx^2PolyLog\left(2, -\frac{ae^{i(c+dx^2)}}{\sqrt{b^2-a^2}+b}\right)}{ad^2\sqrt{b^2-a^2}} + \frac{ibPolyLog\left(3, -\frac{ae^{i(c+dx^2)}}{b-\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}} - \frac{ibPolyLog\left(3, -\frac{ae^{i(c+dx^2)}}{\sqrt{b^2-a^2}+b}\right)}{ad^3\sqrt{b^2-a^2}}$$

Antiderivative was successfully verified.

[In] Int[x^5/(a + b\*Sec[c + d\*x^2]),x]

[Out]  $x^6/(6*a) + ((I/2)*b*x^4*Log[1 + (a*E^(I*(c + d*x^2)))/(b - Sqrt[-a^2 + b^2]])/(a*Sqrt[-a^2 + b^2]*d) - ((I/2)*b*x^4*Log[1 + (a*E^(I*(c + d*x^2)))/(b + Sqrt[-a^2 + b^2]])/(a*Sqrt[-a^2 + b^2]*d) + (b*x^2*PolyLog[2, -((a*E^(I*(c + d*x^2)))/(b - Sqrt[-a^2 + b^2]])]/(a*Sqrt[-a^2 + b^2]*d^2) - (b*x^2*PolyLog[2, -((a*E^(I*(c + d*x^2)))/(b + Sqrt[-a^2 + b^2]])]/(a*Sqrt[-a^2 + b^2]*d^2) + (I*b*PolyLog[3, -((a*E^(I*(c + d*x^2)))/(b - Sqrt[-a^2 + b^2]])]/(a*Sqrt[-a^2 + b^2]*d^3) - (I*b*PolyLog[3, -((a*E^(I*(c + d*x^2)))/(b + Sqrt[-a^2 + b^2]])]/(a*Sqrt[-a^2 + b^2]*d^3)$

**Rule 2190**

Int[(((F\_)^(g\_)\*((e\_) + (f\_)\*(x\_)))^(n\_)\*((c\_) + (d\_)\*(x\_))^(m\_))/((a\_) + (b\_)\*((F\_)^(g\_)\*((e\_) + (f\_)\*(x\_)))^(n\_)), x\_Symbol] := Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a]]/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a]], x]



))<sup>n</sup>)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2264

Int[((F\_)^(u\_)\*((f\_.) + (g\_.)\*(x\_)^(m\_.)))/((a\_.) + (b\_.)\*(F\_)^(u\_) + (c\_.)\*(F\_)^(v\_)), x\_Symbol] :> With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[(2\*c)/q, Int[(f + g\*x)^m\*F^u/(b - q + 2\*c\*F^u), x], x] - Dist[(2\*c)/q, Int[(f + g\*x)^m\*F^u/(b + q + 2\*c\*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2\*u] && LinearQ[u, x] && NeQ[b^2 - 4\*a\*c, 0] && IGtQ[m, 0]

#### Rule 2282

Int[u\_, x\_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_.)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_.)\*((a\_.) + (b\_.)\*x))\*(F\_) [v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

#### Rule 2531

Int[Log[1 + (e\_.)\*((F\_)^(c\_.)\*((a\_.) + (b\_.)\*(x\_)))^(n\_.)]\*((f\_.) + (g\_.)\*(x\_)^(m\_.)), x\_Symbol] :> -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x))))^n])/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m - 1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

#### Rule 3321

Int[((c\_.) + (d\_.)\*(x\_)^(m\_.))/((a\_) + (b\_.)\*sin[(e\_.) + Pi\*(k\_.) + (f\_.)\*(x\_)]), x\_Symbol] :> Dist[2, Int[((c + d\*x)^m\*E^(I\*Pi\*(k - 1/2))\*E^(I\*(e + f\*x)))/(b + 2\*a\*E^(I\*Pi\*(k - 1/2))\*E^(I\*(e + f\*x)) - b\*E^(2\*I\*k\*Pi)\*E^(2\*I\*(e + f\*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2\*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]

#### Rule 4191

Int[(csc[(e\_.) + (f\_.)\*(x\_)]\*(b\_.) + (a\_.))^(n\_.)\*((c\_.) + (d\_.)\*(x\_)^(m\_.)), x\_Symbol] :> Int[ExpandIntegrand[(c + d\*x)^m, 1/(Sin[e + f\*x]^n/(b + a\*Sin[e + f\*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGtQ[m, 0]

#### Rule 4204

Int[(x\_)^(m\_.)\*((a\_.) + (b\_.)\*Sec[(c\_.) + (d\_.)\*(x\_)^(n\_)])^(p\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*Sec[c + d\*x])^p

, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_.))^(p\_.)]/((d\_.) + (e\_.)\*(x\_.)), x\_Symbol] :> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rubi steps

$$\begin{aligned}
 \int \frac{x^5}{a + b \sec(c + dx^2)} dx &= \frac{1}{2} \text{Subst} \left( \int \frac{x^2}{a + b \sec(c + dx)} dx, x, x^2 \right) \\
 &= \frac{1}{2} \text{Subst} \left( \int \left( \frac{x^2}{a} - \frac{bx^2}{a(b + a \cos(c + dx))} \right) dx, x, x^2 \right) \\
 &= \frac{x^6}{6a} - \frac{b \text{Subst} \left( \int \frac{x^2}{b + a \cos(c + dx)} dx, x, x^2 \right)}{2a} \\
 &= \frac{x^6}{6a} - \frac{b \text{Subst} \left( \int \frac{e^{i(c+dx)} x^2}{a + 2be^{i(c+dx)} + ae^{2i(c+dx)}} dx, x, x^2 \right)}{2a} \\
 &= \frac{x^6}{6a} - \frac{a}{2b - 2\sqrt{-a^2 + b^2} + 2ae^{i(c+dx)}} \text{Subst} \left( \int \frac{e^{i(c+dx)} x^2}{\sqrt{-a^2 + b^2}} dx, x, x^2 \right) + \frac{b \text{Subst} \left( \int \frac{e^{i(c+dx)} x^2}{2b + 2\sqrt{-a^2 + b^2} + 2ae^{i(c+dx)}} dx, x, x^2 \right)}{\sqrt{-a^2 + b^2}} \\
 &= \frac{x^6}{6a} + \frac{ibx^4 \log \left( 1 + \frac{ae^{i(c+dx^2)}}{b - \sqrt{-a^2 + b^2}} \right)}{2a\sqrt{-a^2 + b^2} d} - \frac{ibx^4 \log \left( 1 + \frac{ae^{i(c+dx^2)}}{b + \sqrt{-a^2 + b^2}} \right)}{2a\sqrt{-a^2 + b^2} d} - \frac{(ib) \text{Subst} \left( \int x \log \left( 1 + \frac{ae^{i(c+dx^2)}}{b - \sqrt{-a^2 + b^2}} \right) dx, x, x^2 \right)}{a\sqrt{-a^2 + b^2} d^2} \\
 &= \frac{x^6}{6a} + \frac{ibx^4 \log \left( 1 + \frac{ae^{i(c+dx^2)}}{b - \sqrt{-a^2 + b^2}} \right)}{2a\sqrt{-a^2 + b^2} d} - \frac{ibx^4 \log \left( 1 + \frac{ae^{i(c+dx^2)}}{b + \sqrt{-a^2 + b^2}} \right)}{2a\sqrt{-a^2 + b^2} d} + \frac{bx^2 \text{Li}_2 \left( -\frac{ae^{i(c+dx^2)}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d^2} \\
 &= \frac{x^6}{6a} + \frac{ibx^4 \log \left( 1 + \frac{ae^{i(c+dx^2)}}{b - \sqrt{-a^2 + b^2}} \right)}{2a\sqrt{-a^2 + b^2} d} - \frac{ibx^4 \log \left( 1 + \frac{ae^{i(c+dx^2)}}{b + \sqrt{-a^2 + b^2}} \right)}{2a\sqrt{-a^2 + b^2} d} + \frac{bx^2 \text{Li}_2 \left( -\frac{ae^{i(c+dx^2)}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d^2} \\
 &= \frac{x^6}{6a} + \frac{ibx^4 \log \left( 1 + \frac{ae^{i(c+dx^2)}}{b - \sqrt{-a^2 + b^2}} \right)}{2a\sqrt{-a^2 + b^2} d} - \frac{ibx^4 \log \left( 1 + \frac{ae^{i(c+dx^2)}}{b + \sqrt{-a^2 + b^2}} \right)}{2a\sqrt{-a^2 + b^2} d} + \frac{bx^2 \text{Li}_2 \left( -\frac{ae^{i(c+dx^2)}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d^2}
 \end{aligned}$$

**Mathematica [A]** time = 1.12, size = 472, normalized size = 1.24

$$d^3 x^6 \sqrt{e^{2ic} (b^2 - a^2)} + 3ibe^{ic} d^2 x^4 \log \left( 1 + \frac{ae^{i(2c+dx^2)}}{be^{ic} - \sqrt{e^{2ic} (b^2 - a^2)}} \right) - 3ibe^{ic} d^2 x^4 \log \left( 1 + \frac{ae^{i(2c+dx^2)}}{\sqrt{e^{2ic} (b^2 - a^2)} + be^{ic}} \right) + 6be^{ic} dx^2 \text{Li}_2 \left( \frac{6ad^3 \sqrt{e^{2ic}}}{\dots} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^5/(a + b\*Sec[c + d\*x^2]),x]

[Out] (d^3\*sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)]\*x^6 + (3\*I)\*b\*d^2\*E^(I\*c)\*x^4\*Log[1 + (a\*E^(I\*(2\*c + d\*x^2)))/(b\*E^(I\*c) - sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)])] - (3\*I)\*b\*d^2\*E^(I\*c)\*x^4\*Log[1 + (a\*E^(I\*(2\*c + d\*x^2)))/(b\*E^(I\*c) + sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)])] + 6\*b\*d\*E^(I\*c)\*x^2\*PolyLog[2, -((a\*E^(I\*(2\*c + d\*x^2)))/(b\*E^(I\*c) - sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)])] - 6\*b\*d\*E^(I\*c)\*x^2\*PolyLog[2, -((a\*E^(I\*(2\*c + d\*x^2)))/(b\*E^(I\*c) + sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)])] + (6\*I)\*b\*E^(I\*c)\*PolyLog[3, -((a\*E^(I\*(2\*c + d\*x^2)))/(b\*E^(I\*c) - sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)])] - (6\*I)\*b\*E^(I\*c)\*PolyLog[3, -((a\*E^(I\*(2\*c + d\*x^2)))/(b\*E^(I\*c) + sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)])] + (6\*a\*d^3\*sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)])]

**fricas [C]** time = 0.95, size = 1457, normalized size = 3.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(a+b\*sec(d\*x^2+c)),x, algorithm="fricas")

[Out] 1/24\*(4\*(a^2 - b^2)\*d^3\*x^6 - 12\*a\*b\*d\*x^2\*sqrt(-(a^2 - b^2)/a^2)\*dilog(-(b\*cos(d\*x^2 + c) + I\*b\*sin(d\*x^2 + c) + (a\*cos(d\*x^2 + c) + I\*a\*sin(d\*x^2 + c))\*sqrt(-(a^2 - b^2)/a^2) + a)/a + 1) + 12\*a\*b\*d\*x^2\*sqrt(-(a^2 - b^2)/a^2)\*dilog(-(b\*cos(d\*x^2 + c) + I\*b\*sin(d\*x^2 + c) - (a\*cos(d\*x^2 + c) + I\*a\*sin(d\*x^2 + c))\*sqrt(-(a^2 - b^2)/a^2) + a)/a + 1) - 12\*a\*b\*d\*x^2\*sqrt(-(a^2 - b^2)/a^2)\*dilog(-(b\*cos(d\*x^2 + c) - I\*b\*sin(d\*x^2 + c) + (a\*cos(d\*x^2 + c) - I\*a\*sin(d\*x^2 + c))\*sqrt(-(a^2 - b^2)/a^2) + a)/a + 1) + 12\*a\*b\*d\*x^2\*sqrt(-(a^2 - b^2)/a^2)\*dilog(-(b\*cos(d\*x^2 + c) - I\*b\*sin(d\*x^2 + c) - (a\*cos(d\*x^2 + c) - I\*a\*sin(d\*x^2 + c))\*sqrt(-(a^2 - b^2)/a^2) + a)/a + 1) + 6\*I\*a\*b\*c^2\*sqrt(-(a^2 - b^2)/a^2)\*log(2\*a\*cos(d\*x^2 + c) + 2\*I\*a\*sin(d\*x^2 + c) + 2\*a\*sqrt(-(a^2 - b^2)/a^2) + 2\*b) - 6\*I\*a\*b\*c^2\*sqrt(-(a^2 - b^2)/a^2)\*log(2\*a\*cos(d\*x^2 + c) - 2\*I\*a\*sin(d\*x^2 + c) + 2\*a\*sqrt(-(a^2 - b^2)/a^2) + 2\*b) + 6\*I\*a\*b\*c^2\*sqrt(-(a^2 - b^2)/a^2)\*log(-2\*a\*cos(d\*x^2 + c) + 2\*I\*a\*sin(d\*x^2 + c) + 2\*a\*sqrt(-(a^2 - b^2)/a^2) - 2\*b) - 6\*I\*a\*b\*c^2\*sqrt(-(a^2 - b^2)/a^2)\*log(-2\*a\*cos(d\*x^2 + c) - 2\*I\*a\*sin(d\*x^2 + c) + 2\*a\*sqrt(-(a^2 - b^2)/a^2) - 2\*b) - 12\*I\*a\*b\*sqrt(-(a^2 - b^2)/a^2)\*polylog(3, -(b\*c

```

os(d*x^2 + c) + I*b*sin(d*x^2 + c) + (a*cos(d*x^2 + c) + I*a*sin(d*x^2 + c)
)*sqrt(-(a^2 - b^2)/a^2))/a) + 12*I*a*b*sqrt(-(a^2 - b^2)/a^2)*polylog(3, -
(b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c) - (a*cos(d*x^2 + c) + I*a*sin(d*x^2
+ c))*sqrt(-(a^2 - b^2)/a^2))/a) + 12*I*a*b*sqrt(-(a^2 - b^2)/a^2)*polylog(
3, -(b*cos(d*x^2 + c) - I*b*sin(d*x^2 + c) + (a*cos(d*x^2 + c) - I*a*sin(d*
x^2 + c))*sqrt(-(a^2 - b^2)/a^2))/a) - 12*I*a*b*sqrt(-(a^2 - b^2)/a^2)*poly
log(3, -(b*cos(d*x^2 + c) - I*b*sin(d*x^2 + c) - (a*cos(d*x^2 + c) - I*a*si
n(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2))/a) - 2*(3*I*a*b*d^2*x^4 - 3*I*a*b*c^2
)*sqrt(-(a^2 - b^2)/a^2)*log((b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c) + (a*co
s(d*x^2 + c) + I*a*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2) + a)/a) - 2*(-3*I
*a*b*d^2*x^4 + 3*I*a*b*c^2)*sqrt(-(a^2 - b^2)/a^2)*log((b*cos(d*x^2 + c) +
I*b*sin(d*x^2 + c) - (a*cos(d*x^2 + c) + I*a*sin(d*x^2 + c))*sqrt(-(a^2 - b
^2)/a^2) + a)/a) - 2*(-3*I*a*b*d^2*x^4 + 3*I*a*b*c^2)*sqrt(-(a^2 - b^2)/a^2
)*log((b*cos(d*x^2 + c) - I*b*sin(d*x^2 + c) + (a*cos(d*x^2 + c) - I*a*sin(
d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2) + a)/a) - 2*(3*I*a*b*d^2*x^4 - 3*I*a*b*c
^2)*sqrt(-(a^2 - b^2)/a^2)*log((b*cos(d*x^2 + c) - I*b*sin(d*x^2 + c) - (a*
cos(d*x^2 + c) - I*a*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2) + a)/a))/((a^3
- a*b^2)*d^3)

```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{b \sec(dx^2 + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b*sec(d*x^2+c)),x, algorithm="giac")
```

```
[Out] integrate(x^5/(b*sec(d*x^2 + c) + a), x)
```

**maple** [F] time = 0.85, size = 0, normalized size = 0.00

$$\int \frac{x^5}{a + b \sec(dx^2 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a+b*sec(d*x^2+c)),x)
```

```
[Out] int(x^5/(a+b*sec(d*x^2+c)),x)
```

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(a+b\*sec(d\*x^2+c)),x, algorithm="maxima")

[Out] Timed out

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5}{a + \frac{b}{\cos(dx^2+c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(a + b/cos(c + d\*x^2)),x)

[Out] int(x^5/(a + b/cos(c + d\*x^2)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{a + b \sec(c + dx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*5/(a+b\*sec(d\*x\*\*2+c)),x)

[Out] Integral(x\*\*5/(a + b\*sec(c + d\*x\*\*2)), x)

$$3.17 \quad \int \frac{x^4}{a+b \sec(c+dx^2)} dx$$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{x^4}{a+b \sec(c+dx^2)}, x\right)$$

[Out] Unintegrable(x^4/(a+b\*sec(d\*x^2+c)), x)

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{x^4}{a+b \sec(c+dx^2)} dx$$

Verification is Not applicable to the result.

[In] Int[x^4/(a + b\*Sec[c + d\*x^2]), x]

[Out] Defer[Int][x^4/(a + b\*Sec[c + d\*x^2]), x]

Rubi steps

$$\int \frac{x^4}{a+b \sec(c+dx^2)} dx = \int \frac{x^4}{a+b \sec(c+dx^2)} dx$$

Mathematica [A] time = 1.40, size = 0, normalized size = 0.00

$$\int \frac{x^4}{a+b \sec(c+dx^2)} dx$$

Verification is Not applicable to the result.

[In] Integrate[x^4/(a + b\*Sec[c + d\*x^2]), x]

[Out] Integrate[x^4/(a + b\*Sec[c + d\*x^2]), x]

fricas [A] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^4}{b \sec(dx^2 + c) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(a+b*sec(d*x^2+c)),x, algorithm="fricas")`

[Out] `integral(x^4/(b*sec(d*x^2 + c) + a), x)`

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{b \sec(dx^2 + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(a+b*sec(d*x^2+c)),x, algorithm="giac")`

[Out] `integrate(x^4/(b*sec(d*x^2 + c) + a), x)`

**maple** [A] time = 0.80, size = 0, normalized size = 0.00

$$\int \frac{x^4}{a + b \sec(dx^2 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4/(a+b*sec(d*x^2+c)),x)`

[Out] `int(x^4/(a+b*sec(d*x^2+c)),x)`

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(a+b*sec(d*x^2+c)),x, algorithm="maxima")`

[Out] Timed out

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{x^4}{a + \frac{b}{\cos(dx^2+c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4/(a + b/cos(c + d*x^2)),x)`

```
[Out] int(x^4/(a + b/cos(c + d*x^2)), x)
```

```
sympy [A] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{x^4}{a + b \sec(c + dx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(a+b*sec(d*x**2+c)),x)
```

```
[Out] Integral(x**4/(a + b*sec(c + d*x**2)), x)
```



$$3.18 \quad \int \frac{x^3}{a+b \sec(c+dx^2)} dx$$

**Optimal.** Leaf size=261

$$\frac{b \operatorname{Li}_2\left(-\frac{ae^{i(dx^2+c)}}{b-\sqrt{b^2-a^2}}\right)}{2ad^2\sqrt{b^2-a^2}} - \frac{b \operatorname{Li}_2\left(-\frac{ae^{i(dx^2+c)}}{b+\sqrt{b^2-a^2}}\right)}{2ad^2\sqrt{b^2-a^2}} + \frac{ibx^2 \log\left(1 + \frac{ae^{i(c+dx^2)}}{b-\sqrt{b^2-a^2}}\right)}{2ad\sqrt{b^2-a^2}} - \frac{ibx^2 \log\left(1 + \frac{ae^{i(c+dx^2)}}{\sqrt{b^2-a^2}+b}\right)}{2ad\sqrt{b^2-a^2}} + \frac{x^4}{4a}$$

[Out]  $1/4*x^4/a+1/2*I*b*x^2*\ln(1+a*\exp(I*(d*x^2+c)))/(b-(-a^2+b^2)^(1/2))/a/d/(-a^2+b^2)^(1/2)-1/2*I*b*x^2*\ln(1+a*\exp(I*(d*x^2+c)))/(b+(-a^2+b^2)^(1/2))/a/d/(-a^2+b^2)^(1/2)+1/2*b*polylog(2,-a*\exp(I*(d*x^2+c)))/(b-(-a^2+b^2)^(1/2))/a/d^2/(-a^2+b^2)^(1/2)-1/2*b*polylog(2,-a*\exp(I*(d*x^2+c)))/(b+(-a^2+b^2)^(1/2))/a/d^2/(-a^2+b^2)^(1/2)$

**Rubi [A]** time = 0.54, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$ , Rules used = {4204, 4191, 3321, 2264, 2190, 2279, 2391}

$$\frac{b \operatorname{PolyLog}\left(2, -\frac{ae^{i(c+dx^2)}}{b-\sqrt{b^2-a^2}}\right)}{2ad^2\sqrt{b^2-a^2}} - \frac{b \operatorname{PolyLog}\left(2, -\frac{ae^{i(c+dx^2)}}{\sqrt{b^2-a^2}+b}\right)}{2ad^2\sqrt{b^2-a^2}} + \frac{ibx^2 \log\left(1 + \frac{ae^{i(c+dx^2)}}{b-\sqrt{b^2-a^2}}\right)}{2ad\sqrt{b^2-a^2}} - \frac{ibx^2 \log\left(1 + \frac{ae^{i(c+dx^2)}}{\sqrt{b^2-a^2}+b}\right)}{2ad\sqrt{b^2-a^2}} + \frac{x^4}{4a}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^3/(a + b*\operatorname{Sec}[c + d*x^2]), x]$

[Out]  $x^4/(4*a) + ((I/2)*b*x^2*\operatorname{Log}[1 + (a*E^{(I*(c + d*x^2))})/(b - \operatorname{Sqrt}[-a^2 + b^2])])/(a*\operatorname{Sqrt}[-a^2 + b^2]*d) - ((I/2)*b*x^2*\operatorname{Log}[1 + (a*E^{(I*(c + d*x^2))})/(b + \operatorname{Sqrt}[-a^2 + b^2])])/(a*\operatorname{Sqrt}[-a^2 + b^2]*d) + (b*\operatorname{PolyLog}[2, -((a*E^{(I*(c + d*x^2))})/(b - \operatorname{Sqrt}[-a^2 + b^2]))])/(2*a*\operatorname{Sqrt}[-a^2 + b^2]*d^2) - (b*\operatorname{PolyLog}[2, -((a*E^{(I*(c + d*x^2))})/(b + \operatorname{Sqrt}[-a^2 + b^2]))])/(2*a*\operatorname{Sqrt}[-a^2 + b^2]*d^2)$

**Rule 2190**

$\operatorname{Int}[(((F_)^\wedge((g_)*(e_)+(f_)*(x_)))^\wedge(n_))*((c_)+(d_)*(x_))^\wedge(m_)]/((a_)+(b_)*((F_)^\wedge((g_)*(e_)+(f_)*(x_)))^\wedge(n_)), x\_Symbol] \rightarrow \operatorname{Simp}[(c+d*x)^\wedge m*\operatorname{Log}[1+(b*(F^\wedge(g*(e+f*x)))^\wedge n)/a]/(b*f*g*n*\operatorname{Log}[F]), x] - \operatorname{Dist}[(d*m)/(b*f*g*n*\operatorname{Log}[F]), \operatorname{Int}[(c+d*x)^\wedge(m-1)*\operatorname{Log}[1+(b*(F^\wedge(g*(e+f*x)))^\wedge n)/a], x], x] /; \operatorname{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x] \&\& \operatorname{IGtQ}[m, 0]$

**Rule 2264**

$\operatorname{Int}[(F_)^\wedge(u_)*((f_)+(g_)*(x_))^\wedge(m_)]/((a_)+(b_)*(F_)^\wedge(u_)+(c_)*(F_)^\wedge(v_)), x\_Symbol] \rightarrow \operatorname{With}\{q = \operatorname{Rt}[b^2 - 4*a*c, 2]\}, \operatorname{Dist}[(2*c)/q, \operatorname{Int}[\dots]]$

$((f + gx)^m F^u) / (b - q + 2c F^u), x], x] - \text{Dist}[(2c)/q, \text{Int}[(f + gx)^m F^u / (b + q + 2c F^u), x], x] /;$  FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2\*u] && LinearQ[u, x] && NeQ[b^2 - 4\*a\*c, 0] && IGtQ[m, 0]

### Rule 2279

$\text{Int}[\text{Log}[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^{(n_)}], x\_Symbol]$   
 $:= \text{Dist}[1/(d*e*n*\text{Log}[F]), \text{Subst}[\text{Int}[\text{Log}[a + b*x]/x, x], x, (F^{(e*(c + d*x))})^n], x] /;$  FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

### Rule 2391

$\text{Int}[\text{Log}[(c_)*((d_) + (e_)*(x_)^{(n_)})]/(x_), x\_Symbol] := -\text{Simp}[\text{PolyLog}[2, -(c*e*x^n)]/n, x] /;$  FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rule 3321

$\text{Int}[(c_ + (d_)*(x_))^{(m_)} / ((a_ + (b_)*\sin[(e_ + \text{Pi}*(k_ + (f_)*(x_))]), x\_Symbol] := \text{Dist}[2, \text{Int}[(c + d*x)^m E^{(I*\text{Pi}*(k - 1/2))} E^{(I*(e + f*x))} / (b + 2*a*E^{(I*\text{Pi}*(k - 1/2))} E^{(I*(e + f*x))} - b*E^{(2*I*k*\text{Pi})} E^{(2*I*(e + f*x))}), x], x] /;$  FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2\*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]

### Rule 4191

$\text{Int}[(\text{csc}[(e_ + (f_)*(x_)]*(b_ + (a_))^{(n_)})*((c_ + (d_)*(x_))^{(m_)}), x\_Symbol] := \text{Int}[\text{ExpandIntegrand}[(c + d*x)^m, 1/(\text{Sin}[e + f*x]^n / (b + a*\text{Sin}[e + f*x]^n)), x], x] /;$  FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGtQ[m, 0]

### Rule 4204

$\text{Int}[(x_)^{(m_)}*((a_ + (b_)*\text{Sec}[(c_ + (d_)*(x_))^{(n_)}])^{(p_)}), x\_Symbol] := \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)*(a + b*\text{Sec}[c + d*x])^p}, x], x, x^n], x] /;$  FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]

### Rubi steps

$$\begin{aligned}
\int \frac{x^3}{a + b \sec(c + dx^2)} dx &= \frac{1}{2} \text{Subst} \left( \int \frac{x}{a + b \sec(c + dx)} dx, x, x^2 \right) \\
&= \frac{1}{2} \text{Subst} \left( \int \left( \frac{x}{a} - \frac{bx}{a(b + a \cos(c + dx))} \right) dx, x, x^2 \right) \\
&= \frac{x^4}{4a} - \frac{b \text{Subst} \left( \int \frac{x}{b + a \cos(c + dx)} dx, x, x^2 \right)}{2a} \\
&= \frac{x^4}{4a} - \frac{b \text{Subst} \left( \int \frac{e^{i(c+dx)} x}{a + 2be^{i(c+dx)} + ae^{2i(c+dx)}} dx, x, x^2 \right)}{4a} \\
&= \frac{x^4}{4a} - \frac{b \text{Subst} \left( \int \frac{e^{i(c+dx)} x}{2b - 2\sqrt{-a^2 + b^2} + 2ae^{i(c+dx)}} dx, x, x^2 \right)}{4a} + \frac{b \text{Subst} \left( \int \frac{e^{i(c+dx)} x}{2b + 2\sqrt{-a^2 + b^2} + 2ae^{i(c+dx)}} dx, x, x^2 \right)}{4a} \\
&= \frac{x^4}{4a} + \frac{ibx^2 \log \left( 1 + \frac{ae^{i(c+dx^2)}}{b - \sqrt{-a^2 + b^2}} \right)}{2a\sqrt{-a^2 + b^2} d} - \frac{ibx^2 \log \left( 1 + \frac{ae^{i(c+dx^2)}}{b + \sqrt{-a^2 + b^2}} \right)}{2a\sqrt{-a^2 + b^2} d} - \frac{(ib) \text{Subst} \left( \int \log \left( 1 + \frac{ae^{i(c+dx^2)}}{b - \sqrt{-a^2 + b^2}} \right) dx, x, x^2 \right)}{2a\sqrt{-a^2 + b^2}} \\
&= \frac{x^4}{4a} + \frac{ibx^2 \log \left( 1 + \frac{ae^{i(c+dx^2)}}{b - \sqrt{-a^2 + b^2}} \right)}{2a\sqrt{-a^2 + b^2} d} - \frac{ibx^2 \log \left( 1 + \frac{ae^{i(c+dx^2)}}{b + \sqrt{-a^2 + b^2}} \right)}{2a\sqrt{-a^2 + b^2} d} - \frac{b \text{Subst} \left( \int \frac{\log \left( 1 + \frac{ae^{i(c+dx^2)}}{b - \sqrt{-a^2 + b^2}} \right)}{x} dx, x, x^2 \right)}{2a\sqrt{-a^2 + b^2}} \\
&= \frac{x^4}{4a} + \frac{ibx^2 \log \left( 1 + \frac{ae^{i(c+dx^2)}}{b - \sqrt{-a^2 + b^2}} \right)}{2a\sqrt{-a^2 + b^2} d} - \frac{ibx^2 \log \left( 1 + \frac{ae^{i(c+dx^2)}}{b + \sqrt{-a^2 + b^2}} \right)}{2a\sqrt{-a^2 + b^2} d} + \frac{b \text{Li}_2 \left( -\frac{ae^{i(c+dx^2)}}{b - \sqrt{-a^2 + b^2}} \right)}{2a\sqrt{-a^2 + b^2} d^2} - \frac{b \text{Li}_2 \left( -\frac{ae^{i(c+dx^2)}}{b + \sqrt{-a^2 + b^2}} \right)}{2a\sqrt{-a^2 + b^2} d^2}
\end{aligned}$$

**Mathematica [B]** time = 1.40, size = 845, normalized size = 3.24

$$(b + a \cos(dx^2 + c)) \left( x^4 - \frac{2b \left( 2(dx^2 + c) \tanh^{-1} \left( \frac{(a+b) \cot\left(\frac{1}{2}(dx^2 + c)\right)}{\sqrt{a^2 - b^2}} \right) - 2 \left( c + \cos^{-1}\left(-\frac{b}{a}\right) \right) \tanh^{-1} \left( \frac{(a-b) \tan\left(\frac{1}{2}(dx^2 + c)\right)}{\sqrt{a^2 - b^2}} \right) \right) + \left( \cos^{-1}\left(-\frac{b}{a}\right) - 2i \tanh^{-1} \left( \frac{ae^{i(c+dx^2)}}{b - \sqrt{-a^2 + b^2}} \right) \right) \right)}{2a\sqrt{-a^2 + b^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(a + b\*Sec[c + d\*x^2]), x]

[Out] ((b + a\*Cos[c + d\*x^2])\*(x^4 - (2\*b\*(2\*(c + d\*x^2)\*ArcTanh[((a + b)\*Cot[(c + d\*x^2)/2]])/Sqrt[a^2 - b^2]] - 2\*(c + ArcCos[-(b/a)])\*ArcTanh[((a - b)\*Tan[(c + d\*x^2)/2]])/Sqrt[a^2 - b^2]] + (ArcCos[-(b/a)] - (2\*I)\*ArcTanh[((a + b

$$\begin{aligned} & ) * \text{Cot}[(c + d*x^2)/2] / \text{Sqrt}[a^2 - b^2]] + (2*I) * \text{ArcTanh}[\frac{(a - b) * \text{Tan}[(c + d*x^2)/2]}{\text{Sqrt}[a^2 - b^2}}] / \text{Sqrt}[a^2 - b^2]] * \text{Log}[\frac{\text{Sqrt}[a^2 - b^2]}{(\text{Sqrt}[2] * \text{Sqrt}[a] * E^{(I/2) * (c + d*x^2)}) * \text{Sqrt}[b + a * \text{Cos}[c + d*x^2]]}] + (\text{ArcCos}[-(b/a)] + (2*I) * (\text{ArcTanh}[\frac{(a + b) * \text{Cot}[(c + d*x^2)/2]}{\text{Sqrt}[a^2 - b^2}}] - \text{ArcTanh}[\frac{(a - b) * \text{Tan}[(c + d*x^2)/2]}{\text{Sqrt}[a^2 - b^2}}]) * \text{Log}[\frac{(\text{Sqrt}[a^2 - b^2] * E^{(I/2) * (c + d*x^2)})}{(\text{Sqrt}[2] * \text{Sqrt}[a] * \text{Sqrt}[b + a * \text{Cos}[c + d*x^2]])}] - (\text{ArcCos}[-(b/a)] - (2*I) * \text{ArcTanh}[\frac{(a - b) * \text{Tan}[(c + d*x^2)/2]}{\text{Sqrt}[a^2 - b^2}}]) * \text{Log}[\frac{(a + b) * (a - b - I * \text{Sqrt}[a^2 - b^2]) * (1 + I * \text{Tan}[(c + d*x^2)/2])}{(a * (a + b + \text{Sqrt}[a^2 - b^2]) * \text{Tan}[(c + d*x^2)/2])}] - (\text{ArcCos}[-(b/a)] + (2*I) * \text{ArcTanh}[\frac{(a - b) * \text{Tan}[(c + d*x^2)/2]}{\text{Sqrt}[a^2 - b^2}}]) * \text{Log}[\frac{(a + b) * ((-I) * a + I * b + \text{Sqrt}[a^2 - b^2]) * (I + \text{Tan}[(c + d*x^2)/2])}{(a * (a + b + \text{Sqrt}[a^2 - b^2]) * \text{Tan}[(c + d*x^2)/2])}] + I * (\text{PolyLog}[2, ((b - I * \text{Sqrt}[a^2 - b^2]) * (a + b - \text{Sqrt}[a^2 - b^2]) * \text{Tan}[(c + d*x^2)/2])}{(a * (a + b + \text{Sqrt}[a^2 - b^2]) * \text{Tan}[(c + d*x^2)/2])}] - \text{PolyLog}[2, ((b + I * \text{Sqrt}[a^2 - b^2]) * (a + b - \text{Sqrt}[a^2 - b^2]) * \text{Tan}[(c + d*x^2)/2])}{(a * (a + b + \text{Sqrt}[a^2 - b^2]) * \text{Tan}[(c + d*x^2)/2])}])]) / (\text{Sqrt}[a^2 - b^2] * d^2) * \text{Sec}[c + d*x^2] / (4 * a * (a + b * \text{Sec}[c + d*x^2])) \end{aligned}$$

**fricas [B]** time = 1.10, size = 1063, normalized size = 4.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+b\*sec(d\*x^2+c)),x, algorithm="fricas")

[Out]  $\frac{1}{8} * (2 * (a^2 - b^2) * d^2 * x^4 - 2 * I * a * b * c * \text{sqrt}(-(a^2 - b^2)/a^2) * \log(2 * a * \text{cos}(d * x^2 + c) + 2 * I * a * \text{sin}(d * x^2 + c) + 2 * a * \text{sqrt}(-(a^2 - b^2)/a^2) + 2 * b) + 2 * I * a * b * c * \text{sqrt}(-(a^2 - b^2)/a^2) * \log(2 * a * \text{cos}(d * x^2 + c) - 2 * I * a * \text{sin}(d * x^2 + c) + 2 * a * \text{sqrt}(-(a^2 - b^2)/a^2) + 2 * b) - 2 * I * a * b * c * \text{sqrt}(-(a^2 - b^2)/a^2) * \log(-2 * a * \text{cos}(d * x^2 + c) + 2 * I * a * \text{sin}(d * x^2 + c) + 2 * a * \text{sqrt}(-(a^2 - b^2)/a^2) - 2 * b) + 2 * I * a * b * c * \text{sqrt}(-(a^2 - b^2)/a^2) * \log(-2 * a * \text{cos}(d * x^2 + c) - 2 * I * a * \text{sin}(d * x^2 + c) + 2 * a * \text{sqrt}(-(a^2 - b^2)/a^2) - 2 * b) - 2 * a * b * \text{sqrt}(-(a^2 - b^2)/a^2) * \text{dilog}(-(b * \text{cos}(d * x^2 + c) + I * b * \text{sin}(d * x^2 + c) + (a * \text{cos}(d * x^2 + c) + I * a * \text{sin}(d * x^2 + c)) * \text{sqrt}(-(a^2 - b^2)/a^2) + a) / a + 1) + 2 * a * b * \text{sqrt}(-(a^2 - b^2)/a^2) * \text{dilog}(-(b * \text{cos}(d * x^2 + c) + I * b * \text{sin}(d * x^2 + c) - (a * \text{cos}(d * x^2 + c) + I * a * \text{sin}(d * x^2 + c)) * \text{sqrt}(-(a^2 - b^2)/a^2) + a) / a + 1) - 2 * a * b * \text{sqrt}(-(a^2 - b^2)/a^2) * \text{dilog}(-(b * \text{cos}(d * x^2 + c) - I * b * \text{sin}(d * x^2 + c) + (a * \text{cos}(d * x^2 + c) - I * a * \text{sin}(d * x^2 + c)) * \text{sqrt}(-(a^2 - b^2)/a^2) + a) / a + 1) + 2 * a * b * \text{sqrt}(-(a^2 - b^2)/a^2) * \text{dilog}(-(b * \text{cos}(d * x^2 + c) - I * b * \text{sin}(d * x^2 + c) - (a * \text{cos}(d * x^2 + c) - I * a * \text{sin}(d * x^2 + c)) * \text{sqrt}(-(a^2 - b^2)/a^2) + a) / a + 1) - 2 * (I * a * b * d * x^2 + I * a * b * c) * \text{sqrt}(-(a^2 - b^2)/a^2) * \log((b * \text{cos}(d * x^2 + c) + I * b * \text{sin}(d * x^2 + c) + (a * \text{cos}(d * x^2 + c) + I * a * \text{sin}(d * x^2 + c)) * \text{sqrt}(-(a^2 - b^2)/a^2) + a) / a) - 2 * (-I * a * b * d * x^2 - I * a * b * c) * \text{sqrt}(-(a^2 - b^2)/a^2) * \log((b * \text{cos}(d * x^2 + c) + I * b * \text{sin}(d * x^2 + c) - (a * \text{cos}(d * x^2 + c) + I * a * \text{sin}(d * x^2 + c)) * \text{sqrt}(-(a^2 - b^2)/a^2) + a) / a) - 2 * (-I * a * b * d * x^2 - I * a * b * c) * \text{sqrt}(-(a^2 - b^2)/a^2) * \log((b * \text{cos}(d * x^2 + c) - I * b * \text{sin}(d * x^2 + c) + (a * \text{cos}(d * x^2 + c) - I * a * \text{sin}(d * x^2 + c)) * \text{sqrt}(-(a^2 - b^2)/a^2) + a) / a) - 2 * (-I * a * b * d * x^2 - I * a * b * c) * \text{sqrt}(-(a^2 - b^2)/a^2) * \log((b * \text{cos}(d * x^2 + c) - I * b * \text{sin}(d * x^2 + c) + (a * \text{cos}(d * x^2 + c) - I * a * \text{sin}(d * x^2 + c)) * \text{sqrt}(-(a^2 - b^2)/a^2) + a) / a)$

$x^2 + c))\sqrt{-(a^2 - b^2)/a^2 + a}/a - 2*(I*a*b*d*x^2 + I*a*b*c)\sqrt{-(a^2 - b^2)/a^2}\log((b*\cos(d*x^2 + c) - I*b*\sin(d*x^2 + c) - (a*\cos(d*x^2 + c) - I*a*\sin(d*x^2 + c))\sqrt{-(a^2 - b^2)/a^2 + a}/a))/((a^3 - a*b^2)*d^2)$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{b \sec(dx^2 + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+b\*sec(d\*x^2+c)),x, algorithm="giac")

[Out] integrate(x^3/(b\*sec(d\*x^2 + c) + a), x)

**maple** [F] time = 0.82, size = 0, normalized size = 0.00

$$\int \frac{x^3}{a + b \sec(dx^2 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(a+b\*sec(d\*x^2+c)),x)

[Out] int(x^3/(a+b\*sec(d\*x^2+c)),x)

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+b\*sec(d\*x^2+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*b^2-4\*a^2>0)', see `assume?` for more details)Is 4\*b^2-4\*a^2 positive or negative?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{a + \frac{b}{\cos(dx^2+c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b/cos(c + d*x^2)),x)
```

```
[Out] int(x^3/(a + b/cos(c + d*x^2)), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{a + b \sec(c + dx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*sec(d*x**2+c)),x)
```

```
[Out] Integral(x**3/(a + b*sec(c + d*x**2)), x)
```

$$3.19 \quad \int \frac{x^2}{a+b \sec(c+dx^2)} dx$$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{x^2}{a+b \sec(c+dx^2)}, x\right)$$

[Out] Unintegrable(x^2/(a+b\*sec(d\*x^2+c)), x)

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{x^2}{a+b \sec(c+dx^2)} dx$$

Verification is Not applicable to the result.

[In] Int[x^2/(a + b\*Sec[c + d\*x^2]), x]

[Out] Defer[Int][x^2/(a + b\*Sec[c + d\*x^2]), x]

Rubi steps

$$\int \frac{x^2}{a+b \sec(c+dx^2)} dx = \int \frac{x^2}{a+b \sec(c+dx^2)} dx$$

Mathematica [A] time = 1.17, size = 0, normalized size = 0.00

$$\int \frac{x^2}{a+b \sec(c+dx^2)} dx$$

Verification is Not applicable to the result.

[In] Integrate[x^2/(a + b\*Sec[c + d\*x^2]), x]

[Out] Integrate[x^2/(a + b\*Sec[c + d\*x^2]), x]

fricas [A] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^2}{b \sec(dx^2+c)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b\*sec(d\*x^2+c)),x, algorithm="fricas")

[Out] integral(x^2/(b\*sec(d\*x^2 + c) + a), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{b \sec(dx^2 + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b\*sec(d\*x^2+c)),x, algorithm="giac")

[Out] integrate(x^2/(b\*sec(d\*x^2 + c) + a), x)

maple [A] time = 0.83, size = 0, normalized size = 0.00

$$\int \frac{x^2}{a + b \sec(dx^2 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a+b\*sec(d\*x^2+c)),x)

[Out] int(x^2/(a+b\*sec(d\*x^2+c)),x)

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b\*sec(d\*x^2+c)),x, algorithm="maxima")

[Out] Timed out

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{x^2}{a + \frac{b}{\cos(dx^2+c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a + b/cos(c + d\*x^2)),x)



```
[Out] int(x^2/(a + b/cos(c + d*x^2)), x)
```

```
sympy [A] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{x^2}{a + b \sec(c + dx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*sec(d*x**2+c)),x)
```

```
[Out] Integral(x**2/(a + b*sec(c + d*x**2)), x)
```

$$3.20 \quad \int \frac{x}{a+b \sec(c+dx^2)} dx$$

**Optimal.** Leaf size=66

$$\frac{x^2}{2a} - \frac{b \tanh^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx^2)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}$$

[Out]  $1/2*x^2/a-b*\arctanh((a-b)^{(1/2)}*\tan(1/2*d*x^2+1/2*c)/(a+b)^{(1/2)})/a/d/(a-b)^{(1/2)}/(a+b)^{(1/2)}$

**Rubi [A]** time = 0.11, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {4204, 3783, 2659, 208}

$$\frac{x^2}{2a} - \frac{b \tanh^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx^2)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}$$

Antiderivative was successfully verified.

[In] Int[x/(a + b\*Sec[c + d\*x^2]),x]

[Out]  $x^2/(2*a) - (b*\text{ArcTanh}[(\text{Sqrt}[a - b]*\text{Tan}[(c + d*x^2)/2])/(\text{Sqrt}[a + b])]/(a*\text{Sqrt}[a - b]*\text{Sqrt}[a + b]*d)$

### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

### Rule 2659

Int[((a\_) + (b\_.)\*sin[Pi/2 + (c\_.) + (d\_.)\*(x\_)])^(-1), x\_Symbol] := With[{e = FreeFactors[Tan[(c + d\*x)/2], x]}, Dist[(2\*e)/d, Subst[Int[1/(a + b + (a - b)\*e^2\*x^2), x], x, Tan[(c + d\*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

### Rule 3783

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.) + (a\_))(-1), x\_Symbol] := Simp[x/a, x] - Dist[1/a, Int[1/(1 + (a\*Sin[c + d\*x])/b), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 4204

Int[(x\_)^(m\_.)\*((a\_.) + (b\_.)\*Sec[(c\_.) + (d\_.)\*(x\_)^(n\_)])^(p\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*Sec[c + d\*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \frac{x}{a + b \sec(c + dx^2)} dx &= \frac{1}{2} \text{Subst} \left( \int \frac{1}{a + b \sec(c + dx)} dx, x, x^2 \right) \\ &= \frac{x^2}{2a} - \frac{\text{Subst} \left( \int \frac{1}{1 + \frac{a \cos(c+dx)}{b}} dx, x, x^2 \right)}{2a} \\ &= \frac{x^2}{2a} - \frac{\text{Subst} \left( \int \frac{1}{1 + \frac{a}{b} + (1 - \frac{a}{b})x^2} dx, x, \tan \left( \frac{1}{2} (c + dx^2) \right) \right)}{ad} \\ &= \frac{x^2}{2a} - \frac{b \tanh^{-1} \left( \frac{\sqrt{a-b} \tan \left( \frac{1}{2} (c + dx^2) \right)}{\sqrt{a+b}} \right)}{a\sqrt{a-b} \sqrt{a+b} d} \end{aligned}$$

**Mathematica [A]** time = 0.18, size = 67, normalized size = 1.02

$$\frac{2b \tanh^{-1} \left( \frac{(b-a) \tan \left( \frac{1}{2} (c + dx^2) \right)}{\sqrt{a^2 - b^2}} \right)}{d\sqrt{a^2 - b^2}} + \frac{c}{d} + x^2}{2a}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a + b\*Sec[c + d\*x^2]), x]

[Out] (c/d + x^2 + (2\*b\*ArcTanh[(-a + b)\*Tan[(c + d\*x^2)/2]]/Sqrt[a^2 - b^2]))/(Sqrt[a^2 - b^2]\*d)/(2\*a)

**fricas [A]** time = 0.75, size = 251, normalized size = 3.80

$$\left[ \frac{2(a^2 - b^2)dx^2 + \sqrt{a^2 - b^2} b \log \left( \frac{2ab \cos(dx^2+c) - (a^2 - 2b^2) \cos(dx^2+c)^2 - 2\sqrt{a^2 - b^2} (b \cos(dx^2+c) + a) \sin(dx^2+c) + 2a^2 - b^2}{a^2 \cos(dx^2+c)^2 + 2ab \cos(dx^2+c) + b^2} \right)}{4(a^3 - ab^2)d} \right], (a^2 - b^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*sec(d\*x^2+c)),x, algorithm="fricas")

[Out] [1/4\*(2\*(a^2 - b^2)\*d\*x^2 + sqrt(a^2 - b^2)\*b\*log((2\*a\*b\*cos(d\*x^2 + c) - (a^2 - 2\*b^2)\*cos(d\*x^2 + c)^2 - 2\*sqrt(a^2 - b^2)\*(b\*cos(d\*x^2 + c) + a)\*sin(d\*x^2 + c) + 2\*a^2 - b^2)/(a^2\*cos(d\*x^2 + c)^2 + 2\*a\*b\*cos(d\*x^2 + c) + b^2)))/((a^3 - a\*b^2)\*d), 1/2\*((a^2 - b^2)\*d\*x^2 - sqrt(-a^2 + b^2)\*b\*arctan(-sqrt(-a^2 + b^2)\*(b\*cos(d\*x^2 + c) + a)/((a^2 - b^2)\*sin(d\*x^2 + c)))/((a^3 - a\*b^2)\*d)]

**giac** [B] time = 0.50, size = 278, normalized size = 4.21

$$\frac{\left(\sqrt{-a^2 + b^2} (a - 2b)d|-a + b| - \sqrt{-a^2 + b^2} |a||-a + b||d\right) \left( \pi \left[ \frac{dx^2+c}{2\pi} + \frac{1}{2} \right] + \arctan \left( \frac{\tan\left(\frac{1}{2} dx^2 + \frac{1}{2} c\right)}{\sqrt{\frac{bd + \sqrt{b^2 d^2 + (ad+bd)(ad-bd)}}{ad-bd}}} \right) \right)}{2 \left( (a^2 - 2ab + b^2) a^2 d^2 + (a^2 b - 2ab^2 + b^3) d|a||d| \right)} +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*sec(d\*x^2+c)),x, algorithm="giac")

[Out] 1/2\*(sqrt(-a^2 + b^2)\*(a - 2\*b)\*d\*abs(-a + b) - sqrt(-a^2 + b^2)\*abs(a)\*abs(-a + b)\*abs(d))\*(pi\*floor(1/2\*(d\*x^2 + c)/pi + 1/2) + arctan(tan(1/2\*d\*x^2 + 1/2\*c)/sqrt(-(b\*d + sqrt(b^2\*d^2 + (a\*d + b\*d)\*(a\*d - b\*d)))/(a\*d - b\*d))))/((a^2 - 2\*a\*b + b^2)\*a^2\*d^2 + (a^2\*b - 2\*a\*b^2 + b^3)\*d\*abs(a)\*abs(d) + 1/2\*(a\*d - 2\*b\*d + abs(a)\*abs(d))\*(pi\*floor(1/2\*(d\*x^2 + c)/pi + 1/2) + arctan(tan(1/2\*d\*x^2 + 1/2\*c)/sqrt(-(b\*d - sqrt(b^2\*d^2 + (a\*d + b\*d)\*(a\*d - b\*d)))/(a\*d - b\*d))))/((a^2\*d^2 - b\*d\*abs(a)\*abs(d)))

**maple** [A] time = 0.54, size = 70, normalized size = 1.06

$$-\frac{b \operatorname{arctanh} \left( \frac{(a-b) \tan \left( \frac{dx^2}{2} + \frac{c}{2} \right)}{\sqrt{(a-b)(a+b)}} \right)}{da \sqrt{(a-b)(a+b)}} + \frac{\operatorname{arctan} \left( \tan \left( \frac{dx^2}{2} + \frac{c}{2} \right) \right)}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a+b\*sec(d\*x^2+c)),x)

[Out] -1/d/a\*b/((a-b)\*(a+b))^(1/2)\*arctanh((a-b)\*tan(1/2\*d\*x^2+1/2\*c)/((a-b)\*(a+b)))^(1/2))+1/d/a\*arctan(tan(1/2\*d\*x^2+1/2\*c))

**maxima** [B] time = 63.90, size = 7945, normalized size = 120.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*sec(d\*x^2+c)),x, algorithm="maxima")

[Out]  $\frac{1}{2} \cdot (\sqrt{-a^2 + b^2} \cdot d \cdot x^2 - b \cdot \arctan(2 \cdot (4 \cdot (a^6 - a^4 \cdot b^2) \cdot \cos(d \cdot x^2 + 2 \cdot c))^4 \cdot \cos(c) \cdot \sin(c) - 4 \cdot (a^6 - a^4 \cdot b^2) \cdot \cos(c) \cdot \sin(d \cdot x^2 + 2 \cdot c))^4 \cdot \sin(c) + 4 \cdot (3 \cdot (a^5 \cdot b - a^3 \cdot b^3) \cdot \cos(c))^2 \cdot \sin(c) + (a^5 \cdot b - a^3 \cdot b^3) \cdot \sin(c)^3) \cdot \cos(d \cdot x^2 + 2 \cdot c)^3 - 4 \cdot ((a^5 \cdot b - a^3 \cdot b^3) \cdot \cos(c))^3 + 3 \cdot (a^5 \cdot b - a^3 \cdot b^3) \cdot \cos(c) \cdot \sin(c)^2 + ((a^6 - a^4 \cdot b^2) \cdot \cos(c))^2 - (a^6 - a^4 \cdot b^2) \cdot \sin(c)^2) \cdot \cos(d \cdot x^2 + 2 \cdot c)) \cdot \sin(d \cdot x^2 + 2 \cdot c)^3 - 4 \cdot ((a^6 - 5 \cdot a^4 \cdot b^2 + 4 \cdot a^2 \cdot b^4) \cdot \cos(c))^3 \cdot \sin(c) + (a^6 - 5 \cdot a^4 \cdot b^2 + 4 \cdot a^2 \cdot b^4) \cdot \cos(c) \cdot \sin(c)^3) \cdot \cos(d \cdot x^2 + 2 \cdot c)^2 + 4 \cdot ((a^6 - 5 \cdot a^4 \cdot b^2 + 4 \cdot a^2 \cdot b^4) \cdot \cos(c))^3 \cdot \sin(c) + (a^6 - 5 \cdot a^4 \cdot b^2 + 4 \cdot a^2 \cdot b^4) \cdot \cos(c) \cdot \sin(c)^3 - 3 \cdot ((a^5 \cdot b - a^3 \cdot b^3) \cdot \cos(c))^2 \cdot \sin(c) - (a^5 \cdot b - a^3 \cdot b^3) \cdot \sin(c)^3) \cdot \cos(d \cdot x^2 + 2 \cdot c)) \cdot \sin(d \cdot x^2 + 2 \cdot c)^2 - 4 \cdot ((a^5 \cdot b - 3 \cdot a^3 \cdot b^3 + 2 \cdot a \cdot b^5) \cdot \cos(c))^4 \cdot \sin(c) + 2 \cdot (a^5 \cdot b - 3 \cdot a^3 \cdot b^3 + 2 \cdot a \cdot b^5) \cdot \cos(c))^2 \cdot \sin(c)^3 + (a^5 \cdot b - 3 \cdot a^3 \cdot b^3 + 2 \cdot a \cdot b^5) \cdot \sin(c)^5) \cdot \cos(d \cdot x^2 + 2 \cdot c) + 4 \cdot ((a^5 \cdot b - 3 \cdot a^3 \cdot b^3 + 2 \cdot a \cdot b^5) \cdot \cos(c))^5 + 2 \cdot (a^5 \cdot b - 3 \cdot a^3 \cdot b^3 + 2 \cdot a \cdot b^5) \cdot \cos(c))^3 \cdot \sin(c)^2 + (a^5 \cdot b - 3 \cdot a^3 \cdot b^3 + 2 \cdot a \cdot b^5) \cdot \cos(c) \cdot \sin(c)^4 - ((a^6 - a^4 \cdot b^2) \cdot \cos(c))^2 - (a^6 - a^4 \cdot b^2) \cdot \sin(c)^2) \cdot \cos(d \cdot x^2 + 2 \cdot c)^3 - 3 \cdot ((a^5 \cdot b - a^3 \cdot b^3) \cdot \cos(c))^3 - (a^5 \cdot b - a^3 \cdot b^3) \cdot \cos(c) \cdot \sin(c)^2) \cdot \cos(d \cdot x^2 + 2 \cdot c)^2 + ((a^6 - 5 \cdot a^4 \cdot b^2 + 4 \cdot a^2 \cdot b^4) \cdot \cos(c))^4 - (a^6 - 5 \cdot a^4 \cdot b^2 + 4 \cdot a^2 \cdot b^4) \cdot \sin(c)^4) \cdot \cos(d \cdot x^2 + 2 \cdot c)) \cdot \sin(d \cdot x^2 + 2 \cdot c) + (a^5 \cdot \cos(c) \cdot \sin(d \cdot x^2 + 2 \cdot c))^5 - a^5 \cdot \cos(d \cdot x^2 + 2 \cdot c))^5 \cdot \sin(c) - 4 \cdot a^4 \cdot b \cdot \cos(d \cdot x^2 + 2 \cdot c))^4 \cdot \cos(c) \cdot \sin(c) - (a^5 \cdot \cos(d \cdot x^2 + 2 \cdot c) \cdot \sin(c) - 4 \cdot a^4 \cdot b \cdot \cos(c) \cdot \sin(c)) \cdot \sin(d \cdot x^2 + 2 \cdot c))^4 + 2 \cdot (3 \cdot (a^5 - 2 \cdot a^3 \cdot b^2) \cdot \cos(c))^2 \cdot \sin(c) + (a^5 - 2 \cdot a^3 \cdot b^2) \cdot \sin(c)^3) \cdot \cos(d \cdot x^2 + 2 \cdot c)^3 + 2 \cdot (a^5 \cdot \cos(d \cdot x^2 + 2 \cdot c))^2 \cdot \cos(c) - (a^5 - 2 \cdot a^3 \cdot b^2) \cdot \cos(c))^3 - 3 \cdot (a^5 - 2 \cdot a^3 \cdot b^2) \cdot \cos(c) \cdot \sin(c)^2 + 2 \cdot (a^4 \cdot b \cdot \cos(c))^2 - a^4 \cdot b \cdot \sin(c)^2) \cdot \cos(d \cdot x^2 + 2 \cdot c)) \cdot \sin(d \cdot x^2 + 2 \cdot c)^3 + 4 \cdot ((3 \cdot a^4 \cdot b - 4 \cdot a^2 \cdot b^3) \cdot \cos(c))^3 \cdot \sin(c) + (3 \cdot a^4 \cdot b - 4 \cdot a^2 \cdot b^3) \cdot \cos(c) \cdot \sin(c)^3) \cdot \cos(d \cdot x^2 + 2 \cdot c)^2 - 2 \cdot (a^5 \cdot \cos(d \cdot x^2 + 2 \cdot c))^3 \cdot \sin(c) + 2 \cdot (3 \cdot a^4 \cdot b - 4 \cdot a^2 \cdot b^3) \cdot \cos(c))^3 \cdot \sin(c) + 2 \cdot (3 \cdot a^4 \cdot b - 4 \cdot a^2 \cdot b^3) \cdot \cos(c) \cdot \sin(c)^3 + 3 \cdot ((a^5 - 2 \cdot a^3 \cdot b^2) \cdot \cos(c))^2 \cdot \sin(c) - (a^5 - 2 \cdot a^3 \cdot b^2) \cdot \sin(c)^3) \cdot \cos(d \cdot x^2 + 2 \cdot c)) \cdot \sin(d \cdot x^2 + 2 \cdot c)^2 - ((a^5 - 8 \cdot a^3 \cdot b^2 + 8 \cdot a \cdot b^4) \cdot \cos(c))^4 \cdot \sin(c) + 2 \cdot (a^5 - 8 \cdot a^3 \cdot b^2 + 8 \cdot a \cdot b^4) \cdot \cos(c))^2 \cdot \sin(c)^3 + (a^5 - 8 \cdot a^3 \cdot b^2 + 8 \cdot a \cdot b^4) \cdot \sin(c)^5) \cdot \cos(d \cdot x^2 + 2 \cdot c) + (a^5 \cdot \cos(d \cdot x^2 + 2 \cdot c))^4 \cdot \cos(c) + (a^5 - 8 \cdot a^3 \cdot b^2 + 8 \cdot a \cdot b^4) \cdot \cos(c))^5 + 2 \cdot (a^5 - 8 \cdot a^3 \cdot b^2 + 8 \cdot a \cdot b^4) \cdot \cos(c))^3 \cdot \sin(c)^2 + (a^5 - 8 \cdot a^3 \cdot b^2 + 8 \cdot a \cdot b^4) \cdot \cos(c) \cdot \sin(c)^4 + 4 \cdot (a^4 \cdot b \cdot \cos(c))^2 - a^4 \cdot b \cdot \sin(c)^2) \cdot \cos(d \cdot x^2 + 2 \cdot c)^3 - 6 \cdot ((a^5 - 2 \cdot a^3 \cdot b^2) \cdot \cos(c))^3 - (a^5 - 2 \cdot a^3 \cdot b^2) \cdot \cos(c) \cdot \sin(c)^2) \cdot \cos(d \cdot x^2 + 2 \cdot c)^2 - 4 \cdot ((3 \cdot a^4 \cdot b - 4 \cdot a^2 \cdot b^3) \cdot \cos(c))^4 - (3 \cdot a^4 \cdot b - 4 \cdot a^2 \cdot b^3) \cdot \sin(c))^4) \cdot \cos(d \cdot x^2 + 2 \cdot c)) \cdot \sin(d \cdot x^2 + 2 \cdot c)) \cdot \sqrt{-a^2 + b^2}) / (a^6 \cdot \cos(d \cdot x^2 + 2 \cdot c))^6 + 6 \cdot a^5 \cdot b \cdot \cos(d \cdot x^2 + 2 \cdot c))^5 \cdot \cos(c) + a^6 \cdot \sin(d \cdot x^2 + 2 \cdot c))^6 + 6 \cdot a^5 \cdot b \cdot \sin(d \cdot x^2 + 2 \cdot c))^5 \cdot \sin(c) - (a^6 - 18 \cdot a^4 \cdot b^2 + 48 \cdot a^2 \cdot b^4 - 32 \cdot b^6) \cdot \cos(c))^6 - 3 \cdot (a^6 - 18 \cdot a^4 \cdot b^2 + 48 \cdot a^2 \cdot b^4 - 32 \cdot b^6) \cdot \cos(c))^4 \cdot \sin(c)^2 - 3 \cdot (a^6 - 18 \cdot a^4 \cdot b^2 + 48 \cdot a^2 \cdot b^4 - 32 \cdot b^6) \cdot \cos(c))^2 \cdot \sin(c)^4 - (a^6 - 18 \cdot a^4 \cdot b^2 + 48 \cdot a^2 \cdot b^4 - 32 \cdot b^6) \cdot \sin(c))^6 - 3 \cdot (5 \cdot (a^6 - 2 \cdot a^4 \cdot b^2) \cdot \cos(c))^2 + (a^6 -$

$$\begin{aligned}
& 2*a^4*b^2)*\sin(c)^2)*\cos(d*x^2 + 2*c)^4 + 3*(a^6*\cos(d*x^2 + 2*c)^2 + 2*a^5 \\
& *b*\cos(d*x^2 + 2*c)*\cos(c) - (a^6 - 2*a^4*b^2)*\cos(c)^2 - 5*(a^6 - 2*a^4*b^ \\
& 2)*\sin(c)^2)*\sin(d*x^2 + 2*c)^4 - 4*(5*(3*a^5*b - 4*a^3*b^3)*\cos(c)^3 + 3*( \\
& 3*a^5*b - 4*a^3*b^3)*\cos(c)*\sin(c)^2)*\cos(d*x^2 + 2*c)^3 + 4*(3*a^5*b*\cos(d \\
& *x^2 + 2*c)^2*\sin(c) - 6*(a^6 - 2*a^4*b^2)*\cos(d*x^2 + 2*c)*\cos(c)*\sin(c) - \\
& 3*(3*a^5*b - 4*a^3*b^3)*\cos(c)^2*\sin(c) - 5*(3*a^5*b - 4*a^3*b^3)*\sin(c)^3 \\
& )*\sin(d*x^2 + 2*c)^3 + 3*(5*(a^6 - 8*a^4*b^2 + 8*a^2*b^4)*\cos(c)^4 + 6*(a^6 \\
& - 8*a^4*b^2 + 8*a^2*b^4)*\cos(c)^2*\sin(c)^2 + (a^6 - 8*a^4*b^2 + 8*a^2*b^4) \\
& *\sin(c)^4)*\cos(d*x^2 + 2*c)^2 + 3*(a^6*\cos(d*x^2 + 2*c)^4 + 4*a^5*b*\cos(d*x \\
& ^2 + 2*c)^3*\cos(c) + (a^6 - 8*a^4*b^2 + 8*a^2*b^4)*\cos(c)^4 + 6*(a^6 - 8*a^ \\
& 4*b^2 + 8*a^2*b^4)*\cos(c)^2*\sin(c)^2 + 5*(a^6 - 8*a^4*b^2 + 8*a^2*b^4)*\sin( \\
& c)^4 - 6*((a^6 - 2*a^4*b^2)*\cos(c)^2 + (a^6 - 2*a^4*b^2)*\sin(c)^2)*\cos(d*x^ \\
& 2 + 2*c)^2 - 4*((3*a^5*b - 4*a^3*b^3)*\cos(c)^3 + 3*(3*a^5*b - 4*a^3*b^3)*\co \\
& s(c)*\sin(c)^2)*\cos(d*x^2 + 2*c))*\sin(d*x^2 + 2*c)^2 + 6*((5*a^5*b - 20*a^3* \\
& b^3 + 16*a*b^5)*\cos(c)^5 + 2*(5*a^5*b - 20*a^3*b^3 + 16*a*b^5)*\cos(c)^3*\sin \\
& (c)^2 + (5*a^5*b - 20*a^3*b^3 + 16*a*b^5)*\cos(c)*\sin(c)^4)*\cos(d*x^2 + 2*c) \\
& + 6*(a^5*b*\cos(d*x^2 + 2*c)^4*\sin(c) - 4*(a^6 - 2*a^4*b^2)*\cos(d*x^2 + 2*c) \\
& )^3*\cos(c)*\sin(c) + (5*a^5*b - 20*a^3*b^3 + 16*a*b^5)*\cos(c)^4*\sin(c) + 2*( \\
& 5*a^5*b - 20*a^3*b^3 + 16*a*b^5)*\cos(c)^2*\sin(c)^3 + (5*a^5*b - 20*a^3*b^3 \\
& + 16*a*b^5)*\sin(c)^5 - 2*(3*(3*a^5*b - 4*a^3*b^3)*\cos(c)^2*\sin(c) + (3*a^5*b \\
& - 4*a^3*b^3)*\sin(c)^3)*\cos(d*x^2 + 2*c)^2 + 4*((a^6 - 8*a^4*b^2 + 8*a^2*b \\
& ^4)*\cos(c)^3*\sin(c) + (a^6 - 8*a^4*b^2 + 8*a^2*b^4)*\cos(c)*\sin(c)^3)*\cos(d* \\
& x^2 + 2*c))*\sin(d*x^2 + 2*c) + 2*(3*a^5*\cos(d*x^2 + 2*c))^5*\cos(c) + 3*a^5*s \\
& in(d*x^2 + 2*c)^5*\sin(c) + (3*a^4*b - 16*a^2*b^3 + 16*b^5)*\cos(c)^6 + 3*(3* \\
& a^4*b - 16*a^2*b^3 + 16*b^5)*\cos(c)^4*\sin(c)^2 + 3*(3*a^4*b - 16*a^2*b^3 + \\
& 16*b^5)*\cos(c)^2*\sin(c)^4 + (3*a^4*b - 16*a^2*b^3 + 16*b^5)*\sin(c)^6 + 3*(5 \\
& *a^4*b*\cos(c)^2 + a^4*b*\sin(c)^2)*\cos(d*x^2 + 2*c)^4 + 3*(a^5*\cos(d*x^2 + 2 \\
& *c)*\cos(c) + a^4*b*\cos(c)^2 + 5*a^4*b*\sin(c)^2)*\sin(d*x^2 + 2*c)^4 - 2*(5*( \\
& a^5 - 4*a^3*b^2)*\cos(c)^3 + 3*(a^5 - 4*a^3*b^2)*\cos(c)*\sin(c)^2)*\cos(d*x^2 \\
& + 2*c)^3 + 2*(3*a^5*\cos(d*x^2 + 2*c)^2*\sin(c) + 12*a^4*b*\cos(d*x^2 + 2*c)* \\
& os(c)*\sin(c) - 3*(a^5 - 4*a^3*b^2)*\cos(c)^2*\sin(c) - 5*(a^5 - 4*a^3*b^2)*\si \\
& n(c)^3)*\sin(d*x^2 + 2*c)^3 - 6*(5*(a^4*b - 2*a^2*b^3)*\cos(c)^4 + 6*(a^4*b - \\
& 2*a^2*b^3)*\cos(c)^2*\sin(c)^2 + (a^4*b - 2*a^2*b^3)*\sin(c)^4)*\cos(d*x^2 + 2 \\
& *c)^2 + 6*(a^5*\cos(d*x^2 + 2*c)^3*\cos(c) - (a^4*b - 2*a^2*b^3)*\cos(c)^4 - 6 \\
& *(a^4*b - 2*a^2*b^3)*\cos(c)^2*\sin(c)^2 - 5*(a^4*b - 2*a^2*b^3)*\sin(c)^4 + 3 \\
& *(a^4*b*\cos(c)^2 + a^4*b*\sin(c)^2)*\cos(d*x^2 + 2*c)^2 - ((a^5 - 4*a^3*b^2)* \\
& \cos(c)^3 + 3*(a^5 - 4*a^3*b^2)*\cos(c)*\sin(c)^2)*\cos(d*x^2 + 2*c))*\sin(d*x^2 \\
& + 2*c)^2 + 3*((a^5 - 12*a^3*b^2 + 16*a*b^4)*\cos(c)^5 + 2*(a^5 - 12*a^3*b^2 \\
& + 16*a*b^4)*\cos(c)^3*\sin(c)^2 + (a^5 - 12*a^3*b^2 + 16*a*b^4)*\cos(c)*\sin(c) \\
& )^4)*\cos(d*x^2 + 2*c) + 3*(a^5*\cos(d*x^2 + 2*c))^4*\sin(c) + 8*a^4*b*\cos(d*x^ \\
& 2 + 2*c)^3*\cos(c)*\sin(c) + (a^5 - 12*a^3*b^2 + 16*a*b^4)*\cos(c)^4*\sin(c) + \\
& 2*(a^5 - 12*a^3*b^2 + 16*a*b^4)*\cos(c)^2*\sin(c)^3 + (a^5 - 12*a^3*b^2 + 16* \\
& a*b^4)*\sin(c)^5 - 2*(3*(a^5 - 4*a^3*b^2)*\cos(c)^2*\sin(c) + (a^5 - 4*a^3*b^2) \\
& )*\sin(c)^3)*\cos(d*x^2 + 2*c)^2 - 16*((a^4*b - 2*a^2*b^3)*\cos(c)^3*\sin(c) + \\
& (a^4*b - 2*a^2*b^3)*\cos(c)*\sin(c)^3)*\cos(d*x^2 + 2*c))*\sin(d*x^2 + 2*c))*sq
\end{aligned}$$

$$\begin{aligned}
& \text{rt}(-a^2 + b^2)), (a^6 \cos(dx^2 + 2c)^6 + 6a^5 b \cos(dx^2 + 2c)^5 \cos(c) \\
& + a^6 \sin(dx^2 + 2c)^6 + 6a^5 b \sin(dx^2 + 2c)^5 \sin(c) + (a^6 - 8a^4 b^2 + 8a^2 b^4) \cos(c)^6 + 3(a^6 - 8a^4 b^2 + 8a^2 b^4) \cos(c)^4 \sin(c)^2 + 3(a^6 - 8a^4 b^2 + 8a^2 b^4) \cos(c)^2 \sin(c)^4 + (a^6 - 8a^4 b^2 + 8a^2 b^4) \sin(c)^6 - (5(a^6 - 4a^4 b^2) \cos(c)^2 + (a^6 - 4a^4 b^2) \sin(c)^2) \cos(dx^2 + 2c)^4 + (3a^6 \cos(dx^2 + 2c)^2 + 6a^5 b \cos(dx^2 + 2c) \cos(c) - (a^6 - 4a^4 b^2) \cos(c)^2 - 5(a^6 - 4a^4 b^2) \sin(c)^2) \sin(dx^2 + 2c)^4 - 4(5(a^5 b - 2a^3 b^3) \cos(c)^3 + 3(a^5 b - 2a^3 b^3) \cos(c) \sin(c)^2) \cos(dx^2 + 2c)^3 + 4(3a^5 b \cos(dx^2 + 2c)^2 \sin(c) - 2(a^6 - 4a^4 b^2) \cos(dx^2 + 2c) \cos(c) \sin(c) - 3(a^5 b - 2a^3 b^3) \cos(c)^2 \sin(c) - 5(a^5 b - 2a^3 b^3) \sin(c)^3) \sin(dx^2 + 2c)^3 - (5(a^6 + 4a^4 b^2 - 8a^2 b^4) \cos(c)^4 + 6(a^6 + 4a^4 b^2 - 8a^2 b^4) \cos(c)^2 \sin(c)^2 + (a^6 + 4a^4 b^2 - 8a^2 b^4) \sin(c)^4) \cos(dx^2 + 2c)^2 + (3a^6 \cos(dx^2 + 2c)^4 + 12a^5 b \cos(dx^2 + 2c)^3 \cos(c) - (a^6 + 4a^4 b^2 - 8a^2 b^4) \cos(c)^4 - 6(a^6 + 4a^4 b^2 - 8a^2 b^4) \cos(c)^2 \sin(c)^2 - 5(a^6 + 4a^4 b^2 - 8a^2 b^4) \sin(c)^4 - 6((a^6 - 4a^4 b^2) \cos(c)^2 + (a^6 - 4a^4 b^2) \sin(c)^2) \cos(dx^2 + 2c)^2 - 12((a^5 b - 2a^3 b^3) \cos(c)^3 + 3(a^5 b - 2a^3 b^3) \cos(c) \sin(c)^2) \cos(dx^2 + 2c)) \sin(dx^2 + 2c)^2 - 2((5a^5 b - 8a^3 b^5) \cos(c)^5 + 2(5a^5 b - 8a^3 b^5) \cos(c)^3 \sin(c)^2 + (5a^5 b - 8a^3 b^5) \cos(c) \sin(c)^4) \cos(dx^2 + 2c) + 2(3a^5 b \cos(dx^2 + 2c)^4 \sin(c) - 4(a^6 - 4a^4 b^2) \cos(dx^2 + 2c)^3 \cos(c) \sin(c) - (5a^5 b - 8a^3 b^5) \cos(c)^4 \sin(c) - 2(5a^5 b - 8a^3 b^5) \cos(c)^2 \sin(c)^3 - (5a^5 b - 8a^3 b^5) \sin(c)^5 - 6(3(a^5 b - 2a^3 b^3) \cos(c)^2 \sin(c) + (a^5 b - 2a^3 b^3) \sin(c)^3) \cos(dx^2 + 2c)^2 - 4((a^6 + 4a^4 b^2 - 8a^2 b^4) \cos(c)^3 \sin(c) + (a^6 + 4a^4 b^2 - 8a^2 b^4) \cos(c) \sin(c)^3) \cos(dx^2 + 2c)) \sin(dx^2 + 2c) + 4(a^5 \cos(dx^2 + 2c)^5 \cos(c) + a^5 \sin(dx^2 + 2c)^5 \sin(c) - (a^4 b - 2a^2 b^3) \cos(c)^6 - 3(a^4 b - 2a^2 b^3) \cos(c)^4 \sin(c)^2 - 3(a^4 b - 2a^2 b^3) \cos(c)^2 \sin(c)^4 - (a^4 b - 2a^2 b^3) \sin(c)^6 + (5a^4 b \cos(c))^2 + a^4 b \sin(c)^2) \cos(dx^2 + 2c)^4 + (a^5 \cos(dx^2 + 2c) \cos(c) + a^4 b \cos(c)^2 + 5a^4 b \sin(c)^2) \sin(dx^2 + 2c)^4 + 2(5a^3 b^2 \cos(c)^3 + 3a^3 b^2 \cos(c) \sin(c)^2) \cos(dx^2 + 2c)^3 + 2(a^5 \cos(dx^2 + 2c)^2 \sin(c) + 4a^4 b \cos(dx^2 + 2c) \cos(c) \sin(c) + 3a^3 b^2 \cos(c)^2 \sin(c) + 5a^3 b^2 \sin(c)^3) \sin(dx^2 + 2c)^3 + 2(5a^2 b^3 \cos(c)^4 + 6a^2 b^3 \cos(c)^2 \sin(c)^2 + a^2 b^3 \sin(c)^4) \cos(dx^2 + 2c)^2 + 2(a^5 \cos(dx^2 + 2c)^3 \cos(c) + a^2 b^3 \cos(c)^4 + 6a^2 b^3 \cos(c)^2 \sin(c)^2 + 5a^2 b^3 \sin(c)^4 + 3(a^4 b \cos(c)^2 + a^4 b \sin(c)^2) \cos(dx^2 + 2c)^2 + 3(a^3 b^2 \cos(c)^3 + 3a^3 b^2 \cos(c) \sin(c)^2) \cos(dx^2 + 2c)) \sin(dx^2 + 2c)^2 - ((a^5 - 2a^3 b^2 - 4a^2 b^4) \cos(c)^5 + 2(a^5 - 2a^3 b^2 - 4a^2 b^4) \cos(c)^3 \sin(c)^2 + (a^5 - 2a^3 b^2 - 4a^2 b^4) \cos(c) \sin(c)^4) \cos(dx^2 + 2c) + (a^5 \cos(dx^2 + 2c)^4 \sin(c) + 8a^4 b \cos(dx^2 + 2c)^3 \cos(c) \sin(c) - (a^5 - 2a^3 b^2 - 4a^2 b^4) \cos(c)^4 \sin(c) - 2(a^5 - 2a^3 b^2 - 4a^2 b^4) \cos(c)^2 \sin(c)^3 - (a^5 - 2a^3 b^2 - 4a^2 b^4) \sin(c)^5 + 6(3a^3 b^2 \cos(c)^2 \sin(c) + a^3 b^2 \sin(c)^3) \cos(dx^2 + 2c)^2 + 16(a^2 b^3 \cos(c)^3 \sin(c) + a^2 b^3 \cos(c) \sin(c)^3) \cos(dx^2 + 2c)) \sin
\end{aligned}$$

$$\begin{aligned}
& n(dx^2 + 2c))\sqrt{-a^2 + b^2})/(a^6\cos(dx^2 + 2c)^6 + 6a^5b\cos(dx^2 + 2c)^5\cos(c) + a^6\sin(dx^2 + 2c)^6 + 6a^5b\sin(dx^2 + 2c)^5\sin(c) - (a^6 - 18a^4b^2 + 48a^2b^4 - 32b^6)\cos(c)^6 - 3(a^6 - 18a^4b^2 + 48a^2b^4 - 32b^6)\cos(c)^4\sin(c)^2 - 3(a^6 - 18a^4b^2 + 48a^2b^4 - 32b^6)\cos(c)^2\sin(c)^4 - (a^6 - 18a^4b^2 + 48a^2b^4 - 32b^6)\sin(c)^6 - 3(5(a^6 - 2a^4b^2)\cos(c)^2 + (a^6 - 2a^4b^2)\sin(c)^2)\cos(dx^2 + 2c)^4 + 3(a^6\cos(dx^2 + 2c)^2 + 2a^5b\cos(dx^2 + 2c)\cos(c) - (a^6 - 2a^4b^2)\cos(c)^2 - 5(a^6 - 2a^4b^2)\sin(c)^2)\sin(dx^2 + 2c)^4 - 4(5(3a^5b - 4a^3b^3)\cos(c)^3 + 3(3a^5b - 4a^3b^3)\cos(c)\sin(c)^2)\cos(dx^2 + 2c)^3 + 4(3a^5b\cos(dx^2 + 2c)^2\sin(c) - 6(a^6 - 2a^4b^2)\cos(dx^2 + 2c)\cos(c)\sin(c) - 3(3a^5b - 4a^3b^3)\cos(c)^2\sin(c) - 5(3a^5b - 4a^3b^3)\sin(c)^3)\sin(dx^2 + 2c)^3 + 3(5(a^6 - 8a^4b^2 + 8a^2b^4)\cos(c)^4 + 6(a^6 - 8a^4b^2 + 8a^2b^4)\cos(c)^2\sin(c)^2 + (a^6 - 8a^4b^2 + 8a^2b^4)\sin(c)^4)\cos(dx^2 + 2c)^2 + 3(a^6\cos(dx^2 + 2c)^4 + 4a^5b\cos(dx^2 + 2c)^3\cos(c) + (a^6 - 8a^4b^2 + 8a^2b^4)\cos(c)^4 + 6(a^6 - 8a^4b^2 + 8a^2b^4)\cos(c)^2\sin(c)^2 + 5(a^6 - 8a^4b^2 + 8a^2b^4)\sin(c)^4 - 6((a^6 - 2a^4b^2)\cos(c)^2 + (a^6 - 2a^4b^2)\sin(c)^2)\cos(dx^2 + 2c)^2 - 4((3a^5b - 4a^3b^3)\cos(c)^3 + 3(3a^5b - 4a^3b^3)\cos(c)\sin(c)^2)\cos(dx^2 + 2c))\sin(dx^2 + 2c)^2 + 6((5a^5b - 20a^3b^3 + 16a^2b^5)\cos(c)^5 + 2(5a^5b - 20a^3b^3 + 16a^2b^5)\cos(c)^3\sin(c)^2 + (5a^5b - 20a^3b^3 + 16a^2b^5)\cos(c)\sin(c)^4)\cos(dx^2 + 2c) + 6(a^5b\cos(dx^2 + 2c)^4\sin(c) - 4(a^6 - 2a^4b^2)\cos(dx^2 + 2c)^3\cos(c)\sin(c) + (5a^5b - 20a^3b^3 + 16a^2b^5)\cos(c)^4\sin(c) + 2(5a^5b - 20a^3b^3 + 16a^2b^5)\cos(c)^2\sin(c)^3 + (5a^5b - 20a^3b^3 + 16a^2b^5)\sin(c)^5 - 2(3(3a^5b - 4a^3b^3)\cos(c)^2\sin(c) + (3a^5b - 4a^3b^3)\sin(c)^3)\cos(dx^2 + 2c)^2 + 4((a^6 - 8a^4b^2 + 8a^2b^4)\cos(c)^3\sin(c) + (a^6 - 8a^4b^2 + 8a^2b^4)\cos(c)\sin(c)^3)\cos(dx^2 + 2c))\sin(dx^2 + 2c) + 2(3a^5\cos(dx^2 + 2c)^5\cos(c) + 3a^5\sin(dx^2 + 2c)^5\sin(c) + (3a^4b - 16a^2b^3 + 16b^5)\cos(c)^6 + 3(3a^4b - 16a^2b^3 + 16b^5)\cos(c)^4\sin(c)^2 + 3(3a^4b - 16a^2b^3 + 16b^5)\cos(c)^2\sin(c)^4 + (3a^4b - 16a^2b^3 + 16b^5)\sin(c)^6 + 3(5a^4b\cos(c)^2 + a^4b\sin(c)^2)\cos(dx^2 + 2c)^4 + 3(a^5\cos(dx^2 + 2c)\cos(c) + a^4b\cos(c)^2 + 5a^4b\sin(c)^2)\sin(dx^2 + 2c)^4 - 2(5(a^5 - 4a^3b^2)\cos(c)^3 + 3(a^5 - 4a^3b^2)\cos(c)\sin(c)^2)\cos(dx^2 + 2c)^3 + 2(3a^5\cos(dx^2 + 2c)^2\sin(c) + 12a^4b\cos(dx^2 + 2c)\cos(c)\sin(c) - 3(a^5 - 4a^3b^2)\cos(c)^2\sin(c) - 5(a^5 - 4a^3b^2)\sin(c)^3)\sin(dx^2 + 2c)^3 - 6(5(a^4b - 2a^2b^3)\cos(c)^4 + 6(a^4b - 2a^2b^3)\cos(c)^2\sin(c)^2 + (a^4b - 2a^2b^3)\sin(c)^4)\cos(dx^2 + 2c)^2 + 6(a^5\cos(dx^2 + 2c)^3\cos(c) - (a^4b - 2a^2b^3)\cos(c)^4 - 6(a^4b - 2a^2b^3)\cos(c)^2\sin(c)^2 - 5(a^4b - 2a^2b^3)\sin(c)^4 + 3(a^4b\cos(c)^2 + a^4b\sin(c)^2)\cos(dx^2 + 2c)^2 - ((a^5 - 4a^3b^2)\cos(c)^3 + 3(a^5 - 4a^3b^2)\cos(c)\sin(c)^2)\cos(dx^2 + 2c))\sin(dx^2 + 2c)^2 + 3((a^5 - 12a^3b^2 + 16a^2b^4)\cos(c)^5 + 2(a^5 - 12a^3b^2 + 16a^2b^4)\cos(c)^3\sin(c)^2 + (a^5 - 12a^3b^2 + 16a^2b^4)\cos(c)\sin(c)^4)\cos(dx^2 + 2c)
\end{aligned}$$



+ 3\*(a^5\*cos(d\*x^2 + 2\*c)^4\*sin(c) + 8\*a^4\*b\*cos(d\*x^2 + 2\*c)^3\*cos(c)\*sin(c) + (a^5 - 12\*a^3\*b^2 + 16\*a\*b^4)\*cos(c)^4\*sin(c) + 2\*(a^5 - 12\*a^3\*b^2 + 16\*a\*b^4)\*cos(c)^2\*sin(c)^3 + (a^5 - 12\*a^3\*b^2 + 16\*a\*b^4)\*sin(c)^5 - 2\*(3\*(a^5 - 4\*a^3\*b^2)\*cos(c)^2\*sin(c) + (a^5 - 4\*a^3\*b^2)\*sin(c)^3)\*cos(d\*x^2 + 2\*c)^2 - 16\*((a^4\*b - 2\*a^2\*b^3)\*cos(c)^3\*sin(c) + (a^4\*b - 2\*a^2\*b^3)\*cos(c)\*sin(c)^3)\*cos(d\*x^2 + 2\*c))\*sin(d\*x^2 + 2\*c))\*sqrt(-a^2 + b^2)))/(sqrt(-a^2 + b^2)\*a\*d)

**mupad [B]** time = 2.24, size = 157, normalized size = 2.38

$$\frac{x^2}{2a} + \frac{b \ln\left(2bx e^{dx^2 1i} e^{c 1i} - \frac{bx(a+b e^{dx^2 1i} e^{c 1i}) 2i}{\sqrt{a+b} \sqrt{a-b}}\right)}{2ad \sqrt{a+b} \sqrt{a-b}} - \frac{b \ln\left(2bx e^{dx^2 1i} e^{c 1i} + \frac{bx(a+b e^{dx^2 1i} e^{c 1i}) 2i}{\sqrt{a+b} \sqrt{a-b}}\right)}{2ad \sqrt{a+b} \sqrt{a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a + b/cos(c + d\*x^2)),x)

[Out] x^2/(2\*a) + (b\*log(2\*b\*x\*exp(d\*x^2\*1i)\*exp(c\*1i) - (b\*x\*(a + b\*exp(d\*x^2\*1i)\*exp(c\*1i))\*2i)/((a + b)^(1/2)\*(a - b)^(1/2))))/(2\*a\*d\*(a + b)^(1/2)\*(a - b)^(1/2)) - (b\*log(2\*b\*x\*exp(d\*x^2\*1i)\*exp(c\*1i) + (b\*x\*(a + b\*exp(d\*x^2\*1i)\*exp(c\*1i))\*2i)/((a + b)^(1/2)\*(a - b)^(1/2))))/(2\*a\*d\*(a + b)^(1/2)\*(a - b)^(1/2))

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{a + b \sec(c + dx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*sec(d\*x\*\*2+c)),x)

[Out] Integral(x/(a + b\*sec(c + d\*x\*\*2)), x)

$$3.21 \quad \int \frac{1}{x(a+b \sec(c+dx^2))} dx$$

Optimal. Leaf size=21

$$\text{Int} \left( \frac{1}{x(a+b \sec(c+dx^2))}, x \right)$$

[Out] Unintegrable(1/x/(a+b\*sec(d\*x^2+c)), x)

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{x(a+b \sec(c+dx^2))} dx$$

Verification is Not applicable to the result.

[In] Int[1/(x\*(a + b\*Sec[c + d\*x^2])), x]

[Out] Defer[Int][1/(x\*(a + b\*Sec[c + d\*x^2])), x]

Rubi steps

$$\int \frac{1}{x(a+b \sec(c+dx^2))} dx = \int \frac{1}{x(a+b \sec(c+dx^2))} dx$$

Mathematica [A] time = 0.97, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a+b \sec(c+dx^2))} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x\*(a + b\*Sec[c + d\*x^2])), x]

[Out] Integrate[1/(x\*(a + b\*Sec[c + d\*x^2])), x]

fricas [A] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{1}{bx \sec(dx^2 + c) + ax}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*sec(d\*x^2+c)),x, algorithm="fricas")

[Out] integral(1/(b\*x\*sec(d\*x^2 + c) + a\*x), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \sec(dx^2 + c) + a)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*sec(d\*x^2+c)),x, algorithm="giac")

[Out] integrate(1/((b\*sec(d\*x^2 + c) + a)\*x), x)

**maple** [A] time = 0.70, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a + b \sec(dx^2 + c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(a+b\*sec(d\*x^2+c)),x)

[Out] int(1/x/(a+b\*sec(d\*x^2+c)),x)

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*sec(d\*x^2+c)),x, algorithm="maxima")

[Out] Timed out

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x \left( a + \frac{b}{\cos(dx^2+c)} \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x\*(a + b/cos(c + d\*x^2))),x)

[Out] int(1/(x\*(a + b/cos(c + d\*x^2))), x)

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a + b \sec(c + dx^2))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*sec(d\*x\*\*2+c)),x)

[Out] Integral(1/(x\*(a + b\*sec(c + d\*x\*\*2))), x)

$$3.22 \quad \int \frac{a+b \sec(c+dx^2)}{x^2} dx$$

Optimal. Leaf size=24

$$b \operatorname{Int} \left( \frac{\sec(c+dx^2)}{x^2}, x \right) - \frac{a}{x}$$

[Out]  $-a/x + b \cdot \operatorname{Unintegrable}(\sec(d \cdot x^2 + c)/x^2, x)$

**Rubi** [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{a + b \sec(c + dx^2)}{x^2} dx$$

Verification is Not applicable to the result.

[In]  $\operatorname{Int}[(a + b \cdot \operatorname{Sec}[c + d \cdot x^2])/x^2, x]$

[Out]  $-(a/x) + b \cdot \operatorname{Defer}[\operatorname{Int}][\operatorname{Sec}[c + d \cdot x^2]/x^2, x]$

Rubi steps

$$\begin{aligned} \int \frac{a + b \sec(c + dx^2)}{x^2} dx &= \int \left( \frac{a}{x^2} + \frac{b \sec(c + dx^2)}{x^2} \right) dx \\ &= -\frac{a}{x} + b \int \frac{\sec(c + dx^2)}{x^2} dx \end{aligned}$$

**Mathematica** [A] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{a + b \sec(c + dx^2)}{x^2} dx$$

Verification is Not applicable to the result.

[In]  $\operatorname{Integrate}[(a + b \cdot \operatorname{Sec}[c + d \cdot x^2])/x^2, x]$

[Out]  $\operatorname{Integrate}[(a + b \cdot \operatorname{Sec}[c + d \cdot x^2])/x^2, x]$

**fricas** [A] time = 0.59, size = 0, normalized size = 0.00

$$\operatorname{integral} \left( \frac{b \sec(dx^2 + c) + a}{x^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sec(d\*x^2+c))/x^2,x, algorithm="fricas")

[Out] integral((b\*sec(d\*x^2 + c) + a)/x^2, x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \sec(dx^2 + c) + a}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sec(d\*x^2+c))/x^2,x, algorithm="giac")

[Out] integrate((b\*sec(d\*x^2 + c) + a)/x^2, x)

**maple** [A] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{a + b \sec(dx^2 + c)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*sec(d\*x^2+c))/x^2,x)

[Out] int((a+b\*sec(d\*x^2+c))/x^2,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$2b \int \frac{\cos(2dx^2 + 2c) \cos(dx^2 + c) + \sin(2dx^2 + 2c) \sin(dx^2 + c) + \cos(dx^2 + c)}{x^2 \cos(2dx^2 + 2c)^2 + x^2 \sin(2dx^2 + 2c)^2 + 2x^2 \cos(2dx^2 + 2c) + x^2} dx - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sec(d\*x^2+c))/x^2,x, algorithm="maxima")

[Out] 2\*b\*integrate((cos(2\*d\*x^2 + 2\*c)\*cos(d\*x^2 + c) + sin(2\*d\*x^2 + 2\*c)\*sin(d\*x^2 + c) + cos(d\*x^2 + c))/(x^2\*cos(2\*d\*x^2 + 2\*c)^2 + x^2\*sin(2\*d\*x^2 + 2\*c)^2 + 2\*x^2\*cos(2\*d\*x^2 + 2\*c) + x^2), x) - a/x

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a + \frac{b}{\cos(dx^2+c)}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b/cos(c + d*x^2))/x^2,x)`

[Out] `int((a + b/cos(c + d*x^2))/x^2, x)`

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \sec(c + dx^2)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sec(d*x**2+c))/x**2,x)`

[Out] `Integral((a + b*sec(c + d*x**2))/x**2, x)`

$$3.23 \quad \int \frac{x^5}{(a+b \sec(c+dx^2))^2} dx$$

**Optimal.** Leaf size=1092

$$\frac{x^6}{6a^2} + \frac{ib \log\left(\frac{e^{i(dx^2+c)}_a}{b-\sqrt{b^2-a^2}} + 1\right) x^4}{a^2 \sqrt{b^2-a^2} d} - \frac{ib^3 \log\left(\frac{e^{i(dx^2+c)}_a}{b-\sqrt{b^2-a^2}} + 1\right) x^4}{2a^2 (b^2-a^2)^{3/2} d} - \frac{ib \log\left(\frac{e^{i(dx^2+c)}_a}{b+\sqrt{b^2-a^2}} + 1\right) x^4}{a^2 \sqrt{b^2-a^2} d} + \frac{ib^3 \log\left(\frac{e^{i(dx^2+c)}_a}{b+\sqrt{b^2-a^2}} + 1\right) x^4}{2a^2 (b^2-a^2)^{3/2} d} + \dots$$

[Out]  $I*b*x^4*\ln(1+a*\exp(I*(d*x^2+c)))/(b-(-a^2+b^2)^{(1/2)})/a^2/d/(-a^2+b^2)^{(1/2)} + 1/6*x^6/a^2+b^2*x^2*\ln(1+a*\exp(I*(d*x^2+c)))/(b-I*(a^2-b^2)^{(1/2)})/a^2/(a^2-b^2)/d^2+b^2*x^2*\ln(1+a*\exp(I*(d*x^2+c)))/(b+I*(a^2-b^2)^{(1/2)})/a^2/(a^2-b^2)/d^2+2*I*b*polylog(3,-a*\exp(I*(d*x^2+c)))/(b-(-a^2+b^2)^{(1/2)})/a^2/d^3/(-a^2+b^2)^{(1/2)}+I*b^3*polylog(3,-a*\exp(I*(d*x^2+c)))/(b+(-a^2+b^2)^{(1/2)})/a^2/(-a^2+b^2)^{(3/2)}/d^3-I*b^2*polylog(2,-a*\exp(I*(d*x^2+c)))/(b+I*(a^2-b^2)^{(1/2)})/a^2/(a^2-b^2)/d^3-1/2*I*b^3*x^4*\ln(1+a*\exp(I*(d*x^2+c)))/(b-(-a^2+b^2)^{(1/2)})/a^2/(-a^2+b^2)^{(3/2)}/d-b^3*x^2*polylog(2,-a*\exp(I*(d*x^2+c)))/(b-(-a^2+b^2)^{(1/2)})/a^2/(-a^2+b^2)^{(3/2)}/d^2+b^3*x^2*polylog(2,-a*\exp(I*(d*x^2+c)))/(b+(-a^2+b^2)^{(1/2)})/a^2/(-a^2+b^2)^{(3/2)}/d^2-I*b*x^4*\ln(1+a*\exp(I*(d*x^2+c)))/(b+(-a^2+b^2)^{(1/2)})/a^2/d/(-a^2+b^2)^{(1/2)}-2*I*b*polylog(3,-a*\exp(I*(d*x^2+c)))/(b+(-a^2+b^2)^{(1/2)})/a^2/d^3/(-a^2+b^2)^{(1/2)}+1/2*b^2*x^4*\sin(d*x^2+c)/a/(a^2-b^2)/d/(b+a*\cos(d*x^2+c))-I*b^3*polylog(3,-a*\exp(I*(d*x^2+c)))/(b-(-a^2+b^2)^{(1/2)})/a^2/(-a^2+b^2)^{(3/2)}/d^3+1/2*I*b^3*x^4*\ln(1+a*\exp(I*(d*x^2+c)))/(b+(-a^2+b^2)^{(1/2)})/a^2/(-a^2+b^2)^{(3/2)}/d+2*b*x^2*polylog(2,-a*\exp(I*(d*x^2+c)))/(b-(-a^2+b^2)^{(1/2)})/a^2/d^2/(-a^2+b^2)^{(1/2)}-2*b*x^2*polylog(2,-a*\exp(I*(d*x^2+c)))/(b+(-a^2+b^2)^{(1/2)})/a^2/d^2/(-a^2+b^2)^{(1/2)}-1/2*I*b^2*x^4/a^2/(a^2-b^2)/d-I*b^2*polylog(2,-a*\exp(I*(d*x^2+c)))/(b-I*(a^2-b^2)^{(1/2)})/a^2/(a^2-b^2)/d^3$

**Rubi [A]** time = 2.29, antiderivative size = 1092, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 12, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$ , Rules used = {4204, 4191, 3324, 3321, 2264, 2190, 2531, 2282, 6589, 4522, 2279, 2391}

$$\frac{x^6}{6a^2} + \frac{ib \log\left(\frac{e^{i(dx^2+c)}_a}{b-\sqrt{b^2-a^2}} + 1\right) x^4}{a^2 \sqrt{b^2-a^2} d} - \frac{ib^3 \log\left(\frac{e^{i(dx^2+c)}_a}{b-\sqrt{b^2-a^2}} + 1\right) x^4}{2a^2 (b^2-a^2)^{3/2} d} - \frac{ib \log\left(\frac{e^{i(dx^2+c)}_a}{b+\sqrt{b^2-a^2}} + 1\right) x^4}{a^2 \sqrt{b^2-a^2} d} + \frac{ib^3 \log\left(\frac{e^{i(dx^2+c)}_a}{b+\sqrt{b^2-a^2}} + 1\right) x^4}{2a^2 (b^2-a^2)^{3/2} d} + \dots$$

Antiderivative was successfully verified.

[In] Int[x^5/(a + b\*Sec[c + d\*x^2])^2,x]

[Out]  $((-1/2)*b^2*x^4)/(a^2*(a^2 - b^2)*d) + x^6/(6*a^2) + (b^2*x^2*Log[1 + (a*E^{(I*(c + d*x^2))})/(b - I*sqrt[a^2 - b^2])])/(a^2*(a^2 - b^2)*d^2) + (b^2*x^2$



```

*Log[1 + (a*E^(I*(c + d*x^2)))/(b + I*Sqrt[a^2 - b^2])]/(a^2*(a^2 - b^2)*d
^2) - ((I/2)*b^3*x^4*Log[1 + (a*E^(I*(c + d*x^2)))/(b - Sqrt[-a^2 + b^2])])
/(a^2*(-a^2 + b^2)^(3/2)*d) + (I*b*x^4*Log[1 + (a*E^(I*(c + d*x^2)))/(b - S
qrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d) + ((I/2)*b^3*x^4*Log[1 + (a*E^(
I*(c + d*x^2)))/(b + Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d) - (I*b*
x^4*Log[1 + (a*E^(I*(c + d*x^2)))/(b + Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 +
b^2]*d) - (I*b^2*PolyLog[2, -((a*E^(I*(c + d*x^2)))/(b - I*Sqrt[a^2 - b^2]
))])/(a^2*(a^2 - b^2)*d^3) - (I*b^2*PolyLog[2, -((a*E^(I*(c + d*x^2)))/(b +
I*Sqrt[a^2 - b^2]))])/(a^2*(a^2 - b^2)*d^3) - (b^3*x^2*PolyLog[2, -((a*E^(
I*(c + d*x^2)))/(b - Sqrt[-a^2 + b^2])])])/(a^2*(-a^2 + b^2)^(3/2)*d^2) + (2
*b*x^2*PolyLog[2, -((a*E^(I*(c + d*x^2)))/(b - Sqrt[-a^2 + b^2])])])/(a^2*Sq
rt[-a^2 + b^2]*d^2) + (b^3*x^2*PolyLog[2, -((a*E^(I*(c + d*x^2)))/(b + Sqrt
[-a^2 + b^2])])])/(a^2*(-a^2 + b^2)^(3/2)*d^2) - (2*b*x^2*PolyLog[2, -((a*E^(
I*(c + d*x^2)))/(b + Sqrt[-a^2 + b^2])])])/(a^2*Sqrt[-a^2 + b^2]*d^2) - (I*
b^3*PolyLog[3, -((a*E^(I*(c + d*x^2)))/(b - Sqrt[-a^2 + b^2])])])/(a^2*(-a^2
+ b^2)^(3/2)*d^3) + ((2*I)*b*PolyLog[3, -((a*E^(I*(c + d*x^2)))/(b - Sqrt[
-a^2 + b^2])])])/(a^2*Sqrt[-a^2 + b^2]*d^3) + (I*b^3*PolyLog[3, -((a*E^(I*(c
+ d*x^2)))/(b + Sqrt[-a^2 + b^2])])])/(a^2*(-a^2 + b^2)^(3/2)*d^3) - ((2*I)
*b*PolyLog[3, -((a*E^(I*(c + d*x^2)))/(b + Sqrt[-a^2 + b^2])])])/(a^2*Sqrt[
-a^2 + b^2]*d^3) + (b^2*x^4*Sin[c + d*x^2])/(2*a*(a^2 - b^2)*d*(b + a*Cos[c
+ d*x^2]))

```

### Rule 2190

```

Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

```

### Rule 2264

```

Int[((F_)^(u_)*((f_) + (g_)*(x_))^(m_))/((a_) + (b_)*(F_)^(u_) + (c_)
*(F_)^(v_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]

```

### Rule 2279

```

Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_)))^(n_))], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

```

### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x],
Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)*(x_)^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m-1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n]], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

### Rule 3321

```
Int[((c_.) + (d_.)*(x_)^(m_.))/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)]), x_Symbol] := Dist[2, Int[((c + d*x)^m*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

### Rule 3324

```
Int[((c_.) + (d_.)*(x_)^(m_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] := Simp[(b*(c + d*x)^m*Cos[e + f*x])/(f*(a^2 - b^2)*(a + b*Sin[e + f*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x], x] - Dist[(b*d*m)/(f*(a^2 - b^2)), Int[((c + d*x)^(m-1)*Cos[e + f*x])/(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

### Rule 4191

```
Int[(csc[(e_.) + (f_.)*(x_)])*(b_.) + (a_.))^(n_.)*((c_.) + (d_.)*(x_)^(m_.), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGtQ[m, 0]
```

### Rule 4204

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol]
  := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]
```

### Rule 4522

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)])/(Cos[(c_.) + (d_.)*(x_)]*(b_.) + (a_.)), x_Symbol]
  := Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1)), x] + (Int[((e + f*x)^m*E^(I*(c + d*x)))/(I*a - Rt[-a^2 + b^2, 2] + I*b*E^(I*(c + d*x))), x] + Int[((e + f*x)^m*E^(I*(c + d*x)))/(I*a + Rt[-a^2 + b^2, 2] + I*b*E^(I*(c + d*x))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NegQ[a^2 - b^2]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol]
  := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps

$$\begin{aligned}
\int \frac{x^5}{(a + b \sec(c + dx^2))^2} dx &= \frac{1}{2} \text{Subst} \left( \int \frac{x^2}{(a + b \sec(c + dx))^2} dx, x, x^2 \right) \\
&= \frac{1}{2} \text{Subst} \left( \int \left( \frac{x^2}{a^2} + \frac{b^2 x^2}{a^2 (b + a \cos(c + dx))^2} - \frac{2bx^2}{a^2 (b + a \cos(c + dx))} \right) dx, x, x^2 \right) \\
&= \frac{x^6}{6a^2} - \frac{b \text{Subst} \left( \int \frac{x^2}{b + a \cos(c + dx)} dx, x, x^2 \right)}{a^2} + \frac{b^2 \text{Subst} \left( \int \frac{x^2}{(b + a \cos(c + dx))^2} dx, x, x^2 \right)}{2a^2} \\
&= \frac{x^6}{6a^2} + \frac{b^2 x^4 \sin(c + dx^2)}{2a(a^2 - b^2)d(b + a \cos(c + dx^2))} - \frac{(2b) \text{Subst} \left( \int \frac{e^{i(c+dx)} x^2}{a + 2be^{i(c+dx)} + ae^{2i(c+dx)}} dx, x, x^2 \right)}{a^2} \\
&= -\frac{ib^2 x^4}{2a^2(a^2 - b^2)d} + \frac{x^6}{6a^2} + \frac{b^2 x^4 \sin(c + dx^2)}{2a(a^2 - b^2)d(b + a \cos(c + dx^2))} - \frac{b^3 \text{Subst} \left( \int \frac{1}{a + 2be^{i(c+dx)}} dx, x, x^2 \right)}{a^2} \\
&= -\frac{ib^2 x^4}{2a^2(a^2 - b^2)d} + \frac{x^6}{6a^2} + \frac{b^2 x^2 \log \left( 1 + \frac{ae^{i(c+dx^2)}}{b - i\sqrt{a^2 - b^2}} \right)}{a^2(a^2 - b^2)d^2} + \frac{b^2 x^2 \log \left( 1 + \frac{ae^{i(c+dx^2)}}{b + i\sqrt{a^2 - b^2}} \right)}{a^2(a^2 - b^2)d^2} + \dots \\
&= -\frac{ib^2 x^4}{2a^2(a^2 - b^2)d} + \frac{x^6}{6a^2} + \frac{b^2 x^2 \log \left( 1 + \frac{ae^{i(c+dx^2)}}{b - i\sqrt{a^2 - b^2}} \right)}{a^2(a^2 - b^2)d^2} + \frac{b^2 x^2 \log \left( 1 + \frac{ae^{i(c+dx^2)}}{b + i\sqrt{a^2 - b^2}} \right)}{a^2(a^2 - b^2)d^2} - \dots \\
&= -\frac{ib^2 x^4}{2a^2(a^2 - b^2)d} + \frac{x^6}{6a^2} + \frac{b^2 x^2 \log \left( 1 + \frac{ae^{i(c+dx^2)}}{b - i\sqrt{a^2 - b^2}} \right)}{a^2(a^2 - b^2)d^2} + \frac{b^2 x^2 \log \left( 1 + \frac{ae^{i(c+dx^2)}}{b + i\sqrt{a^2 - b^2}} \right)}{a^2(a^2 - b^2)d^2} - \dots \\
&= -\frac{ib^2 x^4}{2a^2(a^2 - b^2)d} + \frac{x^6}{6a^2} + \frac{b^2 x^2 \log \left( 1 + \frac{ae^{i(c+dx^2)}}{b - i\sqrt{a^2 - b^2}} \right)}{a^2(a^2 - b^2)d^2} + \frac{b^2 x^2 \log \left( 1 + \frac{ae^{i(c+dx^2)}}{b + i\sqrt{a^2 - b^2}} \right)}{a^2(a^2 - b^2)d^2} - \dots
\end{aligned}$$

**Mathematica [A]** time = 6.58, size = 895, normalized size = 0.82

$$(b + a \cos(dx^2 + c)) \sec^2(dx^2 + c) \left( (b + a \cos(dx^2 + c)) x^6 + \frac{3b^2(a \sin(dx^2) - b \sin(c)) x^4}{(a-b)(a+b)d(\cos(\frac{c}{2}) - \sin(\frac{c}{2}))(\cos(\frac{c}{2}) + \sin(\frac{c}{2}))} - \frac{3b(b+a \cos(d$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^5/(a + b\*Sec[c + d\*x^2])^2,x]

[Out] ((b + a\*Cos[c + d\*x^2])\*Sec[c + d\*x^2]^2\*(x^6\*(b + a\*Cos[c + d\*x^2]) - (3\*b\*(b + a\*Cos[c + d\*x^2])\*(2\*(1 + E^((2\*I)\*c))\*(I\*b\*Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)] - 2\*a^2\*d\*E^(I\*c)\*x^2 + b^2\*d\*E^(I\*c)\*x^2)\*PolyLog[2, -((a\*E^(I\*(2\*c + d\*x^2)))/(b\*E^(I\*c) - Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)]))]) + 2\*(1 + E^((2\*I)\*c))\*(I\*b\*Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)] + 2\*a^2\*d\*E^(I\*c)\*x^2 - b^2\*d\*E^(I\*c)\*x^2)\*PolyLog[2, -((a\*E^(I\*(2\*c + d\*x^2)))/(b\*E^(I\*c) + Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)]))]) + I\*(d\*x^2\*(2\*b\*d\*E^((2\*I)\*c)\*Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)]\*x^2 + (1 + E^((2\*I)\*c))\*((2\*I)\*b\*Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)] - 2\*a^2\*d\*E^(I\*c)\*x^2 + b^2\*d\*E^(I\*c)\*x^2)\*Log[1 + (a\*E^(I\*(2\*c + d\*x^2)))/(b\*E^(I\*c) - Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)]]) + (1 + E^((2\*I)\*c))\*((2\*I)\*b\*Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)] + 2\*a^2\*d\*E^(I\*c)\*x^2 - b^2\*d\*E^(I\*c)\*x^2)\*Log[1 + (a\*E^(I\*(2\*c + d\*x^2)))/(b\*E^(I\*c) + Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)]]) - 2\*(2\*a^2 - b^2)\*E^(I\*c)\*(1 + E^((2\*I)\*c))\*PolyLog[3, -((a\*E^(I\*(2\*c + d\*x^2)))/(b\*E^(I\*c) - Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)]))]) + 2\*(2\*a^2 - b^2)\*E^(I\*c)\*(1 + E^((2\*I)\*c))\*PolyLog[3, -((a\*E^(I\*(2\*c + d\*x^2)))/(b\*E^(I\*c) + Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)]))])])/(a^2 - b^2)\*d^3\*Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)]\*(1 + E^((2\*I)\*c)) + (3\*b^2\*x^4\*(-(b\*Sin[c]) + a\*Sin[d\*x^2]))/(a - b)\*(a + b)\*d\*(Cos[c/2] - Sin[c/2])\*(Cos[c/2] + Sin[c/2]))/(6\*a^2\*(a + b\*Sec[c + d\*x^2])^2)

**fricas [C]** time = 1.07, size = 3048, normalized size = 2.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(a+b\*sec(d\*x^2+c))^2,x, algorithm="fricas")

[Out] 1/24\*(4\*(a^5 - 2\*a^3\*b^2 + a\*b^4)\*d^3\*x^6\*cos(d\*x^2 + c) + 4\*(a^4\*b - 2\*a^2\*b^3 + b^5)\*d^3\*x^6 + 12\*(a^3\*b^2 - a\*b^4)\*d^2\*x^4\*sin(d\*x^2 + c) + 2\*(-12\*I\*a^3\*b^2 + 6\*I\*a\*b^4 + (-12\*I\*a^4\*b + 6\*I\*a^2\*b^3)\*cos(d\*x^2 + c))\*sqrt(-(a^2 - b^2)/a^2)\*polylog(3, -(b\*cos(d\*x^2 + c) + I\*b\*sin(d\*x^2 + c) + (a\*cos(d\*x^2 + c) + I\*a\*sin(d\*x^2 + c))\*sqrt(-(a^2 - b^2)/a^2))/a) + 2\*(12\*I\*a^3\*

$$\begin{aligned}
& b^2 - 6Ia^3b^4 + (12Ia^4b - 6Ia^2b^3)\cos(dx^2 + c)\sqrt{-(a^2 - b^2)/a^2} \\
& \text{polylog}(3, -(b\cos(dx^2 + c) + Ib\sin(dx^2 + c) - (a\cos(dx^2 + c) + Ia\sin(dx^2 + c))\sqrt{-(a^2 - b^2)/a^2})/a) \\
& + 2*(12Ia^3b^2 - 6Ia^4b + (12Ia^4b - 6Ia^2b^3)\cos(dx^2 + c))\sqrt{-(a^2 - b^2)/a^2} \\
& \text{polylog}(3, -(b\cos(dx^2 + c) - Ib\sin(dx^2 + c) + (a\cos(dx^2 + c) - Ia\sin(dx^2 + c))\sqrt{-(a^2 - b^2)/a^2})/a) \\
& + 2*(-12Ia^3b^2 + 6Ia^4b + (-12Ia^4b + 6Ia^2b^3)\cos(dx^2 + c))\sqrt{-(a^2 - b^2)/a^2} \\
& \text{polylog}(3, -(b\cos(dx^2 + c) - Ib\sin(dx^2 + c) - (a\cos(dx^2 + c) - Ia\sin(dx^2 + c))\sqrt{-(a^2 - b^2)/a^2})/a) \\
& + (-12Ia^2b^3 + 12Ib^5 + (-12Ia^3b^2 + 12Ia^4b)\cos(dx^2 + c) - 12*((2a^4b - a^2b^3)*dx^2\cos(dx^2 + c) \\
& + (2a^3b^2 - ab^4)*dx^2)\sqrt{-(a^2 - b^2)/a^2})\text{dilog}(-(b\cos(dx^2 + c) + Ib\sin(dx^2 + c) + (a\cos(dx^2 + c) + Ia\sin(dx^2 + c))\sqrt{-(a^2 - b^2)/a^2} + a)/a + 1) \\
& + (-12Ia^2b^3 + 12Ib^5 + (-12Ia^3b^2 + 12Ia^4b)\cos(dx^2 + c) + 12*((2a^4b - a^2b^3)*dx^2\cos(dx^2 + c) + (2a^3b^2 - ab^4)*dx^2)\sqrt{-(a^2 - b^2)/a^2})\text{dilog}(-(b\cos(dx^2 + c) + Ib\sin(dx^2 + c) - (a\cos(dx^2 + c) + Ia\sin(dx^2 + c))\sqrt{-(a^2 - b^2)/a^2} + a)/a + 1) \\
& + (12Ia^2b^3 - 12Ib^5 + (12Ia^3b^2 - 12Ia^4b)\cos(dx^2 + c) - 12*((2a^4b - a^2b^3)*dx^2\cos(dx^2 + c) + (2a^3b^2 - ab^4)*dx^2)\sqrt{-(a^2 - b^2)/a^2})\text{dilog}(-(b\cos(dx^2 + c) - Ib\sin(dx^2 + c) + (a\cos(dx^2 + c) - Ia\sin(dx^2 + c))\sqrt{-(a^2 - b^2)/a^2} + a)/a + 1) \\
& + (12Ia^2b^3 - 12Ib^5 + (12Ia^3b^2 - 12Ia^4b)\cos(dx^2 + c) + 12*((2a^4b - a^2b^3)*dx^2\cos(dx^2 + c) + (2a^3b^2 - ab^4)*dx^2)\sqrt{-(a^2 - b^2)/a^2})\text{dilog}(-(b\cos(dx^2 + c) - Ib\sin(dx^2 + c) - (a\cos(dx^2 + c) - Ia\sin(dx^2 + c))\sqrt{-(a^2 - b^2)/a^2} + a)/a + 1) \\
& - 2*(6*(a^3b^2 - ab^4)*c\cos(dx^2 + c) + 6*(a^2b^3 - b^5)*c - (3I*(2a^4b - a^2b^3)*c^2\cos(dx^2 + c) + 3I*(2a^3b^2 - ab^4)*c^2)\sqrt{-(a^2 - b^2)/a^2})\log(2a\cos(dx^2 + c) + 2Ia\sin(dx^2 + c) + 2a\sqrt{-(a^2 - b^2)/a^2} + 2b) \\
& - 2*(6*(a^3b^2 - ab^4)*c\cos(dx^2 + c) + 6*(a^2b^3 - b^5)*c - (3I*(2a^4b - a^2b^3)*c^2\cos(dx^2 + c) + 3I*(2a^3b^2 - ab^4)*c^2)\sqrt{-(a^2 - b^2)/a^2})\log(2a\cos(dx^2 + c) - 2Ia\sin(dx^2 + c) + 2a\sqrt{-(a^2 - b^2)/a^2} + 2b) \\
& - 2*(6*(a^3b^2 - ab^4)*c\cos(dx^2 + c) + 6*(a^2b^3 - b^5)*c - (3I*(2a^4b - a^2b^3)*c^2\cos(dx^2 + c) + 3I*(2a^3b^2 - ab^4)*c^2)\sqrt{-(a^2 - b^2)/a^2})\log(-2a\cos(dx^2 + c) + 2Ia\sin(dx^2 + c) + 2a\sqrt{-(a^2 - b^2)/a^2} - 2b) \\
& - 2*(6*(a^3b^2 - ab^4)*c\cos(dx^2 + c) + 6*(a^2b^3 - b^5)*c - (3I*(2a^4b - a^2b^3)*c^2\cos(dx^2 + c) - 3I*(2a^3b^2 - ab^4)*c^2)\sqrt{-(a^2 - b^2)/a^2})\log(-2a\cos(dx^2 + c) - 2Ia\sin(dx^2 + c) + 2a\sqrt{-(a^2 - b^2)/a^2} - 2b) \\
& + 2*(6*(a^2b^3 - b^5)*c + 6*((a^3b^2 - ab^4)*dx^2 + (a^3b^2 - ab^4)*c)\cos(dx^2 + c) + (-3I*(2a^3b^2 - ab^4)*d^2x^4 + 3I*(2a^3b^2 - ab^4)*c^2 + (-3I*(2a^4b - a^2b^3)*d^2x^4 + 3I*(2a^4b - a^2b^3)*c^2)\cos(dx^2 + c))\sqrt{-(a^2 - b^2)/a^2})\log((b\cos(dx^2 + c) + Ib\sin(dx^2 + c) + (a\cos(dx^2 + c) + Ia\sin(dx^2 + c))\sqrt{-(a^2 - b^2)/a^2} + a)/a) \\
& + 2*(6*(a^2b^3 - b^5)*dx^2 + 6*(a^2b^3 - b^5)*c + 6*((a^3b^2 - ab^4)*dx^2 + (a^3b^2 - ab^4)*c)\cos(dx^2 + c) + (3I*(2a^3b^2 - ab^4)*d^2x^2 + (a^3b^2 - ab^4)*c)\cos(dx^2 + c) + (3I*(2a^3b^2 - ab^4)*d^2x^2 + (a^3b^2 - ab^4)*c)\cos(dx^2 + c)
\end{aligned}$$

$$\begin{aligned}
&^4 - 3*I*(2*a^3*b^2 - a*b^4)*c^2 + (3*I*(2*a^4*b - a^2*b^3)*d^2*x^4 - 3*I*(2*a^4*b - a^2*b^3)*c^2)*\cos(d*x^2 + c))*\sqrt{-(a^2 - b^2)/a^2})*\log((b*\cos(d*x^2 + c) + I*b*\sin(d*x^2 + c) - (a*\cos(d*x^2 + c) + I*a*\sin(d*x^2 + c))*\sqrt{-(a^2 - b^2)/a^2} + a)/a) + 2*(6*(a^2*b^3 - b^5)*d*x^2 + 6*(a^2*b^3 - b^5)*c + 6*((a^3*b^2 - a*b^4)*d*x^2 + (a^3*b^2 - a*b^4)*c)*\cos(d*x^2 + c) + (3*I*(2*a^3*b^2 - a*b^4)*d^2*x^4 - 3*I*(2*a^3*b^2 - a*b^4)*c^2 + (3*I*(2*a^4*b - a^2*b^3)*d^2*x^4 - 3*I*(2*a^4*b - a^2*b^3)*c^2)*\cos(d*x^2 + c))*\sqrt{-(a^2 - b^2)/a^2})*\log((b*\cos(d*x^2 + c) - I*b*\sin(d*x^2 + c) + (a*\cos(d*x^2 + c) - I*a*\sin(d*x^2 + c))*\sqrt{-(a^2 - b^2)/a^2} + a)/a) + 2*(6*(a^2*b^3 - b^5)*d*x^2 + 6*(a^2*b^3 - b^5)*c + 6*((a^3*b^2 - a*b^4)*d*x^2 + (a^3*b^2 - a*b^4)*c)*\cos(d*x^2 + c) + (-3*I*(2*a^3*b^2 - a*b^4)*d^2*x^4 + 3*I*(2*a^3*b^2 - a*b^4)*c^2 + (-3*I*(2*a^4*b - a^2*b^3)*d^2*x^4 + 3*I*(2*a^4*b - a^2*b^3)*c^2)*\cos(d*x^2 + c))*\sqrt{-(a^2 - b^2)/a^2})*\log((b*\cos(d*x^2 + c) - I*b*\sin(d*x^2 + c) - (a*\cos(d*x^2 + c) - I*a*\sin(d*x^2 + c))*\sqrt{-(a^2 - b^2)/a^2} + a)/a))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d^3*\cos(d*x^2 + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d^3)
\end{aligned}$$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(b \sec(dx^2 + c) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(a+b\*sec(d\*x^2+c))^2,x, algorithm="giac")

[Out] integrate(x^5/(b\*sec(d\*x^2 + c) + a)^2, x)

**maple** [F] time = 1.67, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(a + b \sec(dx^2 + c))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(a+b\*sec(d\*x^2+c))^2,x)

[Out] int(x^5/(a+b\*sec(d\*x^2+c))^2,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(a+b\*sec(d\*x^2+c))^2,x, algorithm="maxima")

```
[Out] 1/6*((a^4 - a^2*b^2)*d*x^6*cos(2*d*x^2 + 2*c)^2 + 4*(a^2*b^2 - b^4)*d*x^6*cos(d*x^2 + c)^2 + (a^4 - a^2*b^2)*d*x^6*sin(2*d*x^2 + 2*c)^2 + 4*(a^2*b^2 - b^4)*d*x^6*sin(d*x^2 + c)^2 + 4*(a^3*b - a*b^3)*d*x^6*cos(d*x^2 + c) + 6*a*b^3*x^4*sin(d*x^2 + c) + (a^4 - a^2*b^2)*d*x^6 + 2*(2*(a^3*b - a*b^3)*d*x^6*cos(d*x^2 + c) - 3*a*b^3*x^4*sin(d*x^2 + c) + (a^4 - a^2*b^2)*d*x^6)*cos(2*d*x^2 + 2*c) - 6*((a^6 - a^4*b^2)*d*cos(2*d*x^2 + 2*c)^2 + 4*(a^4*b^2 - a^2*b^4)*d*cos(d*x^2 + c)^2 + (a^6 - a^4*b^2)*d*sin(2*d*x^2 + 2*c)^2 + 4*(a^5*b - a^3*b^3)*d*sin(2*d*x^2 + 2*c)*sin(d*x^2 + c) + 4*(a^4*b^2 - a^2*b^4)*d*sin(d*x^2 + c)^2 + 4*(a^5*b - a^3*b^3)*d*cos(d*x^2 + c) + (a^6 - a^4*b^2)*d + 2*(2*(a^5*b - a^3*b^3)*d*cos(d*x^2 + c) + (a^6 - a^4*b^2)*d)*cos(2*d*x^2 + 2*c))/integrate(2*(2*(2*a^2*b^2 - b^4)*d*x^5*cos(d*x^2 + c)^2 + 2*(2*a^2*b^2 - b^4)*d*x^5*sin(d*x^2 + c)^2 + (2*a^3*b - a*b^3)*d*x^5*cos(d*x^2 + c) + 2*a*b^3*x^3*sin(d*x^2 + c) + ((2*a^3*b - a*b^3)*d*x^5*cos(d*x^2 + c) - 2*a*b^3*x^3*sin(d*x^2 + c))*cos(2*d*x^2 + 2*c) + (2*a*b^3*x^3*cos(d*x^2 + c) + (2*a^3*b - a*b^3)*d*x^5*sin(d*x^2 + c) + 2*a^2*b^2*x^3)*sin(2*d*x^2 + 2*c))/((a^6 - a^4*b^2)*d*cos(2*d*x^2 + 2*c)^2 + 4*(a^4*b^2 - a^2*b^4)*d*cos(d*x^2 + c)^2 + (a^6 - a^4*b^2)*d*sin(2*d*x^2 + 2*c)^2 + 4*(a^5*b - a^3*b^3)*d*sin(2*d*x^2 + 2*c)*sin(d*x^2 + c) + 4*(a^4*b^2 - a^2*b^4)*d*sin(d*x^2 + c)^2 + 4*(a^5*b - a^3*b^3)*d*cos(d*x^2 + c) + (a^6 - a^4*b^2)*d + 2*(2*(a^5*b - a^3*b^3)*d*cos(d*x^2 + c) + (a^6 - a^4*b^2)*d)*cos(2*d*x^2 + 2*c)), x) + 2*(3*a*b^3*x^4*cos(d*x^2 + c) + 2*(a^3*b - a*b^3)*d*x^6*sin(d*x^2 + c) + 3*a^2*b^2*x^4)*sin(2*d*x^2 + 2*c))/((a^6 - a^4*b^2)*d*cos(2*d*x^2 + 2*c)^2 + 4*(a^4*b^2 - a^2*b^4)*d*cos(d*x^2 + c)^2 + (a^6 - a^4*b^2)*d*sin(2*d*x^2 + 2*c)^2 + 4*(a^5*b - a^3*b^3)*d*sin(2*d*x^2 + 2*c)*sin(d*x^2 + c) + 4*(a^4*b^2 - a^2*b^4)*d*sin(d*x^2 + c)^2 + 4*(a^5*b - a^3*b^3)*d*cos(d*x^2 + c) + (a^6 - a^4*b^2)*d + 2*(2*(a^5*b - a^3*b^3)*d*cos(d*x^2 + c) + (a^6 - a^4*b^2)*d)*cos(2*d*x^2 + 2*c))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5}{\left(a + \frac{b}{\cos(dx^2+c)}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b/cos(c + d*x^2))^2,x)
```

```
[Out] int(x^5/(a + b/cos(c + d*x^2))^2, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(a + b \sec(c + dx^2)\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate(x**5/(a+b*sec(d*x**2+c))**2,x)
```

```
[Out] Integral(x**5/(a + b*sec(c + d*x**2))**2, x)
```

$$3.24 \quad \int \frac{x^4}{(a+b \sec(c+dx^2))^2} dx$$

Optimal. Leaf size=21

$$\text{Int} \left( \frac{x^4}{(a+b \sec(c+dx^2))^2}, x \right)$$

[Out] Unintegrable(x^4/(a+b\*sec(d\*x^2+c))^2,x)

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{x^4}{(a+b \sec(c+dx^2))^2} dx$$

Verification is Not applicable to the result.

[In] Int[x^4/(a + b\*Sec[c + d\*x^2])^2,x]

[Out] Defer[Int][x^4/(a + b\*Sec[c + d\*x^2])^2, x]

Rubi steps

$$\int \frac{x^4}{(a+b \sec(c+dx^2))^2} dx = \int \frac{x^4}{(a+b \sec(c+dx^2))^2} dx$$

Mathematica [A] time = 6.88, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(a+b \sec(c+dx^2))^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[x^4/(a + b\*Sec[c + d\*x^2])^2,x]

[Out] Integrate[x^4/(a + b\*Sec[c + d\*x^2])^2, x]

fricas [A] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{x^4}{b^2 \sec(dx^2+c)^2 + 2ab \sec(dx^2+c) + a^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(a+b\*sec(d\*x^2+c))^2,x, algorithm="fricas")

[Out] integral(x^4/(b^2\*sec(d\*x^2 + c)^2 + 2\*a\*b\*sec(d\*x^2 + c) + a^2), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(b \sec(dx^2 + c) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(a+b\*sec(d\*x^2+c))^2,x, algorithm="giac")

[Out] integrate(x^4/(b\*sec(d\*x^2 + c) + a)^2, x)

maple [A] time = 1.41, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(a + b \sec(dx^2 + c))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(a+b\*sec(d\*x^2+c))^2,x)

[Out] int(x^4/(a+b\*sec(d\*x^2+c))^2,x)

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(a+b\*sec(d\*x^2+c))^2,x, algorithm="maxima")

[Out] Timed out

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{x^4}{\left(a + \frac{b}{\cos(dx^2+c)}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4/(a + b/cos(c + d*x^2))^2,x)`

[Out] `int(x^4/(a + b/cos(c + d*x^2))^2, x)`

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(a + b \sec(c + dx^2))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4/(a+b*sec(d*x**2+c))**2,x)`

[Out] `Integral(x**4/(a + b*sec(c + d*x**2))**2, x)`

$$3.25 \quad \int \frac{x^3}{(a+b \sec(c+dx^2))^2} dx$$

**Optimal.** Leaf size=596

$$\frac{b \operatorname{Li}_2\left(-\frac{ae^{i(dx^2+c)}}{b-\sqrt{b^2-a^2}}\right)}{a^2 d^2 \sqrt{b^2-a^2}} - \frac{b \operatorname{Li}_2\left(-\frac{ae^{i(dx^2+c)}}{b+\sqrt{b^2-a^2}}\right)}{a^2 d^2 \sqrt{b^2-a^2}} + \frac{b^2 \log(a \cos(c+dx^2)+b)}{2a^2 d^2 (a^2-b^2)} + \frac{ibx^2 \log\left(1+\frac{ae^{i(c+dx^2)}}{b-\sqrt{b^2-a^2}}\right)}{a^2 d \sqrt{b^2-a^2}} - \frac{ibx^2 \log\left(1+\frac{ae^{i(c+dx^2)}}{b+\sqrt{b^2-a^2}}\right)}{a^2 d \sqrt{b^2-a^2}}$$

[Out]  $1/4*x^4/a^2+1/2*b^2*\ln(b+a*\cos(d*x^2+c))/a^2/(a^2-b^2)/d^2-1/2*I*b^3*x^2*\ln(1+a*\exp(I*(d*x^2+c))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d+1/2*I*b^3*x^2*\ln(1+a*\exp(I*(d*x^2+c))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d-1/2*b^3*\operatorname{polylog}(2,-a*\exp(I*(d*x^2+c))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^2+1/2*b^3*\operatorname{polylog}(2,-a*\exp(I*(d*x^2+c))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^2+1/2*b^2*x^2*\sin(d*x^2+c)/a/(a^2-b^2)/d/(b+a*\cos(d*x^2+c))+I*b*x^2*\ln(1+a*\exp(I*(d*x^2+c))/(b-(-a^2+b^2)^(1/2)))/a^2/d/(-a^2+b^2)^(1/2)-I*b*x^2*\ln(1+a*\exp(I*(d*x^2+c))/(b+(-a^2+b^2)^(1/2)))/a^2/d/(-a^2+b^2)^(1/2)+b*\operatorname{polylog}(2,-a*\exp(I*(d*x^2+c))/(b-(-a^2+b^2)^(1/2)))/a^2/d^2/(-a^2+b^2)^(1/2)-b*\operatorname{polylog}(2,-a*\exp(I*(d*x^2+c))/(b+(-a^2+b^2)^(1/2)))/a^2/d^2/(-a^2+b^2)^(1/2)$

**Rubi [A]** time = 1.20, antiderivative size = 596, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 10, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$ , Rules used = {4204, 4191, 3324, 3321, 2264, 2190, 2279, 2391, 2668, 31}

$$\frac{b \operatorname{PolyLog}\left(2, -\frac{ae^{i(c+dx^2)}}{b-\sqrt{b^2-a^2}}\right)}{a^2 d^2 \sqrt{b^2-a^2}} - \frac{b^3 \operatorname{PolyLog}\left(2, -\frac{ae^{i(c+dx^2)}}{b-\sqrt{b^2-a^2}}\right)}{2a^2 d^2 (b^2-a^2)^{3/2}} - \frac{b \operatorname{PolyLog}\left(2, -\frac{ae^{i(c+dx^2)}}{\sqrt{b^2-a^2}+b}\right)}{a^2 d^2 \sqrt{b^2-a^2}} + \frac{b^3 \operatorname{PolyLog}\left(2, -\frac{ae^{i(c+dx^2)}}{\sqrt{b^2-a^2}+b}\right)}{2a^2 d^2 (b^2-a^2)^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^3/(a + b*\operatorname{Sec}[c + d*x^2])^2, x]$

[Out]  $x^4/(4*a^2) - ((I/2)*b^3*x^2*\operatorname{Log}[1 + (a*E^{(I*(c + d*x^2))})/(b - \operatorname{Sqrt}[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d) + (I*b*x^2*\operatorname{Log}[1 + (a*E^{(I*(c + d*x^2))})/(b - \operatorname{Sqrt}[-a^2 + b^2])])/(a^2*\operatorname{Sqrt}[-a^2 + b^2]*d) + ((I/2)*b^3*x^2*\operatorname{Log}[1 + (a*E^{(I*(c + d*x^2))})/(b + \operatorname{Sqrt}[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d) - (I*b*x^2*\operatorname{Log}[1 + (a*E^{(I*(c + d*x^2))})/(b + \operatorname{Sqrt}[-a^2 + b^2])])/(a^2*\operatorname{Sqrt}[-a^2 + b^2]*d) + (b^2*\operatorname{Log}[b + a*\operatorname{Cos}[c + d*x^2]])/(2*a^2*(a^2 - b^2)*d^2) - (b^3*\operatorname{PolyLog}[2, -((a*E^{(I*(c + d*x^2))})/(b - \operatorname{Sqrt}[-a^2 + b^2]))])/(2*a^2*(-a^2 + b^2)^(3/2)*d^2) + (b*\operatorname{PolyLog}[2, -((a*E^{(I*(c + d*x^2))})/(b - \operatorname{Sqrt}[-a^2 + b^2]))])/(a^2*\operatorname{Sqrt}[-a^2 + b^2]*d^2) + (b^3*\operatorname{PolyLog}[2, -((a*E^{(I*(c + d*x^2))})/(b + \operatorname{Sqrt}[-a^2 + b^2]))])/(2*a^2*(a^2 - b^2)*d^2) - (b*\operatorname{PolyLog}[2, -((a*E^{(I*(c + d*x^2))})/(b + \operatorname{Sqrt}[-a^2 + b^2]))])/(a^2*\operatorname{Sqrt}[-a^2 + b^2]*d^2)$

$$\frac{d*x^2)}{(b + \text{Sqrt}[-a^2 + b^2])})]/(2*a^2*(-a^2 + b^2)^{(3/2)*d^2} - (b*\text{PolyLog}[2, -((a*\text{E}^{(I*(c + d*x^2))})/(b + \text{Sqrt}[-a^2 + b^2])])/(a^2*\text{Sqrt}[-a^2 + b^2]*d^2) + (b^2*x^2*\text{Sin}[c + d*x^2])/(2*a*(a^2 - b^2)*d*(b + a*\text{Cos}[c + d*x^2]))))$$

### Rule 31

$$\text{Int}[(a_) + (b_)*(x_)]^{-1}, x\_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] /; \text{FreeQ}\{a, b\}, x]$$

### Rule 2190

$$\text{Int}[(F_)^{(g_)*((e_) + (f_)*(x_))})^{(n_)*((c_) + (d_)*(x_))^{(m_)}} / ((a_) + (b_)*((F_)^{(g_)*((e_) + (f_)*(x_))})^{(n_)}), x\_Symbol] \rightarrow \text{Simp}[(c + d*x)^m * \text{Log}[1 + (b*(F^{(g*(e + f*x)))^n})/a]] / (b*f*g*n*\text{Log}[F]), x] - \text{Dist}[(d*m) / (b*f*g*n*\text{Log}[F]), \text{Int}[(c + d*x)^{(m-1)} * \text{Log}[1 + (b*(F^{(g*(e + f*x)))^n})/a]], x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x] \&\& \text{IGtQ}[m, 0]$$

### Rule 2264

$$\text{Int}[(F_)^{(u_)*((f_) + (g_)*(x_))^{(m_)}} / ((a_) + (b_)*(F_)^{(u_)} + (c_)* (F_)^{(v_)}), x\_Symbol] \rightarrow \text{With}\{q = \text{Rt}[b^2 - 4*a*c, 2]\}, \text{Dist}[(2*c)/q, \text{Int}[(f + g*x)^m * F^u / (b - q + 2*c * F^u), x], x] - \text{Dist}[(2*c)/q, \text{Int}[(f + g*x)^m * F^u / (b + q + 2*c * F^u), x], x]] /; \text{FreeQ}\{F, a, b, c, f, g\}, x] \&\& \text{EqQ}[v, 2*u] \&\& \text{LinearQ}[u, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{IGtQ}[m, 0]$$

### Rule 2279

$$\text{Int}[\text{Log}[(a_) + (b_)*((F_)^{(e_)*((c_) + (d_)*(x_))})^{(n_)}], x\_Symbol] \rightarrow \text{Dist}[1/(d*e*n*\text{Log}[F]), \text{Subst}[\text{Int}[\text{Log}[a + b*x]/x, x], x, (F^{(e*(c + d*x))})^n], x] /; \text{FreeQ}\{F, a, b, c, d, e, n\}, x] \&\& \text{GtQ}[a, 0]$$

### Rule 2391

$$\text{Int}[\text{Log}[(c_)*((d_) + (e_)*(x_))^{(n_)}] / (x_), x\_Symbol] \rightarrow -\text{Simp}[\text{PolyLog}[2, -(c*e*x^n)]/n, x] /; \text{FreeQ}\{c, d, e, n\}, x] \&\& \text{EqQ}[c*d, 1]$$

### Rule 2668

$$\text{Int}[\cos[(e_) + (f_)*(x_)]^{(p_)*((a_) + (b_)*\text{sin}[(e_) + (f_)*(x_)])^{(m_)}], x\_Symbol] \rightarrow \text{Dist}[1/(b^p*f), \text{Subst}[\text{Int}[(a + x)^m * (b^2 - x^2)^{(p-1)/2}], x], x, b*\text{Sin}[e + f*x]], x] /; \text{FreeQ}\{a, b, e, f, m\}, x] \&\& \text{IntegerQ}[(p - 1)/2] \&\& \text{NeQ}[a^2 - b^2, 0]$$

### Rule 3321

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)])], x_Symbol] := Dist[2, Int[((c + d*x)^m*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

#### Rule 3324

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] := Simp[(b*(c + d*x)^m*Cos[e + f*x])/(f*(a^2 - b^2)*(a + b*Sin[e + f*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x], x] - Dist[(b*d*m)/(f*(a^2 - b^2)), Int[((c + d*x)^(m - 1)*Cos[e + f*x])/(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

#### Rule 4191

```
Int[(csc[(e_.) + (f_.)*(x_)])*(b_.) + (a_.))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGtQ[m, 0]
```

#### Rule 4204

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]
```

#### Rubi steps

$$\begin{aligned}
\int \frac{x^3}{(a + b \sec(c + dx^2))^2} dx &= \frac{1}{2} \text{Subst} \left( \int \frac{x}{(a + b \sec(c + dx))^2} dx, x, x^2 \right) \\
&= \frac{1}{2} \text{Subst} \left( \int \left( \frac{x}{a^2} + \frac{b^2 x}{a^2(b + a \cos(c + dx))^2} - \frac{2bx}{a^2(b + a \cos(c + dx))} \right) dx, x, x^2 \right) \\
&= \frac{x^4}{4a^2} - \frac{b \text{Subst} \left( \int \frac{x}{b + a \cos(c + dx)} dx, x, x^2 \right)}{a^2} + \frac{b^2 \text{Subst} \left( \int \frac{x}{(b + a \cos(c + dx))^2} dx, x, x^2 \right)}{2a^2} \\
&= \frac{x^4}{4a^2} + \frac{b^2 x^2 \sin(c + dx^2)}{2a(a^2 - b^2)d(b + a \cos(c + dx^2))} - \frac{(2b) \text{Subst} \left( \int \frac{e^{i(c+dx)} x}{a + 2be^{i(c+dx)} + ae^{2i(c+dx)}} dx, x, x^2 \right)}{a^2} \\
&= \frac{x^4}{4a^2} + \frac{b^2 x^2 \sin(c + dx^2)}{2a(a^2 - b^2)d(b + a \cos(c + dx^2))} - \frac{b^3 \text{Subst} \left( \int \frac{e^{i(c+dx)} x}{a + 2be^{i(c+dx)} + ae^{2i(c+dx)}} dx, x, x^2 \right)}{a^2(a^2 - b^2)} \\
&= \frac{x^4}{4a^2} + \frac{ibx^2 \log \left( 1 + \frac{ae^{i(c+dx^2)}}{b - \sqrt{-a^2 + b^2}} \right)}{a^2 \sqrt{-a^2 + b^2} d} - \frac{ibx^2 \log \left( 1 + \frac{ae^{i(c+dx^2)}}{b + \sqrt{-a^2 + b^2}} \right)}{a^2 \sqrt{-a^2 + b^2} d} + \frac{b^2 \log(b + a \cos(c + dx^2))}{2a^2(a^2 - b^2)} \\
&= \frac{x^4}{4a^2} - \frac{ib^3 x^2 \log \left( 1 + \frac{ae^{i(c+dx^2)}}{b - \sqrt{-a^2 + b^2}} \right)}{2a^2(-a^2 + b^2)^{3/2} d} + \frac{ibx^2 \log \left( 1 + \frac{ae^{i(c+dx^2)}}{b - \sqrt{-a^2 + b^2}} \right)}{a^2 \sqrt{-a^2 + b^2} d} + \frac{ib^3 x^2 \log \left( 1 + \frac{ae^{i(c+dx^2)}}{b + \sqrt{-a^2 + b^2}} \right)}{2a^2(-a^2 + b^2)^{3/2} d} \\
&= \frac{x^4}{4a^2} - \frac{ib^3 x^2 \log \left( 1 + \frac{ae^{i(c+dx^2)}}{b - \sqrt{-a^2 + b^2}} \right)}{2a^2(-a^2 + b^2)^{3/2} d} + \frac{ibx^2 \log \left( 1 + \frac{ae^{i(c+dx^2)}}{b - \sqrt{-a^2 + b^2}} \right)}{a^2 \sqrt{-a^2 + b^2} d} + \frac{ib^3 x^2 \log \left( 1 + \frac{ae^{i(c+dx^2)}}{b + \sqrt{-a^2 + b^2}} \right)}{2a^2(-a^2 + b^2)^{3/2} d} \\
&= \frac{x^4}{4a^2} - \frac{ib^3 x^2 \log \left( 1 + \frac{ae^{i(c+dx^2)}}{b - \sqrt{-a^2 + b^2}} \right)}{2a^2(-a^2 + b^2)^{3/2} d} + \frac{ibx^2 \log \left( 1 + \frac{ae^{i(c+dx^2)}}{b - \sqrt{-a^2 + b^2}} \right)}{a^2 \sqrt{-a^2 + b^2} d} + \frac{ib^3 x^2 \log \left( 1 + \frac{ae^{i(c+dx^2)}}{b + \sqrt{-a^2 + b^2}} \right)}{2a^2(-a^2 + b^2)^{3/2} d}
\end{aligned}$$

**Mathematica [A]** time = 9.03, size = 1069, normalized size = 1.79

$$\frac{(b + a \cos(dx^2 + c))(b^2 c \sin(dx^2 + c) - b^2(dx^2 + c) \sin(dx^2 + c)) \sec^2(dx^2 + c)}{2a(b - a)(a + b)d^2(a + b \sec(dx^2 + c))^2} + \frac{b \cos^2\left(\frac{1}{2}(dx^2 + c)\right)(b + a \cos(dx^2 + c))}{2a(b - a)(a + b)d^2(a + b \sec(dx^2 + c))^2}$$



Warning: Unable to verify antiderivative.

[In] Integrate[x^3/(a + b\*Sec[c + d\*x^2])^2,x]

[Out] 
$$\begin{aligned} &((-c + d*x^2)*(c + d*x^2)*(b + a*\cos[c + d*x^2])^2*\sec[c + d*x^2]^2)/(4*a^2 \\ &*d^2*(a + b*\sec[c + d*x^2])^2) + ((b + a*\cos[c + d*x^2])* \sec[c + d*x^2]^2*( \\ &b^2*c*\sin[c + d*x^2] - b^2*(c + d*x^2)*\sin[c + d*x^2]))/(2*a*(-a + b)*(a + \\ &b)*d^2*(a + b*\sec[c + d*x^2])^2) + (b*\cos[(c + d*x^2)/2]^2*(b + a*\cos[c + d \\ &*x^2]))*((-2*(2*a^2 - b^2)*c*\text{ArcTan}[(\text{Sqrt}[a - b]*\text{Tan}[(c + d*x^2)/2])/\text{Sqrt}[-a \\ &- b]))/(\text{Sqrt}[-a - b]*\text{Sqrt}[a - b]) + b*(-\text{Log}[\sec[(c + d*x^2)/2]^2] + \text{Log}[-( \\ &(b + a*\cos[c + d*x^2])* \sec[(c + d*x^2)/2]^2])) - (I*(2*a^2 - b^2)*(\text{Log}[1 + \\ &I*\text{Tan}[(c + d*x^2)/2]]*\text{Log}[(I*(\text{Sqrt}[a + b] - \text{Sqrt}[a - b]*\text{Tan}[(c + d*x^2)/2]) \\ &)/(\text{Sqrt}[a - b] + I*\text{Sqrt}[a + b])] - \text{Log}[1 - I*\text{Tan}[(c + d*x^2)/2]]*\text{Log}[(\text{Sqrt}[ \\ &a + b] - \text{Sqrt}[a - b]*\text{Tan}[(c + d*x^2)/2])/(I*\text{Sqrt}[a - b] + \text{Sqrt}[a + b])] + \text{L} \\ &\text{og}[1 - I*\text{Tan}[(c + d*x^2)/2]]*\text{Log}[(I*(\text{Sqrt}[a + b] + \text{Sqrt}[a - b]*\text{Tan}[(c + d*x \\ &^2)/2])]/(\text{Sqrt}[a - b] + I*\text{Sqrt}[a + b])] - \text{Log}[1 + I*\text{Tan}[(c + d*x^2)/2]]*\text{Log} \\ &[(\text{Sqrt}[a + b] + \text{Sqrt}[a - b]*\text{Tan}[(c + d*x^2)/2])/(I*\text{Sqrt}[a - b] + \text{Sqrt}[a + b \\ &])]) - \text{PolyLog}[2, (\text{Sqrt}[a - b]*(1 - I*\text{Tan}[(c + d*x^2)/2]))/(\text{Sqrt}[a - b] - I* \\ &\text{Sqrt}[a + b])] + \text{PolyLog}[2, (\text{Sqrt}[a - b]*(1 - I*\text{Tan}[(c + d*x^2)/2]))/(\text{Sqrt}[a \\ &- b] + I*\text{Sqrt}[a + b])] - \text{PolyLog}[2, (\text{Sqrt}[a - b]*(1 + I*\text{Tan}[(c + d*x^2)/2] \\ &))/(\text{Sqrt}[a - b] - I*\text{Sqrt}[a + b])] + \text{PolyLog}[2, (\text{Sqrt}[a - b]*(1 + I*\text{Tan}[(c + \\ &d*x^2)/2]))/(\text{Sqrt}[a - b] + I*\text{Sqrt}[a + b])])]/(\text{Sqrt}[a - b]*\text{Sqrt}[a + b])* \text{Se} \\ &c[c + d*x^2]^2*((2*a^2 - b^2)*d*x^2 + a*b*\sin[c + d*x^2])*(\text{Sqrt}[a + b] - \text{Sq} \\ &\text{rt}[a - b]*\text{Tan}[(c + d*x^2)/2])*(\text{Sqrt}[a + b] + \text{Sqrt}[a - b]*\text{Tan}[(c + d*x^2)/2] \\ &))/(2*a^2*(a^2 - b^2)*d^2*(a + b*\sec[c + d*x^2])^2*(-((2*a^2 - b^2)*(c - I* \\ &\text{Log}[1 - I*\text{Tan}[(c + d*x^2)/2]] + I*\text{Log}[1 + I*\text{Tan}[(c + d*x^2)/2]])) + a*b*\sin \\ &[c + d*x^2])) \end{aligned}$$

**fricas** [B] time = 1.01, size = 1928, normalized size = 3.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+b\*sec(d\*x^2+c))^2,x, algorithm="fricas")

[Out] 
$$\begin{aligned} &1/4*((a^5 - 2*a^3*b^2 + a*b^4)*d^2*x^4*\cos(d*x^2 + c) + (a^4*b - 2*a^2*b^3 \\ &+ b^5)*d^2*x^4 + 2*(a^3*b^2 - a*b^4)*d*x^2*\sin(d*x^2 + c) - (2*a^3*b^2 - a* \\ &b^4 + (2*a^4*b - a^2*b^3)*\cos(d*x^2 + c))*\text{sqrt}(-(a^2 - b^2)/a^2)*\text{dilog}(-(b* \\ &\cos(d*x^2 + c) + I*b*\sin(d*x^2 + c) + (a*\cos(d*x^2 + c) + I*a*\sin(d*x^2 + c \\ &))*\text{sqrt}(-(a^2 - b^2)/a^2) + a)/a + 1) + (2*a^3*b^2 - a*b^4 + (2*a^4*b - a^2 \\ &*b^3)*\cos(d*x^2 + c))*\text{sqrt}(-(a^2 - b^2)/a^2)*\text{dilog}(-(b*\cos(d*x^2 + c) + I*b \\ &*\sin(d*x^2 + c) - (a*\cos(d*x^2 + c) + I*a*\sin(d*x^2 + c))*\text{sqrt}(-(a^2 - b^2) \\ &/a^2) + a)/a + 1) - (2*a^3*b^2 - a*b^4 + (2*a^4*b - a^2*b^3)*\cos(d*x^2 + c) \\ &)*\text{sqrt}(-(a^2 - b^2)/a^2)*\text{dilog}(-(b*\cos(d*x^2 + c) - I*b*\sin(d*x^2 + c) + (a \\ &*\cos(d*x^2 + c) - I*a*\sin(d*x^2 + c))*\text{sqrt}(-(a^2 - b^2)/a^2) + a)/a + 1) + \end{aligned}$$

```
(2*a^3*b^2 - a*b^4 + (2*a^4*b - a^2*b^3)*cos(d*x^2 + c))*sqrt(-(a^2 - b^2)/
a^2)*dilog(-(b*cos(d*x^2 + c) - I*b*sin(d*x^2 + c) - (a*cos(d*x^2 + c) - I*
a*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2) + a)/a + 1) + (-I*(2*a^3*b^2 - a*b
^4)*d*x^2 - I*(2*a^3*b^2 - a*b^4)*c + (-I*(2*a^4*b - a^2*b^3)*d*x^2 - I*(2*
a^4*b - a^2*b^3)*c)*cos(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2)*log((b*cos(d*x^2
+ c) + I*b*sin(d*x^2 + c) + (a*cos(d*x^2 + c) + I*a*sin(d*x^2 + c))*sqrt(-
(a^2 - b^2)/a^2) + a)/a) + (I*(2*a^3*b^2 - a*b^4)*d*x^2 + I*(2*a^3*b^2 - a*
b^4)*c + (I*(2*a^4*b - a^2*b^3)*d*x^2 + I*(2*a^4*b - a^2*b^3)*c)*cos(d*x^2
+ c))*sqrt(-(a^2 - b^2)/a^2)*log((b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c) - (
a*cos(d*x^2 + c) + I*a*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2) + a)/a) + (I*
(2*a^3*b^2 - a*b^4)*d*x^2 + I*(2*a^3*b^2 - a*b^4)*c + (I*(2*a^4*b - a^2*b^3
)*d*x^2 + I*(2*a^4*b - a^2*b^3)*c)*cos(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2)*l
og((b*cos(d*x^2 + c) - I*b*sin(d*x^2 + c) + (a*cos(d*x^2 + c) - I*a*sin(d*x
^2 + c))*sqrt(-(a^2 - b^2)/a^2) + a)/a) + (-I*(2*a^3*b^2 - a*b^4)*d*x^2 - I
*(2*a^3*b^2 - a*b^4)*c + (-I*(2*a^4*b - a^2*b^3)*d*x^2 - I*(2*a^4*b - a^2*b
^3)*c)*cos(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2)*log((b*cos(d*x^2 + c) - I*b*s
in(d*x^2 + c) - (a*cos(d*x^2 + c) - I*a*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/a
^2) + a)/a) + (a^2*b^3 - b^5 + (a^3*b^2 - a*b^4)*cos(d*x^2 + c) + (-I*(2*a^
4*b - a^2*b^3)*c*cos(d*x^2 + c) - I*(2*a^3*b^2 - a*b^4)*c)*sqrt(-(a^2 - b^2
)/a^2))*log(2*a*cos(d*x^2 + c) + 2*I*a*sin(d*x^2 + c) + 2*a*sqrt(-(a^2 - b^
2)/a^2) + 2*b) + (a^2*b^3 - b^5 + (a^3*b^2 - a*b^4)*cos(d*x^2 + c) + (I*(2*
a^4*b - a^2*b^3)*c*cos(d*x^2 + c) + I*(2*a^3*b^2 - a*b^4)*c)*sqrt(-(a^2 - b
^2)/a^2))*log(2*a*cos(d*x^2 + c) - 2*I*a*sin(d*x^2 + c) + 2*a*sqrt(-(a^2 -
b^2)/a^2) + 2*b) + (a^2*b^3 - b^5 + (a^3*b^2 - a*b^4)*cos(d*x^2 + c) + (-I*
(2*a^4*b - a^2*b^3)*c*cos(d*x^2 + c) - I*(2*a^3*b^2 - a*b^4)*c)*sqrt(-(a^2
- b^2)/a^2))*log(-2*a*cos(d*x^2 + c) + 2*I*a*sin(d*x^2 + c) + 2*a*sqrt(-(a^
2 - b^2)/a^2) - 2*b) + (a^2*b^3 - b^5 + (a^3*b^2 - a*b^4)*cos(d*x^2 + c) +
(I*(2*a^4*b - a^2*b^3)*c*cos(d*x^2 + c) + I*(2*a^3*b^2 - a*b^4)*c)*sqrt(-(a
^2 - b^2)/a^2))*log(-2*a*cos(d*x^2 + c) - 2*I*a*sin(d*x^2 + c) + 2*a*sqrt(-
(a^2 - b^2)/a^2) - 2*b))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d^2*cos(d*x^2 + c) +
(a^6*b - 2*a^4*b^3 + a^2*b^5)*d^2)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(b \sec(dx^2 + c) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+b\*sec(d\*x^2+c))^2,x, algorithm="giac")

[Out] integrate(x^3/(b\*sec(d\*x^2 + c) + a)^2, x)

**maple** [F] time = 1.54, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a + b \sec(dx^2 + c))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/(a+b*sec(d*x^2+c))^2,x)`

[Out] `int(x^3/(a+b*sec(d*x^2+c))^2,x)`

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(a+b*sec(d*x^2+c))^2,x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*b^2-4\*a^2>0)', see `assume?` for more details)Is 4\*b^2-4\*a^2 positive or negative?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\left(a + \frac{b}{\cos(dx^2+c)}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/(a + b/cos(c + d*x^2))^2,x)`

[Out] `int(x^3/(a + b/cos(c + d*x^2))^2, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a + b \sec(c + dx^2))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3/(a+b*sec(d*x**2+c))**2,x)`

[Out] `Integral(x**3/(a + b*sec(c + d*x**2))**2, x)`

$$3.26 \quad \int \frac{x^2}{(a+b \sec(c+dx^2))^2} dx$$

Optimal. Leaf size=21

$$\text{Int} \left( \frac{x^2}{(a+b \sec(c+dx^2))^2}, x \right)$$

[Out] Unintegrable(x^2/(a+b\*sec(d\*x^2+c))^2,x)

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{x^2}{(a+b \sec(c+dx^2))^2} dx$$

Verification is Not applicable to the result.

[In] Int[x^2/(a + b\*Sec[c + d\*x^2])^2,x]

[Out] Defer[Int][x^2/(a + b\*Sec[c + d\*x^2])^2, x]

Rubi steps

$$\int \frac{x^2}{(a+b \sec(c+dx^2))^2} dx = \int \frac{x^2}{(a+b \sec(c+dx^2))^2} dx$$

Mathematica [A] time = 5.80, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a+b \sec(c+dx^2))^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[x^2/(a + b\*Sec[c + d\*x^2])^2,x]

[Out] Integrate[x^2/(a + b\*Sec[c + d\*x^2])^2, x]

fricas [A] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{x^2}{b^2 \sec(dx^2+c)^2 + 2ab \sec(dx^2+c) + a^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b\*sec(d\*x^2+c))^2,x, algorithm="fricas")

[Out] integral(x^2/(b^2\*sec(d\*x^2 + c)^2 + 2\*a\*b\*sec(d\*x^2 + c) + a^2), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(b \sec(dx^2 + c) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b\*sec(d\*x^2+c))^2,x, algorithm="giac")

[Out] integrate(x^2/(b\*sec(d\*x^2 + c) + a)^2, x)

maple [A] time = 1.13, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a + b \sec(dx^2 + c))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a+b\*sec(d\*x^2+c))^2,x)

[Out] int(x^2/(a+b\*sec(d\*x^2+c))^2,x)

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b\*sec(d\*x^2+c))^2,x, algorithm="maxima")

[Out] Timed out

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{x^2}{\left(a + \frac{b}{\cos(dx^2+c)}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(a + b/cos(c + d*x^2))^2,x)`

[Out] `int(x^2/(a + b/cos(c + d*x^2))^2, x)`

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a + b \sec(c + dx^2))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(a+b*sec(d*x**2+c))**2,x)`

[Out] `Integral(x**2/(a + b*sec(c + d*x**2))**2, x)`

$$3.27 \quad \int \frac{x}{(a+b \sec(c+dx^2))^2} dx$$

Optimal. Leaf size=123

$$-\frac{b(2a^2 - b^2) \tanh^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx^2)\right)}{\sqrt{a+b}}\right)}{a^2 d(a-b)^{3/2}(a+b)^{3/2}} + \frac{b^2 \tan(c+dx^2)}{2ad(a^2 - b^2)(a+b \sec(c+dx^2))} + \frac{x^2}{2a^2}$$

[Out]  $1/2*x^2/a^2-b*(2*a^2-b^2)*\operatorname{arctanh}((a-b)^{(1/2)}*\tan(1/2*d*x^2+1/2*c)/(a+b)^{(1/2}))/a^2/(a-b)^{(3/2)}/(a+b)^{(3/2)}/d+1/2*b^2*\tan(d*x^2+c)/a/(a^2-b^2)/d/(a+b*\sec(d*x^2+c))$

**Rubi [A]** time = 0.25, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {4204, 3785, 3919, 3831, 2659, 208}

$$-\frac{b(2a^2 - b^2) \tanh^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx^2)\right)}{\sqrt{a+b}}\right)}{a^2 d(a-b)^{3/2}(a+b)^{3/2}} + \frac{b^2 \tan(c+dx^2)}{2ad(a^2 - b^2)(a+b \sec(c+dx^2))} + \frac{x^2}{2a^2}$$

Antiderivative was successfully verified.

[In] Int[x/(a + b\*Sec[c + d\*x^2])^2,x]

[Out]  $x^2/(2*a^2) - (b*(2*a^2 - b^2)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a - b]*\operatorname{Tan}[(c + d*x^2)/2])/(\operatorname{Sqrt}[a + b]))/(a^2*(a - b)^{(3/2)}*(a + b)^{(3/2)}*d) + (b^2*\operatorname{Tan}[c + d*x^2])/(2*a*(a^2 - b^2)*d*(a + b*\operatorname{Sec}[c + d*x^2]))$

Rule 208

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 2659

Int[((a\_) + (b\_)\*sin[Pi/2 + (c\_) + (d\_)\*(x\_)])^(-1), x\_Symbol] := With[{e = FreeFactors[Tan[(c + d\*x)/2], x]}, Dist[(2\*e)/d, Subst[Int[1/(a + b + (a - b)\*e^2\*x^2), x], x, Tan[(c + d\*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 3785

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_.))^(n_), x_Symbol] := Simp[(b^2*Cot
[c + d*x]*(a + b*Csc[c + d*x])^(n + 1))/(a*d*(n + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

### Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbo
l] := Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

### Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_.)), x_Symbol] := Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
a*d, 0]
```

### Rule 4204

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

### Rubi steps



$$\begin{aligned}
\int \frac{x}{(a + b \sec(c + dx^2))^2} dx &= \frac{1}{2} \text{Subst} \left( \int \frac{1}{(a + b \sec(c + dx))^2} dx, x, x^2 \right) \\
&= \frac{b^2 \tan(c + dx^2)}{2a(a^2 - b^2)d(a + b \sec(c + dx^2))} - \frac{\text{Subst} \left( \int \frac{-a^2 + b^2 + ab \sec(c + dx)}{a + b \sec(c + dx)} dx, x, x^2 \right)}{2a(a^2 - b^2)} \\
&= \frac{x^2}{2a^2} + \frac{b^2 \tan(c + dx^2)}{2a(a^2 - b^2)d(a + b \sec(c + dx^2))} - \frac{(b(2a^2 - b^2)) \text{Subst} \left( \int \frac{\sec(c + dx)}{a + b \sec(c + dx)} dx, x, x^2 \right)}{2a^2(a^2 - b^2)} \\
&= \frac{x^2}{2a^2} + \frac{b^2 \tan(c + dx^2)}{2a(a^2 - b^2)d(a + b \sec(c + dx^2))} - \frac{(2a^2 - b^2) \text{Subst} \left( \int \frac{1}{1 + \frac{a \cos(c + dx)}{b}} dx, x, x^2 \right)}{2a^2(a^2 - b^2)} \\
&= \frac{x^2}{2a^2} + \frac{b^2 \tan(c + dx^2)}{2a(a^2 - b^2)d(a + b \sec(c + dx^2))} - \frac{(2a^2 - b^2) \text{Subst} \left( \int \frac{1}{1 + \frac{a}{b} + (1 - \frac{a}{b})x^2} dx, x, x^2 \right)}{a^2(a^2 - b^2)d} \\
&= \frac{x^2}{2a^2} - \frac{b(2a^2 - b^2) \tanh^{-1} \left( \frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c + dx^2)\right)}{\sqrt{a+b}} \right)}{a^2(a-b)^{3/2}(a+b)^{3/2}d} + \frac{b^2 \tan(c + dx^2)}{2a(a^2 - b^2)d(a + b \sec(c + dx^2))}
\end{aligned}$$

**Mathematica [A]** time = 0.69, size = 153, normalized size = 1.24

$$\frac{\frac{b((a^2 - b^2)(c + dx^2) + ab \sin(c + dx^2)) + a(a^2 - b^2)(c + dx^2) \cos(c + dx^2)}{a \cos(c + dx^2) + b} - \frac{2b(b^2 - 2a^2) \tanh^{-1} \left( \frac{(b-a) \tan\left(\frac{1}{2}(c + dx^2)\right)}{\sqrt{a^2 - b^2}} \right)}{\sqrt{a^2 - b^2}}}{2a^2 d(a - b)(a + b)}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a + b\*Sec[c + d\*x^2])^2,x]

[Out] ((-2\*b\*(-2\*a^2 + b^2)\*ArcTanh[((-a + b)\*Tan[(c + d\*x^2)/2])/Sqrt[a^2 - b^2])]/Sqrt[a^2 - b^2] + (a\*(a^2 - b^2)\*(c + d\*x^2)\*Cos[c + d\*x^2] + b\*((a^2 - b^2)\*(c + d\*x^2) + a\*b\*Sin[c + d\*x^2]))/(b + a\*Cos[c + d\*x^2]))/(2\*a^2\*(a - b)\*(a + b)\*d)

**fricas [B]** time = 2.06, size = 525, normalized size = 4.27

$$\frac{2(a^5 - 2a^3b^2 + ab^4)dx^2 \cos(dx^2 + c) + 2(a^4b - 2a^2b^3 + b^5)dx^2 + (2a^2b^2 - b^4 + (2a^3b - ab^3) \cos(dx^2 + c))\sqrt{4((a^7 - 2a^5b^2 + a^3b^4)d \cos(dx^2 + c))}}{4((a^7 - 2a^5b^2 + a^3b^4)d \cos(dx^2 + c))\sqrt{4((a^7 - 2a^5b^2 + a^3b^4)d \cos(dx^2 + c))}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*sec(d\*x^2+c))^2,x, algorithm="fricas")

[Out] [1/4\*(2\*(a^5 - 2\*a^3\*b^2 + a\*b^4)\*d\*x^2\*cos(d\*x^2 + c) + 2\*(a^4\*b - 2\*a^2\*b^3 + b^5)\*d\*x^2 + (2\*a^2\*b^2 - b^4 + (2\*a^3\*b - a\*b^3)\*cos(d\*x^2 + c))\*sqrt(a^2 - b^2)\*log((2\*a\*b\*cos(d\*x^2 + c) - (a^2 - 2\*b^2)\*cos(d\*x^2 + c))^2 - 2\*sqrt(a^2 - b^2)\*(b\*cos(d\*x^2 + c) + a)\*sin(d\*x^2 + c) + 2\*a^2 - b^2)/(a^2\*cos(d\*x^2 + c)^2 + 2\*a\*b\*cos(d\*x^2 + c) + b^2)) + 2\*(a^3\*b^2 - a\*b^4)\*sin(d\*x^2 + c))/((a^7 - 2\*a^5\*b^2 + a^3\*b^4)\*d\*cos(d\*x^2 + c) + (a^6\*b - 2\*a^4\*b^3 + a^2\*b^5)\*d), 1/2\*((a^5 - 2\*a^3\*b^2 + a\*b^4)\*d\*x^2\*cos(d\*x^2 + c) + (a^4\*b - 2\*a^2\*b^3 + b^5)\*d\*x^2 - (2\*a^2\*b^2 - b^4 + (2\*a^3\*b - a\*b^3)\*cos(d\*x^2 + c))\*sqrt(-a^2 + b^2)\*arctan(-sqrt(-a^2 + b^2)\*(b\*cos(d\*x^2 + c) + a)/((a^2 - b^2)\*sin(d\*x^2 + c))) + (a^3\*b^2 - a\*b^4)\*sin(d\*x^2 + c))/((a^7 - 2\*a^5\*b^2 + a^3\*b^4)\*d\*cos(d\*x^2 + c) + (a^6\*b - 2\*a^4\*b^3 + a^2\*b^5)\*d)]

**giac [A]** time = 0.54, size = 195, normalized size = 1.59

$$\frac{b^2 \tan\left(\frac{1}{2} dx^2 + \frac{1}{2} c\right)}{(a^3d - ab^2d)\left(a \tan\left(\frac{1}{2} dx^2 + \frac{1}{2} c\right)^2 - b \tan\left(\frac{1}{2} dx^2 + \frac{1}{2} c\right)^2 - a - b\right)} + \frac{(2a^2b - b^3)\left(\pi \left[\frac{dx^2+c}{2\pi} + \frac{1}{2}\right] \operatorname{sgn}(2a - 2b) + a\right)}{(a^4d - a^2b^2d)\sqrt{-a^2 + b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*sec(d\*x^2+c))^2,x, algorithm="giac")

[Out] -b^2\*tan(1/2\*d\*x^2 + 1/2\*c)/((a^3\*d - a\*b^2\*d)\*(a\*tan(1/2\*d\*x^2 + 1/2\*c)^2 - b\*tan(1/2\*d\*x^2 + 1/2\*c)^2 - a - b)) + (2\*a^2\*b - b^3)\*(pi\*floor(1/2\*(d\*x^2 + c)/pi + 1/2)\*sgn(2\*a - 2\*b) + arctan((a\*tan(1/2\*d\*x^2 + 1/2\*c) - b\*tan(1/2\*d\*x^2 + 1/2\*c))/sqrt(-a^2 + b^2)))/((a^4\*d - a^2\*b^2\*d)\*sqrt(-a^2 + b^2)) + 1/2\*(d\*x^2 + c)/(a^2\*d)

**maple [A]** time = 0.53, size = 214, normalized size = 1.74

$$\frac{b^2 \tan\left(\frac{dx^2}{2} + \frac{c}{2}\right)}{da(a^2 - b^2)\left(a\left(\tan^2\left(\frac{dx^2}{2} + \frac{c}{2}\right)\right) - \left(\tan^2\left(\frac{dx^2}{2} + \frac{c}{2}\right)\right)b - a - b\right)} - \frac{2b \operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx^2}{2} + \frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{d(a-b)(a+b)\sqrt{(a-b)(a+b)}} + \frac{b^3 \operatorname{arctan}\left(\tan\left(\frac{dx^2}{2} + \frac{c}{2}\right)\right)}{da^2(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a+b\*sec(d\*x^2+c))^2,x)

[Out]  $-1/d*b^2/a/(a^2-b^2)*\tan(1/2*d*x^2+1/2*c)/(a*\tan(1/2*d*x^2+1/2*c)^2-\tan(1/2*d*x^2+1/2*c)^2*b-a-b)-2/d*b/(a-b)/(a+b)/((a-b)*(a+b))^{1/2}*\operatorname{arctanh}((a-b)*\tan(1/2*d*x^2+1/2*c)/((a-b)*(a+b))^{1/2}))+1/d*b^3/a^2/(a-b)/(a+b)/((a-b)*(a+b))^{1/2}*\operatorname{arctanh}((a-b)*\tan(1/2*d*x^2+1/2*c)/((a-b)*(a+b))^{1/2}))+1/d/a^2*\operatorname{arctan}(\tan(1/2*d*x^2+1/2*c))$

**maxima [B]** time = 66.78, size = 8871, normalized size = 72.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*sec(d\*x^2+c))^2,x, algorithm="maxima")

[Out]  $1/2*((a^6 - 2*a^4*b^2 + a^2*b^4)*d*x^2*\cos(2*d*x^2 + 2*c)^2 + 4*(a^4*b^2 - 2*a^2*b^4 + b^6)*d*x^2*\cos(d*x^2 + c)^2 + (a^6 - 2*a^4*b^2 + a^2*b^4)*d*x^2*\sin(2*d*x^2 + 2*c)^2 + 4*(a^4*b^2 - 2*a^2*b^4 + b^6)*d*x^2*\sin(d*x^2 + c)^2 + 4*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x^2*\cos(d*x^2 + c) + (a^6 - 2*a^4*b^2 + a^2*b^4)*d*x^2 + (2*a^4*b - a^2*b^3 + (2*a^4*b - a^2*b^3)*\cos(2*d*x^2 + 2*c)^2 + 4*(2*a^2*b^3 - b^5)*\cos(d*x^2 + c)^2 + (2*a^4*b - a^2*b^3)*\sin(2*d*x^2 + 2*c)^2 + 4*(2*a^3*b^2 - a*b^4)*\sin(2*d*x^2 + 2*c)*\sin(d*x^2 + c) + 4*(2*a^2*b^3 - b^5)*\sin(d*x^2 + c)^2 + 2*(2*a^4*b - a^2*b^3 + 2*(2*a^3*b^2 - a*b^4)*\cos(d*x^2 + c))*\cos(2*d*x^2 + 2*c) + 4*(2*a^3*b^2 - a*b^4)*\cos(d*x^2 + c))*\sqrt{-a^2 + b^2}*\operatorname{arctan}2(2*(4*(a^6 - a^4*b^2)*\cos(d*x^2 + 2*c)^4*\cos(c)*\sin(c) - 4*(a^6 - a^4*b^2)*\cos(c)*\sin(d*x^2 + 2*c)^4*\sin(c) + 4*(3*(a^5*b - a^3*b^3)*\cos(c)^2*\sin(c) + (a^5*b - a^3*b^3)*\sin(c)^3)*\cos(d*x^2 + 2*c)^3 - 4*((a^5*b - a^3*b^3)*\cos(c)^3 + 3*(a^5*b - a^3*b^3)*\cos(c)*\sin(c)^2 + ((a^6 - a^4*b^2)*\cos(c)^2 - (a^6 - a^4*b^2)*\sin(c)^2)*\cos(d*x^2 + 2*c))*\sin(d*x^2 + 2*c)^3 - 4*((a^6 - 5*a^4*b^2 + 4*a^2*b^4)*\cos(c)^3*\sin(c) + (a^6 - 5*a^4*b^2 + 4*a^2*b^4)*\cos(c)*\sin(c)^3)*\cos(d*x^2 + 2*c)^2 + 4*((a^6 - 5*a^4*b^2 + 4*a^2*b^4)*\cos(c)^3*\sin(c) + (a^6 - 5*a^4*b^2 + 4*a^2*b^4)*\cos(c)*\sin(c)^3 - 3*((a^5*b - a^3*b^3)*\cos(c)^2*\sin(c) - (a^5*b - a^3*b^3)*\sin(c)^3)*\cos(d*x^2 + 2*c))*\sin(d*x^2 + 2*c)^2 - 4*((a^5*b - 3*a^3*b^3 + 2*a*b^5)*\cos(c)^4*\sin(c) + 2*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*\cos(c)^2*\sin(c)^3 + (a^5*b$

$$\begin{aligned}
& b - 3a^3b^3 + 2ab^5) \sin(c)^5 \cos(dx^2 + 2c) + 4((a^5b - 3a^3b^3 \\
& + 2ab^5) \cos(c)^5 + 2(a^5b - 3a^3b^3 + 2ab^5) \cos(c)^3 \sin(c)^2 + \\
& (a^5b - 3a^3b^3 + 2ab^5) \cos(c) \sin(c)^4 - ((a^6 - a^4b^2) \cos(c)^2 - \\
& (a^6 - a^4b^2) \sin(c)^2) \cos(dx^2 + 2c)^3 - 3((a^5b - a^3b^3) \cos(c) \\
& ^3 - (a^5b - a^3b^3) \cos(c) \sin(c)^2) \cos(dx^2 + 2c)^2 + ((a^6 - 5a^4b^2 \\
& + 4a^2b^4) \cos(c)^4 - (a^6 - 5a^4b^2 + 4a^2b^4) \sin(c)^4) \cos(dx^2 \\
& + 2c) \sin(dx^2 + 2c) + (a^5 \cos(c) \sin(dx^2 + 2c)^5 - a^5 \cos(dx^2 \\
& + 2c)^5 \sin(c) - 4a^4b \cos(dx^2 + 2c)^4 \cos(c) \sin(c) - (a^5 \cos(dx^2 \\
& + 2c) \sin(c) - 4a^4b \cos(c) \sin(c)) \sin(dx^2 + 2c)^4 + 2(3(a^5 - \\
& 2a^3b^2) \cos(c)^2 \sin(c) + (a^5 - 2a^3b^2) \sin(c)^3) \cos(dx^2 + 2c)^3 \\
& + 2(a^5 \cos(dx^2 + 2c)^2 \cos(c) - (a^5 - 2a^3b^2) \cos(c)^3 - 3(a^5 - \\
& 2a^3b^2) \cos(c) \sin(c)^2 + 2(a^4b \cos(c)^2 - a^4b \sin(c)^2) \cos(dx^2 \\
& + 2c) \sin(dx^2 + 2c)^3 + 4((3a^4b - 4a^2b^3) \cos(c)^3 \sin(c) + (3 \\
& a^4b - 4a^2b^3) \cos(c) \sin(c)^3) \cos(dx^2 + 2c)^2 - 2(a^5 \cos(dx^2 \\
& + 2c)^3 \sin(c) + 2(3a^4b - 4a^2b^3) \cos(c)^3 \sin(c) + 2(3a^4b - 4 \\
& a^2b^3) \cos(c) \sin(c)^3 + 3((a^5 - 2a^3b^2) \cos(c)^2 \sin(c) - (a^5 - 2 \\
& a^3b^2) \sin(c)^3) \cos(dx^2 + 2c) \sin(dx^2 + 2c)^2 - ((a^5 - 8a^3b^2 \\
& + 8ab^4) \cos(c)^4 \sin(c) + 2(a^5 - 8a^3b^2 + 8ab^4) \cos(c)^2 \sin(c) \\
& ^3 + (a^5 - 8a^3b^2 + 8ab^4) \sin(c)^5) \cos(dx^2 + 2c) + (a^5 \cos(dx^2 \\
& + 2c)^4 \cos(c) + (a^5 - 8a^3b^2 + 8ab^4) \cos(c)^5 + 2(a^5 - 8a^3b^2 \\
& + 8ab^4) \cos(c)^3 \sin(c)^2 + (a^5 - 8a^3b^2 + 8ab^4) \cos(c) \sin(c) \\
& ^4 + 4(a^4b \cos(c)^2 - a^4b \sin(c)^2) \cos(dx^2 + 2c)^3 - 6((a^5 - 2a^3 \\
& b^2) \cos(c)^3 - (a^5 - 2a^3b^2) \cos(c) \sin(c)^2) \cos(dx^2 + 2c)^2 - \\
& 4((3a^4b - 4a^2b^3) \cos(c)^4 - (3a^4b - 4a^2b^3) \sin(c)^4) \cos(dx^2 \\
& + 2c) \sin(dx^2 + 2c) \sqrt{-a^2 + b^2}) / (a^6 \cos(dx^2 + 2c)^6 + 6 \\
& a^5b \cos(dx^2 + 2c)^5 \cos(c) + a^6 \sin(dx^2 + 2c)^6 + 6a^5b \sin(dx^2 \\
& + 2c)^5 \sin(c) - (a^6 - 18a^4b^2 + 48a^2b^4 - 32b^6) \cos(c)^6 - 3( \\
& a^6 - 18a^4b^2 + 48a^2b^4 - 32b^6) \cos(c)^4 \sin(c)^2 - 3(a^6 - 18a^4 \\
& b^2 + 48a^2b^4 - 32b^6) \cos(c)^2 \sin(c)^4 - (a^6 - 18a^4b^2 + 48a^2b^4 \\
& - 32b^6) \sin(c)^6 - 3(5(a^6 - 2a^4b^2) \cos(c)^2 + (a^6 - 2a^4b^2 \\
& ) \sin(c)^2) \cos(dx^2 + 2c)^4 + 3(a^6 \cos(dx^2 + 2c)^2 + 2a^5b \cos(dx^2 \\
& + 2c) \cos(c) - (a^6 - 2a^4b^2) \cos(c)^2 - 5(a^6 - 2a^4b^2) \sin(c) \\
& ^2) \sin(dx^2 + 2c)^4 - 4(5(3a^5b - 4a^3b^3) \cos(c)^3 + 3(3a^5b - \\
& 4a^3b^3) \cos(c) \sin(c)^2) \cos(dx^2 + 2c)^3 + 4(3a^5b \cos(dx^2 + 2 \\
& c)^2 \sin(c) - 6(a^6 - 2a^4b^2) \cos(dx^2 + 2c) \cos(c) \sin(c) - 3(3a^5 \\
& b - 4a^3b^3) \cos(c)^2 \sin(c) - 5(3a^5b - 4a^3b^3) \sin(c)^3) \sin(dx^2 \\
& + 2c)^3 + 3(5(a^6 - 8a^4b^2 + 8a^2b^4) \cos(c)^4 + 6(a^6 - 8a^4b^2 \\
& + 8a^2b^4) \cos(c)^2 \sin(c)^2 + (a^6 - 8a^4b^2 + 8a^2b^4) \sin(c)^4) \\
& \cos(dx^2 + 2c)^2 + 3(a^6 \cos(dx^2 + 2c)^4 + 4a^5b \cos(dx^2 + 2c) \\
& ^3 \cos(c) + (a^6 - 8a^4b^2 + 8a^2b^4) \cos(c)^4 + 6(a^6 - 8a^4b^2 + 8 \\
& a^2b^4) \cos(c)^2 \sin(c)^2 + 5(a^6 - 8a^4b^2 + 8a^2b^4) \sin(c)^4 - 6 \\
& ((a^6 - 2a^4b^2) \cos(c)^2 + (a^6 - 2a^4b^2) \sin(c)^2) \cos(dx^2 + 2c)^2 \\
& - 4((3a^5b - 4a^3b^3) \cos(c)^3 + 3(3a^5b - 4a^3b^3) \cos(c) \sin(c) \\
& ^2) \cos(dx^2 + 2c) \sin(dx^2 + 2c)^2 + 6((5a^5b - 20a^3b^3 + 16 \\
& ab^5) \cos(c)^5 + 2(5a^5b - 20a^3b^3 + 16ab^5) \cos(c)^3 \sin(c)^2 + (
\end{aligned}$$

$$\begin{aligned}
& 5*a^5*b - 20*a^3*b^3 + 16*a*b^5)*\cos(c)*\sin(c)^4)*\cos(d*x^2 + 2*c) + 6*(a^5 \\
& *b*\cos(d*x^2 + 2*c)^4*\sin(c) - 4*(a^6 - 2*a^4*b^2)*\cos(d*x^2 + 2*c)^3*\cos(c \\
& )*\sin(c) + (5*a^5*b - 20*a^3*b^3 + 16*a*b^5)*\cos(c)^4*\sin(c) + 2*(5*a^5*b - \\
& 20*a^3*b^3 + 16*a*b^5)*\cos(c)^2*\sin(c)^3 + (5*a^5*b - 20*a^3*b^3 + 16*a*b^ \\
& 5)*\sin(c)^5 - 2*(3*(3*a^5*b - 4*a^3*b^3)*\cos(c)^2*\sin(c) + (3*a^5*b - 4*a^3 \\
& *b^3)*\sin(c)^3)*\cos(d*x^2 + 2*c)^2 + 4*((a^6 - 8*a^4*b^2 + 8*a^2*b^4)*\cos(c \\
& )^3*\sin(c) + (a^6 - 8*a^4*b^2 + 8*a^2*b^4)*\cos(c)*\sin(c)^3)*\cos(d*x^2 + 2*c \\
& ))*\sin(d*x^2 + 2*c) + 2*(3*a^5*\cos(d*x^2 + 2*c)^5*\cos(c) + 3*a^5*\sin(d*x^2 \\
& + 2*c)^5*\sin(c) + (3*a^4*b - 16*a^2*b^3 + 16*b^5)*\cos(c)^6 + 3*(3*a^4*b - 1 \\
& 6*a^2*b^3 + 16*b^5)*\cos(c)^4*\sin(c)^2 + 3*(3*a^4*b - 16*a^2*b^3 + 16*b^5)*\c \\
& \cos(c)^2*\sin(c)^4 + (3*a^4*b - 16*a^2*b^3 + 16*b^5)*\sin(c)^6 + 3*(5*a^4*b*\co \\
& s(c)^2 + a^4*b*\sin(c)^2)*\cos(d*x^2 + 2*c)^4 + 3*(a^5*\cos(d*x^2 + 2*c)*\cos(c \\
& ) + a^4*b*\cos(c)^2 + 5*a^4*b*\sin(c)^2)*\sin(d*x^2 + 2*c)^4 - 2*(5*(a^5 - 4*a \\
& ^3*b^2)*\cos(c)^3 + 3*(a^5 - 4*a^3*b^2)*\cos(c)*\sin(c)^2)*\cos(d*x^2 + 2*c)^3 \\
& + 2*(3*a^5*\cos(d*x^2 + 2*c)^2*\sin(c) + 12*a^4*b*\cos(d*x^2 + 2*c)*\cos(c)*\sin \\
& (c) - 3*(a^5 - 4*a^3*b^2)*\cos(c)^2*\sin(c) - 5*(a^5 - 4*a^3*b^2)*\sin(c)^3)*\s \\
& \sin(d*x^2 + 2*c)^3 - 6*(5*(a^4*b - 2*a^2*b^3)*\cos(c)^4 + 6*(a^4*b - 2*a^2*b^ \\
& 3)*\cos(c)^2*\sin(c)^2 + (a^4*b - 2*a^2*b^3)*\sin(c)^4)*\cos(d*x^2 + 2*c)^2 + 6 \\
& *(a^5*\cos(d*x^2 + 2*c)^3*\cos(c) - (a^4*b - 2*a^2*b^3)*\cos(c)^4 - 6*(a^4*b - \\
& 2*a^2*b^3)*\cos(c)^2*\sin(c)^2 - 5*(a^4*b - 2*a^2*b^3)*\sin(c)^4 + 3*(a^4*b*\c \\
& \cos(c)^2 + a^4*b*\sin(c)^2)*\cos(d*x^2 + 2*c)^2 - ((a^5 - 4*a^3*b^2)*\cos(c)^3 \\
& + 3*(a^5 - 4*a^3*b^2)*\cos(c)*\sin(c)^2)*\cos(d*x^2 + 2*c))*\sin(d*x^2 + 2*c)^2 \\
& + 3*((a^5 - 12*a^3*b^2 + 16*a*b^4)*\cos(c)^5 + 2*(a^5 - 12*a^3*b^2 + 16*a*b \\
& ^4)*\cos(c)^3*\sin(c)^2 + (a^5 - 12*a^3*b^2 + 16*a*b^4)*\cos(c)*\sin(c)^4)*\cos( \\
& d*x^2 + 2*c) + 3*(a^5*\cos(d*x^2 + 2*c)^4*\sin(c) + 8*a^4*b*\cos(d*x^2 + 2*c)^ \\
& 3*\cos(c)*\sin(c) + (a^5 - 12*a^3*b^2 + 16*a*b^4)*\cos(c)^4*\sin(c) + 2*(a^5 - \\
& 12*a^3*b^2 + 16*a*b^4)*\cos(c)^2*\sin(c)^3 + (a^5 - 12*a^3*b^2 + 16*a*b^4)*\si \\
& n(c)^5 - 2*(3*(a^5 - 4*a^3*b^2)*\cos(c)^2*\sin(c) + (a^5 - 4*a^3*b^2)*\sin(c)^ \\
& 3)*\cos(d*x^2 + 2*c)^2 - 16*((a^4*b - 2*a^2*b^3)*\cos(c)^3*\sin(c) + (a^4*b - \\
& 2*a^2*b^3)*\cos(c)*\sin(c)^3)*\cos(d*x^2 + 2*c))*\sin(d*x^2 + 2*c))*\sqrt{-a^2 + \\
& b^2}), (a^6*\cos(d*x^2 + 2*c)^6 + 6*a^5*b*\cos(d*x^2 + 2*c)^5*\cos(c) + a^6*\s \\
& \sin(d*x^2 + 2*c)^6 + 6*a^5*b*\sin(d*x^2 + 2*c)^5*\sin(c) + (a^6 - 8*a^4*b^2 + \\
& 8*a^2*b^4)*\cos(c)^6 + 3*(a^6 - 8*a^4*b^2 + 8*a^2*b^4)*\cos(c)^4*\sin(c)^2 + 3 \\
& *(a^6 - 8*a^4*b^2 + 8*a^2*b^4)*\cos(c)^2*\sin(c)^4 + (a^6 - 8*a^4*b^2 + 8*a^2 \\
& *b^4)*\sin(c)^6 - (5*(a^6 - 4*a^4*b^2)*\cos(c)^2 + (a^6 - 4*a^4*b^2)*\sin(c)^2 \\
& )*\cos(d*x^2 + 2*c)^4 + (3*a^6*\cos(d*x^2 + 2*c)^2 + 6*a^5*b*\cos(d*x^2 + 2*c) \\
& )*\cos(c) - (a^6 - 4*a^4*b^2)*\cos(c)^2 - 5*(a^6 - 4*a^4*b^2)*\sin(c)^2)*\sin(d* \\
& x^2 + 2*c)^4 - 4*(5*(a^5*b - 2*a^3*b^3)*\cos(c)^3 + 3*(a^5*b - 2*a^3*b^3)*\co \\
& s(c)*\sin(c)^2)*\cos(d*x^2 + 2*c)^3 + 4*(3*a^5*b*\cos(d*x^2 + 2*c)^2*\sin(c) - \\
& 2*(a^6 - 4*a^4*b^2)*\cos(d*x^2 + 2*c)*\cos(c)*\sin(c) - 3*(a^5*b - 2*a^3*b^3)* \\
& \cos(c)^2*\sin(c) - 5*(a^5*b - 2*a^3*b^3)*\sin(c)^3)*\sin(d*x^2 + 2*c)^3 - (5*( \\
& a^6 + 4*a^4*b^2 - 8*a^2*b^4)*\cos(c)^4 + 6*(a^6 + 4*a^4*b^2 - 8*a^2*b^4)*\cos \\
& (c)^2*\sin(c)^2 + (a^6 + 4*a^4*b^2 - 8*a^2*b^4)*\sin(c)^4)*\cos(d*x^2 + 2*c)^2 \\
& + (3*a^6*\cos(d*x^2 + 2*c)^4 + 12*a^5*b*\cos(d*x^2 + 2*c)^3*\cos(c) - (a^6 + \\
& 4*a^4*b^2 - 8*a^2*b^4)*\cos(c)^4 - 6*(a^6 + 4*a^4*b^2 - 8*a^2*b^4)*\cos(c)^2*
\end{aligned}$$

$$\begin{aligned}
& \sin(c)^2 - 5*(a^6 + 4*a^4*b^2 - 8*a^2*b^4)*\sin(c)^4 - 6*((a^6 - 4*a^4*b^2)* \\
& \cos(c)^2 + (a^6 - 4*a^4*b^2)*\sin(c)^2)*\cos(d*x^2 + 2*c)^2 - 12*((a^5*b - 2* \\
& a^3*b^3)*\cos(c)^3 + 3*(a^5*b - 2*a^3*b^3)*\cos(c)*\sin(c)^2)*\cos(d*x^2 + 2*c) \\
& )*\sin(d*x^2 + 2*c)^2 - 2*((5*a^5*b - 8*a*b^5)*\cos(c)^5 + 2*(5*a^5*b - 8*a*b \\
& ^5)*\cos(c)^3*\sin(c)^2 + (5*a^5*b - 8*a*b^5)*\cos(c)*\sin(c)^4)*\cos(d*x^2 + 2* \\
& c) + 2*(3*a^5*b*\cos(d*x^2 + 2*c)^4*\sin(c) - 4*(a^6 - 4*a^4*b^2)*\cos(d*x^2 + \\
& 2*c)^3*\cos(c)*\sin(c) - (5*a^5*b - 8*a*b^5)*\cos(c)^4*\sin(c) - 2*(5*a^5*b - \\
& 8*a*b^5)*\cos(c)^2*\sin(c)^3 - (5*a^5*b - 8*a*b^5)*\sin(c)^5 - 6*(3*(a^5*b - 2 \\
& *a^3*b^3)*\cos(c)^2*\sin(c) + (a^5*b - 2*a^3*b^3)*\sin(c)^3)*\cos(d*x^2 + 2*c)^2 \\
& - 4*((a^6 + 4*a^4*b^2 - 8*a^2*b^4)*\cos(c)^3*\sin(c) + (a^6 + 4*a^4*b^2 - 8 \\
& *a^2*b^4)*\cos(c)*\sin(c)^3)*\cos(d*x^2 + 2*c))*\sin(d*x^2 + 2*c) + 4*(a^5*\cos( \\
& d*x^2 + 2*c)^5*\cos(c) + a^5*\sin(d*x^2 + 2*c)^5*\sin(c) - (a^4*b - 2*a^2*b^3) \\
& *\cos(c)^6 - 3*(a^4*b - 2*a^2*b^3)*\cos(c)^4*\sin(c)^2 - 3*(a^4*b - 2*a^2*b^3) \\
& *\cos(c)^2*\sin(c)^4 - (a^4*b - 2*a^2*b^3)*\sin(c)^6 + (5*a^4*b*\cos(c)^2 + a^4 \\
& *b*\sin(c)^2)*\cos(d*x^2 + 2*c)^4 + (a^5*\cos(d*x^2 + 2*c)*\cos(c) + a^4*b*\cos( \\
& c)^2 + 5*a^4*b*\sin(c)^2)*\sin(d*x^2 + 2*c)^4 + 2*(5*a^3*b^2*\cos(c)^3 + 3*a^3 \\
& *b^2*\cos(c)*\sin(c)^2)*\cos(d*x^2 + 2*c)^3 + 2*(a^5*\cos(d*x^2 + 2*c)^2*\sin(c) \\
& + 4*a^4*b*\cos(d*x^2 + 2*c)*\cos(c)*\sin(c) + 3*a^3*b^2*\cos(c)^2*\sin(c) + 5*a \\
& ^3*b^2*\sin(c)^3)*\sin(d*x^2 + 2*c)^3 + 2*(5*a^2*b^3*\cos(c)^4 + 6*a^2*b^3*\cos \\
& (c)^2*\sin(c)^2 + a^2*b^3*\sin(c)^4)*\cos(d*x^2 + 2*c)^2 + 2*(a^5*\cos(d*x^2 + \\
& 2*c)^3*\cos(c) + a^2*b^3*\cos(c)^4 + 6*a^2*b^3*\cos(c)^2*\sin(c)^2 + 5*a^2*b^3* \\
& \sin(c)^4 + 3*(a^4*b*\cos(c)^2 + a^4*b*\sin(c)^2)*\cos(d*x^2 + 2*c)^2 + 3*(a^3* \\
& b^2*\cos(c)^3 + 3*a^3*b^2*\cos(c)*\sin(c)^2)*\cos(d*x^2 + 2*c))*\sin(d*x^2 + 2*c) \\
& )^2 - ((a^5 - 2*a^3*b^2 - 4*a*b^4)*\cos(c)^5 + 2*(a^5 - 2*a^3*b^2 - 4*a*b^4) \\
& *\cos(c)^3*\sin(c)^2 + (a^5 - 2*a^3*b^2 - 4*a*b^4)*\cos(c)*\sin(c)^4)*\cos(d*x^2 \\
& + 2*c) + (a^5*\cos(d*x^2 + 2*c)^4*\sin(c) + 8*a^4*b*\cos(d*x^2 + 2*c)^3*\cos(c) \\
& )*\sin(c) - (a^5 - 2*a^3*b^2 - 4*a*b^4)*\cos(c)^4*\sin(c) - 2*(a^5 - 2*a^3*b^2 \\
& - 4*a*b^4)*\cos(c)^2*\sin(c)^3 - (a^5 - 2*a^3*b^2 - 4*a*b^4)*\sin(c)^5 + 6*(3 \\
& *a^3*b^2*\cos(c)^2*\sin(c) + a^3*b^2*\sin(c)^3)*\cos(d*x^2 + 2*c)^2 + 16*(a^2*b \\
& ^3*\cos(c)^3*\sin(c) + a^2*b^3*\cos(c)*\sin(c)^3)*\cos(d*x^2 + 2*c))*\sin(d*x^2 + \\
& 2*c))*\sqrt{-a^2 + b^2})/(a^6*\cos(d*x^2 + 2*c)^6 + 6*a^5*b*\cos(d*x^2 + 2*c) \\
& ^5*\cos(c) + a^6*\sin(d*x^2 + 2*c)^6 + 6*a^5*b*\sin(d*x^2 + 2*c)^5*\sin(c) - (a \\
& ^6 - 18*a^4*b^2 + 48*a^2*b^4 - 32*b^6)*\cos(c)^6 - 3*(a^6 - 18*a^4*b^2 + 48* \\
& a^2*b^4 - 32*b^6)*\cos(c)^4*\sin(c)^2 - 3*(a^6 - 18*a^4*b^2 + 48*a^2*b^4 - 32 \\
& *b^6)*\cos(c)^2*\sin(c)^4 - (a^6 - 18*a^4*b^2 + 48*a^2*b^4 - 32*b^6)*\sin(c)^6 \\
& - 3*(5*(a^6 - 2*a^4*b^2)*\cos(c)^2 + (a^6 - 2*a^4*b^2)*\sin(c)^2)*\cos(d*x^2 \\
& + 2*c)^4 + 3*(a^6*\cos(d*x^2 + 2*c)^2 + 2*a^5*b*\cos(d*x^2 + 2*c)*\cos(c) - (a \\
& ^6 - 2*a^4*b^2)*\cos(c)^2 - 5*(a^6 - 2*a^4*b^2)*\sin(c)^2)*\sin(d*x^2 + 2*c)^4 \\
& - 4*(5*(3*a^5*b - 4*a^3*b^3)*\cos(c)^3 + 3*(3*a^5*b - 4*a^3*b^3)*\cos(c)*\sin \\
& (c)^2)*\cos(d*x^2 + 2*c)^3 + 4*(3*a^5*b*\cos(d*x^2 + 2*c)^2*\sin(c) - 6*(a^6 - \\
& 2*a^4*b^2)*\cos(d*x^2 + 2*c)*\cos(c)*\sin(c) - 3*(3*a^5*b - 4*a^3*b^3)*\cos(c) \\
& ^2*\sin(c) - 5*(3*a^5*b - 4*a^3*b^3)*\sin(c)^3)*\sin(d*x^2 + 2*c)^3 + 3*(5*(a^6 \\
& - 8*a^4*b^2 + 8*a^2*b^4)*\cos(c)^4 + 6*(a^6 - 8*a^4*b^2 + 8*a^2*b^4)*\cos(c) \\
& )^2*\sin(c)^2 + (a^6 - 8*a^4*b^2 + 8*a^2*b^4)*\sin(c)^4)*\cos(d*x^2 + 2*c)^2 + \\
& 3*(a^6*\cos(d*x^2 + 2*c)^4 + 4*a^5*b*\cos(d*x^2 + 2*c)^3*\cos(c) + (a^6 - 8*a
\end{aligned}$$

$$\begin{aligned}
&^4*b^2 + 8*a^2*b^4)*\cos(c)^4 + 6*(a^6 - 8*a^4*b^2 + 8*a^2*b^4)*\cos(c)^2*\sin \\
&(c)^2 + 5*(a^6 - 8*a^4*b^2 + 8*a^2*b^4)*\sin(c)^4 - 6*((a^6 - 2*a^4*b^2)*\cos \\
&(c)^2 + (a^6 - 2*a^4*b^2)*\sin(c)^2)*\cos(d*x^2 + 2*c)^2 - 4*((3*a^5*b - 4*a^ \\
&3*b^3)*\cos(c)^3 + 3*(3*a^5*b - 4*a^3*b^3)*\cos(c)*\sin(c)^2)*\cos(d*x^2 + 2*c) \\
&)*\sin(d*x^2 + 2*c)^2 + 6*((5*a^5*b - 20*a^3*b^3 + 16*a*b^5)*\cos(c)^5 + 2*(5 \\
&*a^5*b - 20*a^3*b^3 + 16*a*b^5)*\cos(c)^3*\sin(c)^2 + (5*a^5*b - 20*a^3*b^3 + \\
&16*a*b^5)*\cos(c)*\sin(c)^4)*\cos(d*x^2 + 2*c) + 6*(a^5*b*\cos(d*x^2 + 2*c)^4* \\
&\sin(c) - 4*(a^6 - 2*a^4*b^2)*\cos(d*x^2 + 2*c)^3*\cos(c)*\sin(c) + (5*a^5*b - \\
&20*a^3*b^3 + 16*a*b^5)*\cos(c)^4*\sin(c) + 2*(5*a^5*b - 20*a^3*b^3 + 16*a*b^5 \\
&)*\cos(c)^2*\sin(c)^3 + (5*a^5*b - 20*a^3*b^3 + 16*a*b^5)*\sin(c)^5 - 2*(3*(3* \\
&a^5*b - 4*a^3*b^3)*\cos(c)^2*\sin(c) + (3*a^5*b - 4*a^3*b^3)*\sin(c)^3)*\cos(d* \\
&x^2 + 2*c)^2 + 4*((a^6 - 8*a^4*b^2 + 8*a^2*b^4)*\cos(c)^3*\sin(c) + (a^6 - 8* \\
&a^4*b^2 + 8*a^2*b^4)*\cos(c)*\sin(c)^3)*\cos(d*x^2 + 2*c))*\sin(d*x^2 + 2*c) + \\
&2*(3*a^5*\cos(d*x^2 + 2*c)^5*\cos(c) + 3*a^5*\sin(d*x^2 + 2*c)^5*\sin(c) + (3*a \\
&^4*b - 16*a^2*b^3 + 16*b^5)*\cos(c)^6 + 3*(3*a^4*b - 16*a^2*b^3 + 16*b^5)*\co \\
&s(c)^4*\sin(c)^2 + 3*(3*a^4*b - 16*a^2*b^3 + 16*b^5)*\cos(c)^2*\sin(c)^4 + (3* \\
&a^4*b - 16*a^2*b^3 + 16*b^5)*\sin(c)^6 + 3*(5*a^4*b*\cos(c)^2 + a^4*b*\sin(c)^ \\
&2)*\cos(d*x^2 + 2*c)^4 + 3*(a^5*\cos(d*x^2 + 2*c)*\cos(c) + a^4*b*\cos(c)^2 + 5 \\
&*a^4*b*\sin(c)^2)*\sin(d*x^2 + 2*c)^4 - 2*(5*(a^5 - 4*a^3*b^2)*\cos(c)^3 + 3*( \\
&a^5 - 4*a^3*b^2)*\cos(c)*\sin(c)^2)*\cos(d*x^2 + 2*c)^3 + 2*(3*a^5*\cos(d*x^2 + \\
&2*c)^2*\sin(c) + 12*a^4*b*\cos(d*x^2 + 2*c)*\cos(c)*\sin(c) - 3*(a^5 - 4*a^3*b \\
&^2)*\cos(c)^2*\sin(c) - 5*(a^5 - 4*a^3*b^2)*\sin(c)^3)*\sin(d*x^2 + 2*c)^3 - 6* \\
&(5*(a^4*b - 2*a^2*b^3)*\cos(c)^4 + 6*(a^4*b - 2*a^2*b^3)*\cos(c)^2*\sin(c)^2 + \\
&(a^4*b - 2*a^2*b^3)*\sin(c)^4)*\cos(d*x^2 + 2*c)^2 + 6*(a^5*\cos(d*x^2 + 2*c) \\
&^3*\cos(c) - (a^4*b - 2*a^2*b^3)*\cos(c)^4 - 6*(a^4*b - 2*a^2*b^3)*\cos(c)^2*s \\
&\sin(c)^2 - 5*(a^4*b - 2*a^2*b^3)*\sin(c)^4 + 3*(a^4*b*\cos(c)^2 + a^4*b*\sin(c) \\
&^2)*\cos(d*x^2 + 2*c)^2 - ((a^5 - 4*a^3*b^2)*\cos(c)^3 + 3*(a^5 - 4*a^3*b^2)* \\
&\cos(c)*\sin(c)^2)*\cos(d*x^2 + 2*c))*\sin(d*x^2 + 2*c)^2 + 3*((a^5 - 12*a^3*b^ \\
&2 + 16*a*b^4)*\cos(c)^5 + 2*(a^5 - 12*a^3*b^2 + 16*a*b^4)*\cos(c)^3*\sin(c)^2 \\
&+ (a^5 - 12*a^3*b^2 + 16*a*b^4)*\cos(c)*\sin(c)^4)*\cos(d*x^2 + 2*c) + 3*(a^5* \\
&\cos(d*x^2 + 2*c)^4*\sin(c) + 8*a^4*b*\cos(d*x^2 + 2*c)^3*\cos(c)*\sin(c) + (a^5 \\
&- 12*a^3*b^2 + 16*a*b^4)*\cos(c)^4*\sin(c) + 2*(a^5 - 12*a^3*b^2 + 16*a*b^4) \\
&*\cos(c)^2*\sin(c)^3 + (a^5 - 12*a^3*b^2 + 16*a*b^4)*\sin(c)^5 - 2*(3*(a^5 - 4 \\
&*a^3*b^2)*\cos(c)^2*\sin(c) + (a^5 - 4*a^3*b^2)*\sin(c)^3)*\cos(d*x^2 + 2*c)^2 \\
&- 16*((a^4*b - 2*a^2*b^3)*\cos(c)^3*\sin(c) + (a^4*b - 2*a^2*b^3)*\cos(c)*\sin( \\
&c)^3)*\cos(d*x^2 + 2*c))*\sin(d*x^2 + 2*c))*\sqrt{-a^2 + b^2})) + 2*(2*(a^5*b \\
&- 2*a^3*b^3 + a*b^5)*d*x^2*\cos(d*x^2 + c) + (a^6 - 2*a^4*b^2 + a^2*b^4)*d*x \\
&^2 - (a^3*b^3 - a*b^5)*\sin(d*x^2 + c))*\cos(2*d*x^2 + 2*c) + 2*(a^4*b^2 - a^ \\
&2*b^4 + 2*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x^2*\sin(d*x^2 + c) + (a^3*b^3 - a*b \\
&^5)*\cos(d*x^2 + c))*\sin(2*d*x^2 + 2*c) + 2*(a^3*b^3 - a*b^5)*\sin(d*x^2 + c) \\
&)/((a^8 - 2*a^6*b^2 + a^4*b^4)*d*\cos(2*d*x^2 + 2*c)^2 + 4*(a^6*b^2 - 2*a^4* \\
&b^4 + a^2*b^6)*d*\cos(d*x^2 + c)^2 + (a^8 - 2*a^6*b^2 + a^4*b^4)*d*\sin(2*d*x \\
&^2 + 2*c)^2 + 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*d*\sin(2*d*x^2 + 2*c)*\sin(d*x^ \\
&2 + c) + 4*(a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*d*\sin(d*x^2 + c)^2 + 4*(a^7*b - \\
&2*a^5*b^3 + a^3*b^5)*d*\cos(d*x^2 + c) + (a^8 - 2*a^6*b^2 + a^4*b^4)*d + 2*(
\end{aligned}$$

$$2*(a^7*b - 2*a^5*b^3 + a^3*b^5)*d*\cos(d*x^2 + c) + (a^8 - 2*a^6*b^2 + a^4*b^4)*d*\cos(2*d*x^2 + 2*c))$$

**mupad [B]** time = 5.01, size = 340, normalized size = 2.76

$$\frac{\frac{b^2}{d(a b^2 1i - a^3 1i)} + \frac{b^3 e^{1i d x^2 + c 1i}}{a d (a b^2 1i - a^3 1i)}}{a + a e^{2i d x^2 + c 2i} + 2 b e^{1i d x^2 + c 1i}} + \frac{x^2}{2 a^2} + \frac{b \ln \left( 2 b x e^{1i d x^2 + c 1i} (2 a^2 - b^2) - \frac{b x (a^2 - b^2) (2 a^2 - b^2) (a + b e^{1i d x^2 + c 1i})^{2i}}{(a + b)^{3/2} (a - b)^{3/2}} \right)}{2 a^2 d (a + b)^{3/2} (a - b)^{3/2}} (2 a^2 -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a + b/cos(c + d\*x^2))^2,x)

[Out] (b^2/(d\*(a\*b^2\*1i - a^3\*1i)) + (b^3\*exp(c\*1i + d\*x^2\*1i))/(a\*d\*(a\*b^2\*1i - a^3\*1i)))/(a + a\*exp(c\*2i + d\*x^2\*2i) + 2\*b\*exp(c\*1i + d\*x^2\*1i)) + x^2/(2\*a^2) + (b\*log(2\*b\*x\*exp(c\*1i + d\*x^2\*1i)\*(2\*a^2 - b^2) - (b\*x\*(a^2 - b^2)\*(2\*a^2 - b^2)\*(a + b\*exp(c\*1i + d\*x^2\*1i))\*2i)/((a + b)^(3/2)\*(a - b)^(3/2)))\*(2\*a^2 - b^2))/(2\*a^2\*d\*(a + b)^(3/2)\*(a - b)^(3/2)) - (b\*log(2\*b\*x\*exp(c\*1i + d\*x^2\*1i)\*(2\*a^2 - b^2) + (b\*x\*(a^2 - b^2)\*(2\*a^2 - b^2)\*(a + b\*exp(c\*1i + d\*x^2\*1i))\*2i)/((a + b)^(3/2)\*(a - b)^(3/2)))\*(2\*a^2 - b^2))/(2\*a^2\*d\*(a + b)^(3/2)\*(a - b)^(3/2))

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a + b \sec(c + dx^2))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*sec(d\*x\*\*2+c))\*\*2,x)

[Out] Integral(x/(a + b\*sec(c + d\*x\*\*2))\*\*2, x)



$$3.28 \quad \int \frac{1}{x(a+b \sec(c+dx^2))^2} dx$$

Optimal. Leaf size=21

$$\text{Int} \left( \frac{1}{x(a+b \sec(c+dx^2))^2}, x \right)$$

[Out] Unintegrable(1/x/(a+b\*sec(d\*x^2+c))^2,x)

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{x(a+b \sec(c+dx^2))^2} dx$$

Verification is Not applicable to the result.

[In] Int[1/(x\*(a + b\*Sec[c + d\*x^2]))^2],x]

[Out] Defer[Int][1/(x\*(a + b\*Sec[c + d\*x^2]))^2], x]

Rubi steps

$$\int \frac{1}{x(a+b \sec(c+dx^2))^2} dx = \int \frac{1}{x(a+b \sec(c+dx^2))^2} dx$$

Mathematica [A] time = 11.30, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a+b \sec(c+dx^2))^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x\*(a + b\*Sec[c + d\*x^2]))^2],x]

[Out] Integrate[1/(x\*(a + b\*Sec[c + d\*x^2]))^2], x]

fricas [A] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{1}{b^2x \sec(dx^2+c)^2 + 2abx \sec(dx^2+c) + a^2x}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*sec(d\*x^2+c))^2,x, algorithm="fricas")

[Out] integral(1/(b^2\*x\*sec(d\*x^2 + c)^2 + 2\*a\*b\*x\*sec(d\*x^2 + c) + a^2\*x), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \sec(dx^2 + c) + a)^2 x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*sec(d\*x^2+c))^2,x, algorithm="giac")

[Out] integrate(1/((b\*sec(d\*x^2 + c) + a)^2\*x), x)

**maple** [A] time = 1.31, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a + b \sec(dx^2 + c))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(a+b\*sec(d\*x^2+c))^2,x)

[Out] int(1/x/(a+b\*sec(d\*x^2+c))^2,x)

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*sec(d\*x^2+c))^2,x, algorithm="maxima")

[Out] Timed out

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x \left( a + \frac{b}{\cos(dx^2+c)} \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x\*(a + b/cos(c + d\*x^2))^2),x)

[Out] `int(1/(x*(a + b/cos(c + d*x^2))^2), x)`

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a + b \sec(c + dx^2))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(a+b*sec(d*x**2+c))**2,x)`

[Out] `Integral(1/(x*(a + b*sec(c + d*x**2))**2), x)`

$$3.29 \quad \int \frac{1}{x^2(a+b \sec(c+dx^2))^2} dx$$

Optimal. Leaf size=21

$$\text{Int} \left( \frac{1}{x^2 (a + b \sec(c + dx^2))^2}, x \right)$$

[Out] Unintegrable(1/x^2/(a+b\*sec(d\*x^2+c))^2,x)

**Rubi** [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{x^2 (a + b \sec(c + dx^2))^2} dx$$

Verification is Not applicable to the result.

[In] Int[1/(x^2\*(a + b\*Sec[c + d\*x^2])^2),x]

[Out] Defer[Int][1/(x^2\*(a + b\*Sec[c + d\*x^2])^2), x]

Rubi steps

$$\int \frac{1}{x^2 (a + b \sec(c + dx^2))^2} dx = \int \frac{1}{x^2 (a + b \sec(c + dx^2))^2} dx$$

**Mathematica** [A] time = 7.37, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (a + b \sec(c + dx^2))^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x^2\*(a + b\*Sec[c + d\*x^2])^2),x]

[Out] Integrate[1/(x^2\*(a + b\*Sec[c + d\*x^2])^2), x]

**fricas** [A] time = 0.99, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{1}{b^2 x^2 \sec(dx^2 + c)^2 + 2 abx^2 \sec(dx^2 + c) + a^2 x^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b\*sec(d\*x^2+c))^2,x, algorithm="fricas")

[Out] integral(1/(b^2\*x^2\*sec(d\*x^2 + c)^2 + 2\*a\*b\*x^2\*sec(d\*x^2 + c) + a^2\*x^2), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \sec(dx^2 + c) + a)^2 x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b\*sec(d\*x^2+c))^2,x, algorithm="giac")

[Out] integrate(1/((b\*sec(d\*x^2 + c) + a)^2\*x^2), x)

**maple** [A] time = 1.46, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (a + b \sec(dx^2 + c))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(a+b\*sec(d\*x^2+c))^2,x)

[Out] int(1/x^2/(a+b\*sec(d\*x^2+c))^2,x)

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b\*sec(d\*x^2+c))^2,x, algorithm="maxima")

[Out] Timed out

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x^2 \left( a + \frac{b}{\cos(dx^2+c)} \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^2*(a + b/cos(c + d*x^2))^2),x)`

[Out] `int(1/(x^2*(a + b/cos(c + d*x^2))^2), x)`

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (a + b \sec(c + dx^2))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**2/(a+b*sec(d*x**2+c))**2,x)`

[Out] `Integral(1/(x**2*(a + b*sec(c + d*x**2))**2), x)`

$$3.30 \quad \int \frac{1}{x^3 (a + b \sec(c + dx^2))^2} dx$$

Optimal. Leaf size=21

$$\text{Int} \left( \frac{1}{x^3 (a + b \sec(c + dx^2))^2}, x \right)$$

[Out] Unintegrable(1/x^3/(a+b\*sec(d\*x^2+c))^2,x)

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{x^3 (a + b \sec(c + dx^2))^2} dx$$

Verification is Not applicable to the result.

[In] Int[1/(x^3\*(a + b\*Sec[c + d\*x^2])^2),x]

[Out] Defer[Int][1/(x^3\*(a + b\*Sec[c + d\*x^2])^2), x]

Rubi steps

$$\int \frac{1}{x^3 (a + b \sec(c + dx^2))^2} dx = \int \frac{1}{x^3 (a + b \sec(c + dx^2))^2} dx$$

Mathematica [A] time = 7.81, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 (a + b \sec(c + dx^2))^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x^3\*(a + b\*Sec[c + d\*x^2])^2),x]

[Out] Integrate[1/(x^3\*(a + b\*Sec[c + d\*x^2])^2), x]

fricas [A] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{1}{b^2 x^3 \sec(dx^2 + c)^2 + 2 abx^3 \sec(dx^2 + c) + a^2 x^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a+b\*sec(d\*x^2+c))^2,x, algorithm="fricas")

[Out] integral(1/(b^2\*x^3\*sec(d\*x^2 + c)^2 + 2\*a\*b\*x^3\*sec(d\*x^2 + c) + a^2\*x^3), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \sec(dx^2 + c) + a)^2 x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a+b\*sec(d\*x^2+c))^2,x, algorithm="giac")

[Out] integrate(1/((b\*sec(d\*x^2 + c) + a)^2\*x^3), x)

maple [A] time = 1.64, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 (a + b \sec(dx^2 + c))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(a+b\*sec(d\*x^2+c))^2,x)

[Out] int(1/x^3/(a+b\*sec(d\*x^2+c))^2,x)

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a+b\*sec(d\*x^2+c))^2,x, algorithm="maxima")

[Out] Timed out

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x^3 \left( a + \frac{b}{\cos(dx^2+c)} \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.



[In] `int(1/(x^3*(a + b/cos(c + d*x^2))^2), x)`

[Out] `int(1/(x^3*(a + b/cos(c + d*x^2))^2), x)`

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 (a + b \sec(c + dx^2))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**3/(a+b*sec(d*x**2+c))**2, x)`

[Out] `Integral(1/(x**3*(a + b*sec(c + d*x**2))**2), x)`

### 3.31 $\int x^3 (a + b \sec(c + d\sqrt{x})) dx$

**Optimal.** Leaf size=476

$$\frac{ax^4}{4} - \frac{10080ib\text{Li}_8\left(-ie^{i(c+d\sqrt{x})}\right)}{d^8} + \frac{10080ib\text{Li}_8\left(ie^{i(c+d\sqrt{x})}\right)}{d^8} - \frac{10080b\sqrt{x}\text{Li}_7\left(-ie^{i(c+d\sqrt{x})}\right)}{d^7} + \frac{10080b\sqrt{x}\text{Li}_7\left(ie^{i(c+d\sqrt{x})}\right)}{d^7}$$

[Out]  $1/4*a*x^4-420*I*b*x^2*\text{polylog}(4,-I*\exp(I*(c+d*x^{(1/2)})))/d^4+420*I*b*x^2*\text{polylog}(4,I*\exp(I*(c+d*x^{(1/2)})))/d^4-4*I*b*x^{(7/2)}*\arctan(\exp(I*(c+d*x^{(1/2)})))/d-84*b*x^{(5/2)}*\text{polylog}(3,-I*\exp(I*(c+d*x^{(1/2)})))/d^3+84*b*x^{(5/2)}*\text{polylog}(3,I*\exp(I*(c+d*x^{(1/2)})))/d^3+5040*I*b*x*\text{polylog}(6,-I*\exp(I*(c+d*x^{(1/2)})))/d^6-5040*I*b*x*\text{polylog}(6,I*\exp(I*(c+d*x^{(1/2)})))/d^6+1680*b*x^{(3/2)}*\text{polylog}(5,-I*\exp(I*(c+d*x^{(1/2)})))/d^5-1680*b*x^{(3/2)}*\text{polylog}(5,I*\exp(I*(c+d*x^{(1/2)})))/d^5+14*I*b*x^3*\text{polylog}(2,-I*\exp(I*(c+d*x^{(1/2)})))/d^2-14*I*b*x^3*\text{polylog}(2,I*\exp(I*(c+d*x^{(1/2)})))/d^2-10080*I*b*\text{polylog}(8,-I*\exp(I*(c+d*x^{(1/2)})))/d^8+10080*I*b*\text{polylog}(8,I*\exp(I*(c+d*x^{(1/2)})))/d^8-10080*b*\text{polylog}(7,-I*\exp(I*(c+d*x^{(1/2)})))*x^{(1/2)}/d^7+10080*b*\text{polylog}(7,I*\exp(I*(c+d*x^{(1/2)})))*x^{(1/2)}/d^7$

**Rubi [A]** time = 0.46, antiderivative size = 476, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 7, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$ , Rules used = {14, 4204, 4181, 2531, 6609, 2282, 6589}

$$\frac{14ibx^3\text{PolyLog}\left(2,-ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{14ibx^3\text{PolyLog}\left(2,ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{84bx^{5/2}\text{PolyLog}\left(3,-ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{84bx^{5/2}\text{PolyLog}\left(3,ie^{i(c+d\sqrt{x})}\right)}{d^3}$$

Antiderivative was successfully verified.

[In] Int[x^3\*(a + b\*Sec[c + d\*Sqrt[x]]),x]

[Out]  $(a*x^4)/4 - ((4*I)*b*x^{(7/2)}*\text{ArcTan}[E^{(I*(c + d*\text{Sqrt}[x])})])/d + ((14*I)*b*x^3*\text{PolyLog}[2, (-I)*E^{(I*(c + d*\text{Sqrt}[x])})])/d^2 - ((14*I)*b*x^3*\text{PolyLog}[2, I*\text{E}^{(I*(c + d*\text{Sqrt}[x])})])/d^2 - (84*b*x^{(5/2)}*\text{PolyLog}[3, (-I)*E^{(I*(c + d*\text{Sqrt}[x])})])/d^3 + (84*b*x^{(5/2)}*\text{PolyLog}[3, I*\text{E}^{(I*(c + d*\text{Sqrt}[x])})])/d^3 - ((420*I)*b*x^2*\text{PolyLog}[4, (-I)*E^{(I*(c + d*\text{Sqrt}[x])})])/d^4 + ((420*I)*b*x^2*\text{PolyLog}[4, I*\text{E}^{(I*(c + d*\text{Sqrt}[x])})])/d^4 + (1680*b*x^{(3/2)}*\text{PolyLog}[5, (-I)*E^{(I*(c + d*\text{Sqrt}[x])})])/d^5 - (1680*b*x^{(3/2)}*\text{PolyLog}[5, I*\text{E}^{(I*(c + d*\text{Sqrt}[x])})])/d^5 + ((5040*I)*b*x*\text{PolyLog}[6, (-I)*E^{(I*(c + d*\text{Sqrt}[x])})])/d^6 - ((5040*I)*b*x*\text{PolyLog}[6, I*\text{E}^{(I*(c + d*\text{Sqrt}[x])})])/d^6 - (10080*b*\text{Sqrt}[x]*\text{PolyLog}[7, (-I)*E^{(I*(c + d*\text{Sqrt}[x])})])/d^7 + (10080*b*\text{Sqrt}[x]*\text{PolyLog}[7, I*\text{E}^{(I*(c + d*\text{Sqrt}[x])})])/d^7 - ((10080*I)*b*\text{PolyLog}[8, (-I)*E^{(I*(c + d*\text{Sqrt}[x])})])/d^8 + ((10080*I)*b*\text{PolyLog}[8, I*\text{E}^{(I*(c + d*\text{Sqrt}[x])})])/d^8$

Rule 14

```
Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 2531

```
Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_))))^(n_)]*((f_) + (g_)*(x_))^(m_), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

### Rule 4181

```
Int[csc[(e_) + Pi*(k_) + (f_)*(x_)]*((c_) + (d_)*(x_))^(m_), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f, x] + (-Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))], x], x) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

### Rule 4204

```
Int[(x_)^(m_)*((a_) + (b_)*Sec[(c_) + (d_)*(x_)^(n_)])^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]
```

### Rule 6589

```
Int[PolyLog[n_, (c_)*((a_) + (b_)*(x_))^(p_)]/((d_) + (e_)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rule 6609

```
Int[((e_) + (f_)*(x_))^(m_)*PolyLog[n_, (d_)*((F_)^((c_)*((a_) + (b_)*(x_))))^(p_)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
```



**Mathematica** [A] time = 0.15, size = 479, normalized size = 1.01

$$\frac{ax^4}{4} - \frac{10080ib\text{Li}_8\left(-ie^{i(c+d\sqrt{x})}\right)}{d^8} + \frac{10080ib\text{Li}_8\left(ie^{i(c+d\sqrt{x})}\right)}{d^8} - \frac{10080b\sqrt{x}\text{Li}_7\left(-ie^{i(c+d\sqrt{x})}\right)}{d^7} + \frac{10080b\sqrt{x}\text{Li}_7\left(ie^{i(c+d\sqrt{x})}\right)}{d^7}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*(a + b\*Sec[c + d\*Sqrt[x]]), x]

[Out] (a\*x^4)/4 - ((4\*I)\*b\*x^(7/2)\*ArcTan[E^(I\*c + I\*d\*Sqrt[x])])/d + ((14\*I)\*b\*x^3\*PolyLog[2, (-I)\*E^(I\*(c + d\*Sqrt[x]))])/d^2 - ((14\*I)\*b\*x^3\*PolyLog[2, I\*E^(I\*(c + d\*Sqrt[x]))])/d^2 - (84\*b\*x^(5/2)\*PolyLog[3, (-I)\*E^(I\*(c + d\*Sqrt[x]))])/d^3 + (84\*b\*x^(5/2)\*PolyLog[3, I\*E^(I\*(c + d\*Sqrt[x]))])/d^3 - ((420\*I)\*b\*x^2\*PolyLog[4, (-I)\*E^(I\*(c + d\*Sqrt[x]))])/d^4 + ((420\*I)\*b\*x^2\*PolyLog[4, I\*E^(I\*(c + d\*Sqrt[x]))])/d^4 + (1680\*b\*x^(3/2)\*PolyLog[5, (-I)\*E^(I\*(c + d\*Sqrt[x]))])/d^5 - (1680\*b\*x^(3/2)\*PolyLog[5, I\*E^(I\*(c + d\*Sqrt[x]))])/d^5 + ((5040\*I)\*b\*x\*PolyLog[6, (-I)\*E^(I\*(c + d\*Sqrt[x]))])/d^6 - ((5040\*I)\*b\*x\*PolyLog[6, I\*E^(I\*(c + d\*Sqrt[x]))])/d^6 - (10080\*b\*Sqrt[x]\*PolyLog[7, (-I)\*E^(I\*(c + d\*Sqrt[x]))])/d^7 + (10080\*b\*Sqrt[x]\*PolyLog[7, I\*E^(I\*(c + d\*Sqrt[x]))])/d^7 - ((10080\*I)\*b\*PolyLog[8, (-I)\*E^(I\*(c + d\*Sqrt[x]))])/d^8 + ((10080\*I)\*b\*PolyLog[8, I\*E^(I\*(c + d\*Sqrt[x]))])/d^8

**fricas** [F] time = 2.11, size = 0, normalized size = 0.00

$$\text{integral}\left(bx^3 \sec(d\sqrt{x} + c) + ax^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*sec(c+d\*x^(1/2))), x, algorithm="fricas")

[Out] integral(b\*x^3\*sec(d\*sqrt(x) + c) + a\*x^3, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sec(d\sqrt{x} + c) + a)x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*sec(c+d\*x^(1/2))), x, algorithm="giac")

[Out] integrate((b\*sec(d\*sqrt(x) + c) + a)\*x^3, x)

**maple** [F] time = 1.29, size = 0, normalized size = 0.00

$$\int x^3 (a + b \sec(c + d\sqrt{x})) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(x^3*(a+b*\sec(c+d*x^{(1/2)})),x)$

[Out]  $\text{int}(x^3*(a+b*\sec(c+d*x^{(1/2)})),x)$

**maxima** [B] time = 1.07, size = 1512, normalized size = 3.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(x^3*(a+b*\sec(c+d*x^{(1/2)})),x, \text{algorithm}="maxima")$

[Out]  $\frac{1}{4}*((d*\sqrt{x} + c)^8*a - 8*(d*\sqrt{x} + c)^7*a*c + 28*(d*\sqrt{x} + c)^6*a*c^2 - 56*(d*\sqrt{x} + c)^5*a*c^3 + 70*(d*\sqrt{x} + c)^4*a*c^4 - 56*(d*\sqrt{x} + c)^3*a*c^5 + 28*(d*\sqrt{x} + c)^2*a*c^6 - 8*(d*\sqrt{x} + c)*a*c^7 - 8*b*c^7*\log(\sec(d*\sqrt{x} + c) + \tan(d*\sqrt{x} + c)) + 4*(-2*I*(d*\sqrt{x} + c)^7*b + 14*I*(d*\sqrt{x} + c)^6*b*c - 42*I*(d*\sqrt{x} + c)^5*b*c^2 + 70*I*(d*\sqrt{x} + c)^4*b*c^3 - 70*I*(d*\sqrt{x} + c)^3*b*c^4 + 42*I*(d*\sqrt{x} + c)^2*b*c^5 - 14*I*(d*\sqrt{x} + c)*b*c^6)*\arctan2(\cos(d*\sqrt{x} + c), \sin(d*\sqrt{x} + c) + 1) + 4*(-2*I*(d*\sqrt{x} + c)^7*b + 14*I*(d*\sqrt{x} + c)^6*b*c - 42*I*(d*\sqrt{x} + c)^5*b*c^2 + 70*I*(d*\sqrt{x} + c)^4*b*c^3 - 70*I*(d*\sqrt{x} + c)^3*b*c^4 + 42*I*(d*\sqrt{x} + c)^2*b*c^5 - 14*I*(d*\sqrt{x} + c)*b*c^6)*\arctan2(\cos(d*\sqrt{x} + c), -\sin(d*\sqrt{x} + c) + 1) + 4*(-14*I*(d*\sqrt{x} + c)^6*b + 84*I*(d*\sqrt{x} + c)^5*b*c - 210*I*(d*\sqrt{x} + c)^4*b*c^2 + 280*I*(d*\sqrt{x} + c)^3*b*c^3 - 210*I*(d*\sqrt{x} + c)^2*b*c^4 + 84*I*(d*\sqrt{x} + c)*b*c^5 - 14*I*b*c^6)*\text{dilog}(I*e^{(I*d*\sqrt{x} + I*c)}) + 4*(14*I*(d*\sqrt{x} + c)^6*b - 84*I*(d*\sqrt{x} + c)^5*b*c + 210*I*(d*\sqrt{x} + c)^4*b*c^2 - 280*I*(d*\sqrt{x} + c)^3*b*c^3 + 210*I*(d*\sqrt{x} + c)^2*b*c^4 - 84*I*(d*\sqrt{x} + c)*b*c^5 + 14*I*b*c^6)*\text{dilog}(-I*e^{(I*d*\sqrt{x} + I*c)}) + 4*((d*\sqrt{x} + c)^7*b - 7*(d*\sqrt{x} + c)^6*b*c + 21*(d*\sqrt{x} + c)^5*b*c^2 - 35*(d*\sqrt{x} + c)^4*b*c^3 + 35*(d*\sqrt{x} + c)^3*b*c^4 - 21*(d*\sqrt{x} + c)^2*b*c^5 + 7*(d*\sqrt{x} + c)*b*c^6)*\log(\cos(d*\sqrt{x} + c)^2 + \sin(d*\sqrt{x} + c)^2 + 2*\sin(d*\sqrt{x} + c) + 1) - 4*((d*\sqrt{x} + c)^7*b - 7*(d*\sqrt{x} + c)^6*b*c + 21*(d*\sqrt{x} + c)^5*b*c^2 - 35*(d*\sqrt{x} + c)^4*b*c^3 + 35*(d*\sqrt{x} + c)^3*b*c^4 - 21*(d*\sqrt{x} + c)^2*b*c^5 + 7*(d*\sqrt{x} + c)*b*c^6)*\log(\cos(d*\sqrt{x} + c)^2 + \sin(d*\sqrt{x} + c)^2 - 2*\sin(d*\sqrt{x} + c) + 1) + 40320*I*b*\text{polylog}(8, I*e^{(I*d*\sqrt{x} + I*c)}) - 40320*I*b*\text{polylog}(8, -I*e^{(I*d*\sqrt{x} + I*c)}) + 40320*((d*\sqrt{x} + c)*b - b*c)*\text{polylog}(7, I*e^{(I*d*\sqrt{x} + I*c)}) - 40320*((d*\sqrt{x} + c)*b - b*c)*\text{polylog}(7, -I*e^{(I*d*\sqrt{x} + I*c)}) + 4*(-5040*I*(d*\sqrt{x} + c)^2*b + 10080*I*(d*\sqrt{x} + c)*b*c - 5040*I*b*c^2)*\text{polylog}(6, I*e^{(I*d*\sqrt{x} + I*c)}) + 4*(5040*I*(d*\sqrt{x} + c)^2*b - 10080*I*(d*\sqrt{x} + c)*b*c + 5040*I*b*c^2)*\text{polylog}(6, -I*e^{(I*d*\sqrt{x} + I*c)}) - 6720*((d*\sqrt{x} + c)^3*b - 3*(d*\sqrt{x} + c)^2*b*c + 3*(d*\sqrt{x} + c)*b*c^2 - b*c^3)*\text{polylog}(5, I*e^{(I*d*\sqrt{x} + I*c)})$

+ 6720\*((d\*sqrt(x) + c)^3\*b - 3\*(d\*sqrt(x) + c)^2\*b\*c + 3\*(d\*sqrt(x) + c)\*b\*c^2 - b\*c^3)\*polylog(5, -I\*e^(I\*d\*sqrt(x) + I\*c)) + 4\*(420\*I\*(d\*sqrt(x) + c)^4\*b - 1680\*I\*(d\*sqrt(x) + c)^3\*b\*c + 2520\*I\*(d\*sqrt(x) + c)^2\*b\*c^2 - 1680\*I\*(d\*sqrt(x) + c)\*b\*c^3 + 420\*I\*b\*c^4)\*polylog(4, I\*e^(I\*d\*sqrt(x) + I\*c)) + 4\*(-420\*I\*(d\*sqrt(x) + c)^4\*b + 1680\*I\*(d\*sqrt(x) + c)^3\*b\*c - 2520\*I\*(d\*sqrt(x) + c)^2\*b\*c^2 + 1680\*I\*(d\*sqrt(x) + c)\*b\*c^3 - 420\*I\*b\*c^4)\*polylog(4, -I\*e^(I\*d\*sqrt(x) + I\*c)) + 336\*((d\*sqrt(x) + c)^5\*b - 5\*(d\*sqrt(x) + c)^4\*b\*c + 10\*(d\*sqrt(x) + c)^3\*b\*c^2 - 10\*(d\*sqrt(x) + c)^2\*b\*c^3 + 5\*(d\*sqrt(x) + c)\*b\*c^4 - b\*c^5)\*polylog(3, I\*e^(I\*d\*sqrt(x) + I\*c)) - 336\*((d\*sqrt(x) + c)^5\*b - 5\*(d\*sqrt(x) + c)^4\*b\*c + 10\*(d\*sqrt(x) + c)^3\*b\*c^2 - 10\*(d\*sqrt(x) + c)^2\*b\*c^3 + 5\*(d\*sqrt(x) + c)\*b\*c^4 - b\*c^5)\*polylog(3, -I\*e^(I\*d\*sqrt(x) + I\*c)))/d^8

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \left( a + \frac{b}{\cos(c + d\sqrt{x})} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(a + b/cos(c + d\*x^(1/2))),x)

[Out] int(x^3\*(a + b/cos(c + d\*x^(1/2))), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 (a + b \sec(c + d\sqrt{x})) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(a+b\*sec(c+d\*x\*\*(1/2))),x)

[Out] Integral(x\*\*3\*(a + b\*sec(c + d\*sqrt(x))), x)

### 3.32 $\int x^2 (a + b \sec(c + d\sqrt{x})) dx$

**Optimal.** Leaf size=348

$$\frac{ax^3}{3} + \frac{240ib\text{Li}_6\left(-ie^{i(c+d\sqrt{x})}\right)}{d^6} - \frac{240ib\text{Li}_6\left(ie^{i(c+d\sqrt{x})}\right)}{d^6} + \frac{240b\sqrt{x}\text{Li}_5\left(-ie^{i(c+d\sqrt{x})}\right)}{d^5} - \frac{240b\sqrt{x}\text{Li}_5\left(ie^{i(c+d\sqrt{x})}\right)}{d^5} - \frac{120ibx^2\text{PolyLog}\left(2, -ie^{i(c+d\sqrt{x})}\right)}{d^2} + \frac{120ibx^2\text{PolyLog}\left(2, ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{40bx^{3/2}\text{PolyLog}\left(3, -ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{40bx^{3/2}\text{PolyLog}\left(3, ie^{i(c+d\sqrt{x})}\right)}{d^3} - \frac{120ibx\text{PolyLog}\left(4, -ie^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{120ibx\text{PolyLog}\left(4, ie^{i(c+d\sqrt{x})}\right)}{d^4} - \frac{240ib\text{PolyLog}\left(5, -ie^{i(c+d\sqrt{x})}\right)}{d^5} + \frac{240ib\text{PolyLog}\left(5, ie^{i(c+d\sqrt{x})}\right)}{d^5} - \frac{240ib\text{PolyLog}\left(6, -ie^{i(c+d\sqrt{x})}\right)}{d^6} + \frac{240ib\text{PolyLog}\left(6, ie^{i(c+d\sqrt{x})}\right)}{d^6}$$

[Out]  $\frac{1}{3}ax^3 - 4Ib x^{5/2} \arctan(\exp(I(c+d\sqrt{x}))) / d + 10Ib x^2 \text{polylog}(2, -\exp(I(c+d\sqrt{x}))) / d^2 - 10Ib x^2 \text{polylog}(2, \exp(I(c+d\sqrt{x}))) / d^2 - 40Ib x^{3/2} \text{polylog}(3, -\exp(I(c+d\sqrt{x}))) / d^3 + 40Ib x^{3/2} \text{polylog}(3, \exp(I(c+d\sqrt{x}))) / d^3 - 120Ib x \text{polylog}(4, -\exp(I(c+d\sqrt{x}))) / d^4 + 120Ib x \text{polylog}(4, \exp(I(c+d\sqrt{x}))) / d^4 + 240Ib \text{polylog}(6, -\exp(I(c+d\sqrt{x}))) / d^6 - 240Ib \text{polylog}(6, \exp(I(c+d\sqrt{x}))) / d^6 + 240Ib \text{polylog}(5, -\exp(I(c+d\sqrt{x}))) x^{1/2} / d^5 - 240Ib \text{polylog}(5, \exp(I(c+d\sqrt{x}))) x^{1/2} / d^5$

**Rubi [A]** time = 0.31, antiderivative size = 348, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$ , Rules used = {14, 4204, 4181, 2531, 6609, 2282, 6589}

$$\frac{10ibx^2\text{PolyLog}\left(2, -ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{10ibx^2\text{PolyLog}\left(2, ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{40bx^{3/2}\text{PolyLog}\left(3, -ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{40bx^{3/2}\text{PolyLog}\left(3, ie^{i(c+d\sqrt{x})}\right)}{d^3} - \frac{120ibx\text{PolyLog}\left(4, -ie^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{120ibx\text{PolyLog}\left(4, ie^{i(c+d\sqrt{x})}\right)}{d^4} - \frac{240ib\text{PolyLog}\left(5, -ie^{i(c+d\sqrt{x})}\right)}{d^5} + \frac{240ib\text{PolyLog}\left(5, ie^{i(c+d\sqrt{x})}\right)}{d^5} - \frac{240ib\text{PolyLog}\left(6, -ie^{i(c+d\sqrt{x})}\right)}{d^6} + \frac{240ib\text{PolyLog}\left(6, ie^{i(c+d\sqrt{x})}\right)}{d^6}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^2(a + b\text{Sec}[c + d\text{Sqrt}[x]]), x]$

[Out]  $(ax^3)/3 - ((4I)b x^{5/2} \text{ArcTan}[E^{I(c + d\text{Sqrt}[x])}]) / d + ((10I)b x^2 \text{PolyLog}[2, (-I)E^{I(c + d\text{Sqrt}[x])}]) / d^2 - ((10I)b x^2 \text{PolyLog}[2, I E^{I(c + d\text{Sqrt}[x])}]) / d^2 - (40Ib x^{3/2} \text{PolyLog}[3, (-I)E^{I(c + d\text{Sqrt}[x])}]) / d^3 + (40Ib x^{3/2} \text{PolyLog}[3, I E^{I(c + d\text{Sqrt}[x])}]) / d^3 - ((120I)b x \text{PolyLog}[4, (-I)E^{I(c + d\text{Sqrt}[x])}]) / d^4 + ((120I)b x \text{PolyLog}[4, I E^{I(c + d\text{Sqrt}[x])}]) / d^4 + (240Ib \text{Sqrt}[x] \text{PolyLog}[5, (-I)E^{I(c + d\text{Sqrt}[x])}]) / d^5 - (240Ib \text{Sqrt}[x] \text{PolyLog}[5, I E^{I(c + d\text{Sqrt}[x])}]) / d^5 + ((240I)b \text{PolyLog}[6, (-I)E^{I(c + d\text{Sqrt}[x])}]) / d^6 - ((240I)b \text{PolyLog}[6, I E^{I(c + d\text{Sqrt}[x])}]) / d^6$

#### Rule 14

$\text{Int}[(u_*)((c_*) (x_*)^m), x\_Symbol] := \text{Int}[\text{ExpandIntegrand}[(c*x)^m u, x], x] /; \text{FreeQ}[\{c, m\}, x] \&\& \text{SumQ}[u] \&\& !\text{LinearQ}[u, x] \&\& !\text{MatchQ}[u, (a_*) + (b_*) (v_*)] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{InverseFunctionQ}[v]$

#### Rule 2282



```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)^v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_)^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n])]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]), x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

### Rule 4204

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)
*(x_))))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

### Rubi steps

$$\begin{aligned}
\int x^2 (a + b \sec(c + d\sqrt{x})) dx &= \int (ax^2 + bx^2 \sec(c + d\sqrt{x})) dx \\
&= \frac{ax^3}{3} + b \int x^2 \sec(c + d\sqrt{x}) dx \\
&= \frac{ax^3}{3} + (2b) \text{Subst} \left( \int x^5 \sec(c + dx) dx, x, \sqrt{x} \right) \\
&= \frac{ax^3}{3} - \frac{4ibx^{5/2} \tan^{-1} \left( e^{i(c+d\sqrt{x})} \right)}{d} - \frac{(10b) \text{Subst} \left( \int x^4 \log(1 - ie^{i(c+dx)}) dx, x, \sqrt{x} \right)}{d} \\
&= \frac{ax^3}{3} - \frac{4ibx^{5/2} \tan^{-1} \left( e^{i(c+d\sqrt{x})} \right)}{d} + \frac{10ibx^2 \text{Li}_2 \left( -ie^{i(c+d\sqrt{x})} \right)}{d^2} - \frac{10ibx^2 \text{Li}_2 \left( ie^{i(c+d\sqrt{x})} \right)}{d^2} \\
&= \frac{ax^3}{3} - \frac{4ibx^{5/2} \tan^{-1} \left( e^{i(c+d\sqrt{x})} \right)}{d} + \frac{10ibx^2 \text{Li}_2 \left( -ie^{i(c+d\sqrt{x})} \right)}{d^2} - \frac{10ibx^2 \text{Li}_2 \left( ie^{i(c+d\sqrt{x})} \right)}{d^2} \\
&= \frac{ax^3}{3} - \frac{4ibx^{5/2} \tan^{-1} \left( e^{i(c+d\sqrt{x})} \right)}{d} + \frac{10ibx^2 \text{Li}_2 \left( -ie^{i(c+d\sqrt{x})} \right)}{d^2} - \frac{10ibx^2 \text{Li}_2 \left( ie^{i(c+d\sqrt{x})} \right)}{d^2} \\
&= \frac{ax^3}{3} - \frac{4ibx^{5/2} \tan^{-1} \left( e^{i(c+d\sqrt{x})} \right)}{d} + \frac{10ibx^2 \text{Li}_2 \left( -ie^{i(c+d\sqrt{x})} \right)}{d^2} - \frac{10ibx^2 \text{Li}_2 \left( ie^{i(c+d\sqrt{x})} \right)}{d^2} \\
&= \frac{ax^3}{3} - \frac{4ibx^{5/2} \tan^{-1} \left( e^{i(c+d\sqrt{x})} \right)}{d} + \frac{10ibx^2 \text{Li}_2 \left( -ie^{i(c+d\sqrt{x})} \right)}{d^2} - \frac{10ibx^2 \text{Li}_2 \left( ie^{i(c+d\sqrt{x})} \right)}{d^2} \\
&= \frac{ax^3}{3} - \frac{4ibx^{5/2} \tan^{-1} \left( e^{i(c+d\sqrt{x})} \right)}{d} + \frac{10ibx^2 \text{Li}_2 \left( -ie^{i(c+d\sqrt{x})} \right)}{d^2} - \frac{10ibx^2 \text{Li}_2 \left( ie^{i(c+d\sqrt{x})} \right)}{d^2} \\
&= \frac{ax^3}{3} - \frac{4ibx^{5/2} \tan^{-1} \left( e^{i(c+d\sqrt{x})} \right)}{d} + \frac{10ibx^2 \text{Li}_2 \left( -ie^{i(c+d\sqrt{x})} \right)}{d^2} - \frac{10ibx^2 \text{Li}_2 \left( ie^{i(c+d\sqrt{x})} \right)}{d^2}
\end{aligned}$$

**Mathematica [A]** time = 0.08, size = 351, normalized size = 1.01

$$\frac{ax^3}{3} + \frac{240ib\text{Li}_6 \left( -ie^{i(c+d\sqrt{x})} \right)}{d^6} - \frac{240ib\text{Li}_6 \left( ie^{i(c+d\sqrt{x})} \right)}{d^6} + \frac{240b\sqrt{x}\text{Li}_5 \left( -ie^{i(c+d\sqrt{x})} \right)}{d^5} - \frac{240b\sqrt{x}\text{Li}_5 \left( ie^{i(c+d\sqrt{x})} \right)}{d^5} - \frac{120ibx^2}{d^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*(a + b\*Sec[c + d\*Sqrt[x]]),x]

[Out] (a\*x^3)/3 - ((4\*I)\*b\*x^(5/2)\*ArcTan[E^(I\*c + I\*d\*Sqrt[x])])/d + ((10\*I)\*b\*x^2\*PolyLog[2, (-I)\*E^(I\*(c + d\*Sqrt[x]))])/d^2 - ((10\*I)\*b\*x^2\*PolyLog[2, I\*E^(I\*(c + d\*Sqrt[x]))])/d^2 - (40\*b\*x^(3/2)\*PolyLog[3, (-I)\*E^(I\*(c + d\*Sqrt[x]))])/d^3 + (40\*b\*x^(3/2)\*PolyLog[3, I\*E^(I\*(c + d\*Sqrt[x]))])/d^3 - ((120\*I)\*b\*x\*PolyLog[4, (-I)\*E^(I\*(c + d\*Sqrt[x]))])/d^4 + ((120\*I)\*b\*x\*PolyLog[4, I\*E^(I\*(c + d\*Sqrt[x]))])/d^4

og[4, I\*E^(I\*(c + d\*Sqrt[x]))]/d^4 + (240\*b\*Sqrt[x]\*PolyLog[5, (-I)\*E^(I\*(c + d\*Sqrt[x]))]/d^5 - (240\*b\*Sqrt[x]\*PolyLog[5, I\*E^(I\*(c + d\*Sqrt[x]))])/d^5 + ((240\*I)\*b\*PolyLog[6, (-I)\*E^(I\*(c + d\*Sqrt[x]))])/d^6 - ((240\*I)\*b\*PolyLog[6, I\*E^(I\*(c + d\*Sqrt[x]))])/d^6

**fricas** [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}(bx^2 \sec(d\sqrt{x} + c) + ax^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*sec(c+d\*x^(1/2))),x, algorithm="fricas")

[Out] integral(b\*x^2\*sec(d\*sqrt(x) + c) + a\*x^2, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sec(d\sqrt{x} + c) + a)x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*sec(c+d\*x^(1/2))),x, algorithm="giac")

[Out] integrate((b\*sec(d\*sqrt(x) + c) + a)\*x^2, x)

**maple** [F] time = 1.16, size = 0, normalized size = 0.00

$$\int x^2 (a + b \sec(c + d\sqrt{x})) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a+b\*sec(c+d\*x^(1/2))),x)

[Out] int(x^2\*(a+b\*sec(c+d\*x^(1/2))),x)

**maxima** [B] time = 1.01, size = 966, normalized size = 2.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*sec(c+d\*x^(1/2))),x, algorithm="maxima")

[Out] 1/3\*((d\*sqrt(x) + c)^6\*a - 6\*(d\*sqrt(x) + c)^5\*a\*c + 15\*(d\*sqrt(x) + c)^4\*a\*c^2 - 20\*(d\*sqrt(x) + c)^3\*a\*c^3 + 15\*(d\*sqrt(x) + c)^2\*a\*c^4 - 6\*(d\*sqrt(x) + c)\*a\*c^5 - 6\*b\*c^5\*log(sec(d\*sqrt(x) + c) + tan(d\*sqrt(x) + c)) + 3\*(-2\*I\*(d\*sqrt(x) + c)^5\*b + 10\*I\*(d\*sqrt(x) + c)^4\*b\*c - 20\*I\*(d\*sqrt(x) + c)

$$\begin{aligned} &^3*b*c^2 + 20*I*(d*\sqrt{x} + c)^2*b*c^3 - 10*I*(d*\sqrt{x} + c)*b*c^4)*\arctan2(\cos(d*\sqrt{x} + c), \sin(d*\sqrt{x} + c) + 1) + 3*(-2*I*(d*\sqrt{x} + c)^5*b \\ &+ 10*I*(d*\sqrt{x} + c)^4*b*c - 20*I*(d*\sqrt{x} + c)^3*b*c^2 + 20*I*(d*\sqrt{x} + c)^2*b*c^3 - 10*I*(d*\sqrt{x} + c)*b*c^4)*\arctan2(\cos(d*\sqrt{x} + c), \\ &-\sin(d*\sqrt{x} + c) + 1) + 3*(-10*I*(d*\sqrt{x} + c)^4*b + 40*I*(d*\sqrt{x} + c)^3*b*c - 60*I*(d*\sqrt{x} + c)^2*b*c^2 + 40*I*(d*\sqrt{x} + c)*b*c^3 - 10 \\ &*I*b*c^4)*\operatorname{dilog}(I*e^{(I*d*\sqrt{x} + I*c)}) + 3*(10*I*(d*\sqrt{x} + c)^4*b - 40 \\ &*I*(d*\sqrt{x} + c)^3*b*c + 60*I*(d*\sqrt{x} + c)^2*b*c^2 - 40*I*(d*\sqrt{x} + c)*b*c^3 + 10*I*b*c^4)*\operatorname{dilog}(-I*e^{(I*d*\sqrt{x} + I*c)}) + 3*((d*\sqrt{x} + c) \\ &)^5*b - 5*(d*\sqrt{x} + c)^4*b*c + 10*(d*\sqrt{x} + c)^3*b*c^2 - 10*(d*\sqrt{x} + c)^2*b*c^3 + 5*(d*\sqrt{x} + c)*b*c^4)*\log(\cos(d*\sqrt{x} + c)^2 + \sin(d* \\ &\sqrt{x} + c)^2 + 2*\sin(d*\sqrt{x} + c) + 1) - 3*((d*\sqrt{x} + c)^5*b - 5*(d* \\ &\sqrt{x} + c)^4*b*c + 10*(d*\sqrt{x} + c)^3*b*c^2 - 10*(d*\sqrt{x} + c)^2*b*c^3 + 5*(d*\sqrt{x} + c)*b*c^4)*\log(\cos(d*\sqrt{x} + c)^2 + \sin(d*\sqrt{x} + c)^2 \\ &- 2*\sin(d*\sqrt{x} + c) + 1) - 720*I*b*\operatorname{polylog}(6, I*e^{(I*d*\sqrt{x} + I*c)}) \\ &+ 720*I*b*\operatorname{polylog}(6, -I*e^{(I*d*\sqrt{x} + I*c)}) - 720*((d*\sqrt{x} + c)*b - \\ &b*c)*\operatorname{polylog}(5, I*e^{(I*d*\sqrt{x} + I*c)}) + 720*((d*\sqrt{x} + c)*b - b*c)*\operatorname{polylog}(5, -I*e^{(I*d*\sqrt{x} + I*c)}) + 3*(120*I*(d*\sqrt{x} + c)^2*b - 240*I*( \\ &d*\sqrt{x} + c)*b*c + 120*I*b*c^2)*\operatorname{polylog}(4, I*e^{(I*d*\sqrt{x} + I*c)}) + 3*( \\ &-120*I*(d*\sqrt{x} + c)^2*b + 240*I*(d*\sqrt{x} + c)*b*c - 120*I*b*c^2)*\operatorname{polylog}(4, -I*e^{(I*d*\sqrt{x} + I*c)}) + 120*((d*\sqrt{x} + c)^3*b - 3*(d*\sqrt{x} + c) \\ &)^2*b*c + 3*(d*\sqrt{x} + c)*b*c^2 - b*c^3)*\operatorname{polylog}(3, I*e^{(I*d*\sqrt{x} + I*c)}) - 120*((d*\sqrt{x} + c)^3*b - 3*(d*\sqrt{x} + c)^2*b*c + 3*(d*\sqrt{x} + c) \\ &)*b*c^2 - b*c^3)*\operatorname{polylog}(3, -I*e^{(I*d*\sqrt{x} + I*c)})/d^6 \end{aligned}$$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \left( a + \frac{b}{\cos(c + d\sqrt{x})} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(a + b/cos(c + d*x^(1/2))),x)`

[Out] `int(x^2*(a + b/cos(c + d*x^(1/2))), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 (a + b \sec(c + d\sqrt{x})) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(a+b*sec(c+d*x**(1/2))),x)`

[Out] `Integral(x**2*(a + b*sec(c + d*sqrt(x))), x)`

### 3.33 $\int x \left( a + b \sec \left( c + d\sqrt{x} \right) \right) dx$

**Optimal.** Leaf size=220

$$\frac{ax^2}{2} - \frac{12ib\text{Li}_4\left(-ie^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{12ib\text{Li}_4\left(ie^{i(c+d\sqrt{x})}\right)}{d^4} - \frac{12b\sqrt{x}\text{Li}_3\left(-ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{12b\sqrt{x}\text{Li}_3\left(ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{6ibx\text{Li}_2\left(-ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{6ibx\text{Li}_2\left(ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{12b\sqrt{x}\text{PolyLog}\left(3, -ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{12b\sqrt{x}\text{PolyLog}\left(3, ie^{i(c+d\sqrt{x})}\right)}{d^3}$$

[Out]  $\frac{1}{2}ax^2 - 4Ib\sqrt{x} \arctan\left(\exp\left(I(c+d\sqrt{x})\right)\right) + 6Ib\sqrt{x} \text{polylog}\left(2, -\exp\left(I(c+d\sqrt{x})\right)\right) - 6Ib\sqrt{x} \text{polylog}\left(2, \exp\left(I(c+d\sqrt{x})\right)\right) - 12Ib\sqrt{x} \text{polylog}\left(4, -\exp\left(I(c+d\sqrt{x})\right)\right) + 12Ib\sqrt{x} \text{polylog}\left(4, \exp\left(I(c+d\sqrt{x})\right)\right) - 12Ib\sqrt{x} \text{polylog}\left(3, -\exp\left(I(c+d\sqrt{x})\right)\right) \sqrt{x} + 12Ib\sqrt{x} \text{polylog}\left(3, \exp\left(I(c+d\sqrt{x})\right)\right) \sqrt{x}$

**Rubi [A]** time = 0.19, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$ , Rules used = {14, 4204, 4181, 2531, 6609, 2282, 6589}

$$\frac{6ibx\text{PolyLog}\left(2, -ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{6ibx\text{PolyLog}\left(2, ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{12b\sqrt{x}\text{PolyLog}\left(3, -ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{12b\sqrt{x}\text{PolyLog}\left(3, ie^{i(c+d\sqrt{x})}\right)}{d^3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x(a + b\text{Sec}[c + d\text{Sqrt}[x]]), x]$

[Out]  $\frac{ax^2}{2} - \left(\frac{4Ib\sqrt{x} \text{ArcTan}\left[E^{I(c+d\sqrt{x})}\right]}{d} + \frac{6Ib\sqrt{x} \text{PolyLog}\left[2, (-1)E^{I(c+d\sqrt{x})}\right]}{d^2} - \frac{6Ib\sqrt{x} \text{PolyLog}\left[2, E^{I(c+d\sqrt{x})}\right]}{d^2} - \frac{12Ib\sqrt{x} \text{PolyLog}\left[3, (-1)E^{I(c+d\sqrt{x})}\right]}{d^3} + \frac{12Ib\sqrt{x} \text{PolyLog}\left[3, E^{I(c+d\sqrt{x})}\right]}{d^3} - \frac{12Ib\sqrt{x} \text{PolyLog}\left[4, (-1)E^{I(c+d\sqrt{x})}\right]}{d^4} + \frac{12Ib\sqrt{x} \text{PolyLog}\left[4, E^{I(c+d\sqrt{x})}\right]}{d^4}\right)$

#### Rule 14

$\text{Int}[(u_*)((c_*)*(x_*))^{(m_*)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(c_*x_*)^{m_*}, x], x] /; \text{FreeQ}[\{c, m\}, x] \&\& \text{SumQ}[u] \&\& !\text{LinearQ}[u, x] \&\& !\text{MatchQ}[u, (a_*) + (b_*)*(v_*)] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{InverseFunctionQ}[v]$

#### Rule 2282

$\text{Int}[u, x\_Symbol] \rightarrow \text{With}[\{v = \text{FunctionOfExponential}[u, x]\}, \text{Dist}[v/D[v, x], \text{Subst}[\text{Int}[\text{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x] /; \text{FunctionOfExponentialQ}[u, x] \&\& !\text{MatchQ}[u, (w_*)((a_*)*(v_*)^{(n_*)})^{(m_*)} /; \text{FreeQ}[\{a, m, n\}, x] \&\& \text{IntegerQ}[m*n] \&\& !\text{MatchQ}[u, E^{((c_*)*(a_*) + (b_*)*x)}] * (F_*)[v_] /; \text{FreeQ}[\{a, b, c\}, x] \&\& \text{InverseFunctionQ}[F[x]]]$

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_)))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x)
)], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4204

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)
*(x_)))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int x(a + b \sec(c + d\sqrt{x})) dx &= \int (ax + bx \sec(c + d\sqrt{x})) dx \\
&= \frac{ax^2}{2} + b \int x \sec(c + d\sqrt{x}) dx \\
&= \frac{ax^2}{2} + (2b) \text{Subst} \left( \int x^3 \sec(c + dx) dx, x, \sqrt{x} \right) \\
&= \frac{ax^2}{2} - \frac{4ibx^{3/2} \tan^{-1} \left( e^{i(c+d\sqrt{x})} \right)}{d} - \frac{(6b) \text{Subst} \left( \int x^2 \log(1 - ie^{i(c+dx)}) dx, x, \sqrt{x} \right)}{d} \\
&= \frac{ax^2}{2} - \frac{4ibx^{3/2} \tan^{-1} \left( e^{i(c+d\sqrt{x})} \right)}{d} + \frac{6ibx \text{Li}_2 \left( -ie^{i(c+d\sqrt{x})} \right)}{d^2} - \frac{6ibx \text{Li}_2 \left( ie^{i(c+d\sqrt{x})} \right)}{d^2} \\
&= \frac{ax^2}{2} - \frac{4ibx^{3/2} \tan^{-1} \left( e^{i(c+d\sqrt{x})} \right)}{d} + \frac{6ibx \text{Li}_2 \left( -ie^{i(c+d\sqrt{x})} \right)}{d^2} - \frac{6ibx \text{Li}_2 \left( ie^{i(c+d\sqrt{x})} \right)}{d^2} \\
&= \frac{ax^2}{2} - \frac{4ibx^{3/2} \tan^{-1} \left( e^{i(c+d\sqrt{x})} \right)}{d} + \frac{6ibx \text{Li}_2 \left( -ie^{i(c+d\sqrt{x})} \right)}{d^2} - \frac{6ibx \text{Li}_2 \left( ie^{i(c+d\sqrt{x})} \right)}{d^2} \\
&= \frac{ax^2}{2} - \frac{4ibx^{3/2} \tan^{-1} \left( e^{i(c+d\sqrt{x})} \right)}{d} + \frac{6ibx \text{Li}_2 \left( -ie^{i(c+d\sqrt{x})} \right)}{d^2} - \frac{6ibx \text{Li}_2 \left( ie^{i(c+d\sqrt{x})} \right)}{d^2}
\end{aligned}$$

**Mathematica [A]** time = 0.06, size = 223, normalized size = 1.01

$$\frac{ax^2}{2} - \frac{12ib \text{Li}_4 \left( -ie^{i(c+d\sqrt{x})} \right)}{d^4} + \frac{12ib \text{Li}_4 \left( ie^{i(c+d\sqrt{x})} \right)}{d^4} - \frac{12b\sqrt{x} \text{Li}_3 \left( -ie^{i(c+d\sqrt{x})} \right)}{d^3} + \frac{12b\sqrt{x} \text{Li}_3 \left( ie^{i(c+d\sqrt{x})} \right)}{d^3} + \frac{6ibx \text{Li}_2 \left( -ie^{i(c+d\sqrt{x})} \right)}{d^2} - \frac{6ibx \text{Li}_2 \left( ie^{i(c+d\sqrt{x})} \right)}{d^2}$$

Antiderivative was successfully verified.

[In] Integrate[x\*(a + b\*Sec[c + d\*Sqrt[x]]),x]

[Out] (a\*x^2)/2 - ((4\*I)\*b\*x^(3/2)\*ArcTan[E^(I\*c + I\*d\*Sqrt[x])])/d + ((6\*I)\*b\*x\*PolyLog[2, (-I)\*E^(I\*(c + d\*Sqrt[x]))])/d^2 - ((6\*I)\*b\*x\*PolyLog[2, I\*E^(I\*(c + d\*Sqrt[x]))])/d^2 - (12\*b\*Sqrt[x]\*PolyLog[3, (-I)\*E^(I\*(c + d\*Sqrt[x]))])/d^3 + (12\*b\*Sqrt[x]\*PolyLog[3, I\*E^(I\*(c + d\*Sqrt[x]))])/d^3 - ((12\*I)\*b\*PolyLog[4, (-I)\*E^(I\*(c + d\*Sqrt[x]))])/d^4 + ((12\*I)\*b\*PolyLog[4, I\*E^(I\*(c + d\*Sqrt[x]))])/d^4

**fricas [F]** time = 0.82, size = 0, normalized size = 0.00

$$\text{integral} \left( bx \sec(d\sqrt{x} + c) + ax, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*sec(c+d\*x^(1/2))),x, algorithm="fricas")

[Out] integral(b\*x\*sec(d\*sqrt(x) + c) + a\*x, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sec(d\sqrt{x} + c) + a)x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*sec(c+d\*x^(1/2))),x, algorithm="giac")

[Out] integrate((b\*sec(d\*sqrt(x) + c) + a)\*x, x)

**maple** [F] time = 1.15, size = 0, normalized size = 0.00

$$\int x(a + b \sec(c + d\sqrt{x})) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a+b\*sec(c+d\*x^(1/2))),x)

[Out] int(x\*(a+b\*sec(c+d\*x^(1/2))),x)

**maxima** [B] time = 0.89, size = 540, normalized size = 2.45

$$(d\sqrt{x} + c)^4 a - 4(d\sqrt{x} + c)^3 ac + 6(d\sqrt{x} + c)^2 ac^2 - 4(d\sqrt{x} + c)ac^3 - 4bc^3 \log(\sec(d\sqrt{x} + c) + \tan(d\sqrt{x} + c))$$


---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*sec(c+d\*x^(1/2))),x, algorithm="maxima")

[Out] 1/2\*((d\*sqrt(x) + c)^4\*a - 4\*(d\*sqrt(x) + c)^3\*a\*c + 6\*(d\*sqrt(x) + c)^2\*a\*c^2 - 4\*(d\*sqrt(x) + c)\*a\*c^3 - 4\*b\*c^3\*log(sec(d\*sqrt(x) + c) + tan(d\*sqrt(x) + c)) + 2\*(-2\*I\*(d\*sqrt(x) + c)^3\*b + 6\*I\*(d\*sqrt(x) + c)^2\*b\*c - 6\*I\*(d\*sqrt(x) + c)\*b\*c^2)\*arctan2(cos(d\*sqrt(x) + c), sin(d\*sqrt(x) + c) + 1) + 2\*(-2\*I\*(d\*sqrt(x) + c)^3\*b + 6\*I\*(d\*sqrt(x) + c)^2\*b\*c - 6\*I\*(d\*sqrt(x) + c)\*b\*c^2)\*arctan2(cos(d\*sqrt(x) + c), -sin(d\*sqrt(x) + c) + 1) + 2\*(-6\*I\*(d\*sqrt(x) + c)^2\*b + 12\*I\*(d\*sqrt(x) + c)\*b\*c - 6\*I\*b\*c^2)\*dilog(I\*e^(I\*d\*sqrt(x) + I\*c)) + 2\*(6\*I\*(d\*sqrt(x) + c)^2\*b - 12\*I\*(d\*sqrt(x) + c)\*b\*c + 6\*I\*b\*c^2)\*dilog(-I\*e^(I\*d\*sqrt(x) + I\*c)) + 2\*((d\*sqrt(x) + c)^3\*b - 3\*(d\*sqrt(x) + c)^2\*b\*c + 3\*(d\*sqrt(x) + c)\*b\*c^2)\*log(cos(d\*sqrt(x) + c)^2 + sin(d\*sqrt(x) + c)^2 + 2\*sin(d\*sqrt(x) + c) + 1) - 2\*((d\*sqrt(x) + c)^3\*b - 3\*(d\*sqrt(x) + c)^2\*b\*c + 3\*(d\*sqrt(x) + c)\*b\*c^2)\*log(cos(d\*sqrt(x) + c)^2 + sin(d\*sqrt(x) + c)^2 - 2\*sin(d\*sqrt(x) + c) + 1) + 24\*I\*b\*polylog(4, I\*e^(I



```
*d*sqrt(x) + I*c)) - 24*I*b*polylog(4, -I*e^(I*d*sqrt(x) + I*c)) + 24*((d*sqrt(x) + c)*b - b*c)*polylog(3, I*e^(I*d*sqrt(x) + I*c)) - 24*((d*sqrt(x) + c)*b - b*c)*polylog(3, -I*e^(I*d*sqrt(x) + I*c)))/d^4
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x \left( a + \frac{b}{\cos(c + d\sqrt{x})} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b/cos(c + d*x^(1/2))),x)
```

```
[Out] int(x*(a + b/cos(c + d*x^(1/2))), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x (a + b \sec(c + d\sqrt{x})) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*sec(c+d*x**(1/2))),x)
```

```
[Out] Integral(x*(a + b*sec(c + d*sqrt(x))), x)
```

$$3.34 \quad \int \frac{a+b \sec(c+d\sqrt{x})}{x} dx$$

Optimal. Leaf size=24

$$b \operatorname{Int}\left(\frac{\sec(c+d\sqrt{x})}{x}, x\right) + a \log(x)$$

[Out]  $a*\ln(x)+b*\operatorname{Unintegrable}(\sec(c+d*x^{(1/2)})/x, x)$

**Rubi** [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{a+b \sec(c+d\sqrt{x})}{x} dx$$

Verification is Not applicable to the result.

[In]  $\operatorname{Int}[(a + b*\operatorname{Sec}[c + d*\operatorname{Sqrt}[x]])/x, x]$

[Out]  $a*\operatorname{Log}[x] + b*\operatorname{Defer}[\operatorname{Int}][\operatorname{Sec}[c + d*\operatorname{Sqrt}[x]]/x, x]$

Rubi steps

$$\begin{aligned} \int \frac{a+b \sec(c+d\sqrt{x})}{x} dx &= \int \left( \frac{a}{x} + \frac{b \sec(c+d\sqrt{x})}{x} \right) dx \\ &= a \log(x) + b \int \frac{\sec(c+d\sqrt{x})}{x} dx \end{aligned}$$

**Mathematica** [A] time = 2.06, size = 0, normalized size = 0.00

$$\int \frac{a+b \sec(c+d\sqrt{x})}{x} dx$$

Verification is Not applicable to the result.

[In]  $\operatorname{Integrate}[(a + b*\operatorname{Sec}[c + d*\operatorname{Sqrt}[x]])/x, x]$

[Out]  $\operatorname{Integrate}[(a + b*\operatorname{Sec}[c + d*\operatorname{Sqrt}[x]])/x, x]$

**fricas** [A] time = 0.75, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{b \sec(d\sqrt{x} + c) + a}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sec(c+d\*x^(1/2)))/x,x, algorithm="fricas")

[Out] integral((b\*sec(d\*sqrt(x) + c) + a)/x, x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \sec(d\sqrt{x} + c) + a}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sec(c+d\*x^(1/2)))/x,x, algorithm="giac")

[Out] integrate((b\*sec(d\*sqrt(x) + c) + a)/x, x)

**maple** [A] time = 1.35, size = 0, normalized size = 0.00

$$\int \frac{a + b \sec(c + d\sqrt{x})}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*sec(c+d\*x^(1/2)))/x,x)

[Out] int((a+b\*sec(c+d\*x^(1/2)))/x,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$2b \int \frac{\cos(2d\sqrt{x} + 2c) \cos(d\sqrt{x} + c) + \sin(2d\sqrt{x} + 2c) \sin(d\sqrt{x} + c) + \cos(d\sqrt{x} + c)}{(\cos(2d\sqrt{x} + 2c)^2 + \sin(2d\sqrt{x} + 2c)^2 + 2 \cos(2d\sqrt{x} + 2c) + 1)x} dx + a \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sec(c+d\*x^(1/2)))/x,x, algorithm="maxima")

[Out] 2\*b\*integrate((cos(2\*d\*sqrt(x) + 2\*c)\*cos(d\*sqrt(x) + c) + sin(2\*d\*sqrt(x) + 2\*c)\*sin(d\*sqrt(x) + c) + cos(d\*sqrt(x) + c))/((cos(2\*d\*sqrt(x) + 2\*c)^2 + sin(2\*d\*sqrt(x) + 2\*c)^2 + 2\*cos(2\*d\*sqrt(x) + 2\*c) + 1)\*x), x) + a\*log(x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a + \frac{b}{\cos(c+d\sqrt{x})}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b/cos(c + d*x^(1/2)))/x,x)`

[Out] `int((a + b/cos(c + d*x^(1/2)))/x, x)`

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \sec(c + d\sqrt{x})}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sec(c+d*x**(1/2)))/x,x)`

[Out] `Integral((a + b*sec(c + d*sqrt(x)))/x, x)`

$$3.35 \quad \int \frac{a+b \sec(c+d\sqrt{x})}{x^2} dx$$

Optimal. Leaf size=26

$$b \operatorname{Int} \left( \frac{\sec(c+d\sqrt{x})}{x^2}, x \right) - \frac{a}{x}$$

[Out]  $-a/x + b \operatorname{Unintegrable}(\sec(c+d\sqrt{x})/x^2, x)$

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{a + b \sec(c + d\sqrt{x})}{x^2} dx$$

Verification is Not applicable to the result.

[In]  $\operatorname{Int}[(a + b \operatorname{Sec}[c + d \operatorname{Sqrt}[x]])/x^2, x]$

[Out]  $-(a/x) + b \operatorname{Defer}[\operatorname{Int}[\operatorname{Sec}[c + d \operatorname{Sqrt}[x]]/x^2, x]]$

Rubi steps

$$\begin{aligned} \int \frac{a + b \sec(c + d\sqrt{x})}{x^2} dx &= \int \left( \frac{a}{x^2} + \frac{b \sec(c + d\sqrt{x})}{x^2} \right) dx \\ &= -\frac{a}{x} + b \int \frac{\sec(c + d\sqrt{x})}{x^2} dx \end{aligned}$$

Mathematica [A] time = 2.20, size = 0, normalized size = 0.00

$$\int \frac{a + b \sec(c + d\sqrt{x})}{x^2} dx$$

Verification is Not applicable to the result.

[In]  $\operatorname{Integrate}[(a + b \operatorname{Sec}[c + d \operatorname{Sqrt}[x]])/x^2, x]$

[Out]  $\operatorname{Integrate}[(a + b \operatorname{Sec}[c + d \operatorname{Sqrt}[x]])/x^2, x]$

fricas [A] time = 0.48, size = 0, normalized size = 0.00

$$\operatorname{integral} \left( \frac{b \sec(d\sqrt{x} + c) + a}{x^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sec(c+d\*x^(1/2)))/x^2,x, algorithm="fricas")

[Out] integral((b\*sec(d\*sqrt(x) + c) + a)/x^2, x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \sec(d\sqrt{x} + c) + a}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sec(c+d\*x^(1/2)))/x^2,x, algorithm="giac")

[Out] integrate((b\*sec(d\*sqrt(x) + c) + a)/x^2, x)

**maple** [A] time = 1.34, size = 0, normalized size = 0.00

$$\int \frac{a + b \sec(c + d\sqrt{x})}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*sec(c+d\*x^(1/2)))/x^2,x)

[Out] int((a+b\*sec(c+d\*x^(1/2)))/x^2,x)

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sec(c+d\*x^(1/2)))/x^2,x, algorithm="maxima")

[Out] Timed out

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a + \frac{b}{\cos(c + d\sqrt{x})}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cos(c + d\*x^(1/2)))/x^2,x)

[Out] int((a + b/cos(c + d\*x^(1/2)))/x^2, x)

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \sec(c + d\sqrt{x})}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sec(c+d\*x\*\*(1/2)))/x\*\*2,x)

[Out] Integral((a + b\*sec(c + d\*sqrt(x)))/x\*\*2, x)

### 3.36 $\int x^3 \left( a + b \sec \left( c + d\sqrt{x} \right) \right)^2 dx$

**Optimal.** Leaf size=749

$$\frac{a^2 x^4}{4} - \frac{20160iab \operatorname{Li}_8 \left( -ie^{i(c+d\sqrt{x})} \right)}{d^8} + \frac{20160iab \operatorname{Li}_8 \left( ie^{i(c+d\sqrt{x})} \right)}{d^8} - \frac{20160ab\sqrt{x} \operatorname{Li}_7 \left( -ie^{i(c+d\sqrt{x})} \right)}{d^7} + \frac{20160ab\sqrt{x} \operatorname{Li}_7 \left( ie^{i(c+d\sqrt{x})} \right)}{d^7}$$

[Out]  $14*b^2*x^3*\ln(1+\exp(2*I*(c+d*x^(1/2))))/d^2+105*b^2*x^2*\operatorname{polylog}(3,-\exp(2*I*(c+d*x^(1/2))))/d^4-315*b^2*x*\operatorname{polylog}(5,-\exp(2*I*(c+d*x^(1/2))))/d^6+2*b^2*x^(7/2)*\tan(c+d*x^(1/2))/d-2*I*b^2*x^(7/2)/d+315/2*b^2*\operatorname{polylog}(7,-\exp(2*I*(c+d*x^(1/2))))/d^8+1/4*a^2*x^4-8*I*a*b*x^(7/2)*\arctan(\exp(I*(c+d*x^(1/2))))/d-28*I*a*b*x^3*\operatorname{polylog}(2,I*\exp(I*(c+d*x^(1/2))))/d^2-840*I*a*b*x^2*\operatorname{polylog}(4,-I*\exp(I*(c+d*x^(1/2))))/d^4+28*I*a*b*x^3*\operatorname{polylog}(2,-I*\exp(I*(c+d*x^(1/2))))/d^2+840*I*a*b*x^2*\operatorname{polylog}(4,I*\exp(I*(c+d*x^(1/2))))/d^4+10080*I*a*b*x*\operatorname{polylog}(6,-I*\exp(I*(c+d*x^(1/2))))/d^6-10080*I*a*b*x*\operatorname{polylog}(6,I*\exp(I*(c+d*x^(1/2))))/d^6+210*I*b^2*x^(3/2)*\operatorname{polylog}(4,-\exp(2*I*(c+d*x^(1/2))))/d^5+20160*I*a*b*\operatorname{polylog}(8,I*\exp(I*(c+d*x^(1/2))))/d^8-168*a*b*x^(5/2)*\operatorname{polylog}(3,-I*\exp(I*(c+d*x^(1/2))))/d^3+168*a*b*x^(5/2)*\operatorname{polylog}(3,I*\exp(I*(c+d*x^(1/2))))/d^3+3360*a*b*x^(3/2)*\operatorname{polylog}(5,-I*\exp(I*(c+d*x^(1/2))))/d^5-3360*a*b*x^(3/2)*\operatorname{polylog}(5,I*\exp(I*(c+d*x^(1/2))))/d^5-20160*a*b*\operatorname{polylog}(7,-I*\exp(I*(c+d*x^(1/2))))*x^(1/2)/d^7+20160*a*b*\operatorname{polylog}(7,I*\exp(I*(c+d*x^(1/2))))*x^(1/2)/d^7-42*I*b^2*x^(5/2)*\operatorname{polylog}(2,-\exp(2*I*(c+d*x^(1/2))))/d^3-20160*I*a*b*\operatorname{polylog}(8,-I*\exp(I*(c+d*x^(1/2))))/d^8-315*I*b^2*\operatorname{polylog}(6,-\exp(2*I*(c+d*x^(1/2))))*x^(1/2)/d^7$

**Rubi [A]** time = 0.86, antiderivative size = 749, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 10, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {4204, 4190, 4181, 2531, 6609, 2282, 6589, 4184, 3719, 2190}

$$\frac{28iabx^3 \operatorname{PolyLog} \left( 2, -ie^{i(c+d\sqrt{x})} \right)}{d^2} - \frac{28iabx^3 \operatorname{PolyLog} \left( 2, ie^{i(c+d\sqrt{x})} \right)}{d^2} - \frac{168abx^{5/2} \operatorname{PolyLog} \left( 3, -ie^{i(c+d\sqrt{x})} \right)}{d^3} + \frac{168abx^{5/2} \operatorname{PolyLog} \left( 3, ie^{i(c+d\sqrt{x})} \right)}{d^3}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^3*(a + b*\operatorname{Sec}[c + d*\operatorname{Sqrt}[x]])^2, x]$

[Out]  $((-2*I)*b^2*x^(7/2))/d + (a^2*x^4)/4 - ((8*I)*a*b*x^(7/2)*\operatorname{ArcTan}[E^(I*(c + d*\operatorname{Sqrt}[x]))])/d + (14*b^2*x^3*\operatorname{Log}[1 + E^((2*I)*(c + d*\operatorname{Sqrt}[x]))])/d^2 + ((8*I)*a*b*x^3*\operatorname{PolyLog}[2, (-I)*E^(I*(c + d*\operatorname{Sqrt}[x]))])/d^2 - ((28*I)*a*b*x^3*\operatorname{PolyLog}[2, I*E^(I*(c + d*\operatorname{Sqrt}[x]))])/d^2 - ((42*I)*b^2*x^(5/2)*\operatorname{PolyLog}[2, -E^((2*I)*(c + d*\operatorname{Sqrt}[x]))])/d^3 - (168*a*b*x^(5/2)*\operatorname{PolyLog}[3, (-I)*E^(I*(c + d*\operatorname{Sqrt}[x]))])/d^3 + (168*a*b*x^(5/2)*\operatorname{PolyLog}[3, I*E^(I*(c + d*\operatorname{Sqrt}[x]))])/d^3$



$$\begin{aligned} & /d^3 + (105*b^2*x^2*PolyLog[3, -E^((2*I)*(c + d*Sqrt[x]))])/d^4 - ((840*I)* \\ & a*b*x^2*PolyLog[4, (-I)*E^(I*(c + d*Sqrt[x]))])/d^4 + ((840*I)*a*b*x^2*Poly \\ & Log[4, I*E^(I*(c + d*Sqrt[x]))])/d^4 + ((210*I)*b^2*x^(3/2)*PolyLog[4, -E^(( \\ & (2*I)*(c + d*Sqrt[x]))])/d^5 + (3360*a*b*x^(3/2)*PolyLog[5, (-I)*E^(I*(c + \\ & d*Sqrt[x]))])/d^5 - (3360*a*b*x^(3/2)*PolyLog[5, I*E^(I*(c + d*Sqrt[x]))])/ \\ & d^5 - (315*b^2*x*PolyLog[5, -E^((2*I)*(c + d*Sqrt[x]))])/d^6 + ((10080*I)*a \\ & *b*x*PolyLog[6, (-I)*E^(I*(c + d*Sqrt[x]))])/d^6 - ((10080*I)*a*b*x*PolyLog \\ & [6, I*E^(I*(c + d*Sqrt[x]))])/d^6 - ((315*I)*b^2*Sqrt[x]*PolyLog[6, -E^((2* \\ & I)*(c + d*Sqrt[x]))])/d^7 - (20160*a*b*Sqrt[x]*PolyLog[7, (-I)*E^(I*(c + d* \\ & Sqrt[x]))])/d^7 + (20160*a*b*Sqrt[x]*PolyLog[7, I*E^(I*(c + d*Sqrt[x]))])/d \\ & ^7 + (315*b^2*PolyLog[7, -E^((2*I)*(c + d*Sqrt[x]))])/(2*d^8) - ((20160*I)* \\ & a*b*PolyLog[8, (-I)*E^(I*(c + d*Sqrt[x]))])/d^8 + ((20160*I)*a*b*PolyLog[8, \\ & I*E^(I*(c + d*Sqrt[x]))])/d^8 + (2*b^2*x^(7/2)*Tan[c + d*Sqrt[x]])/d \end{aligned}$$

### Rule 2190

```
Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] :> Simp
[(((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n])/a)]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n]/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 2531

```
Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_)))^(n_))]*(f_) + (g_)
*(x_)^(m_), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 3719

```
Int[(((c_) + (d_)*(x_))^(m_))*tan[(e_) + (f_)*(x_)], x_Symbol] :> Simp[[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[(((c + d*x)^m*E^(2*I*(e
+ f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_.)]*((c_.) + (d_.)*(x_.))^(m_.), x_Symbol]
:= Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))], x], x) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_.)]^2*((c_.) + (d_.)*(x_.))^(m_.), x_Symbol] := -Simp[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4190

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(n_.)*((c_.) + (d_.)*(x_.))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 4204

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_.))^(p_.)]/((d_.) + (e_.)*(x_.)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_.))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_.)))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps



[In] Integrate[x^3\*(a + b\*Sec[c + d\*Sqrt[x]])^2,x]

[Out]  $((-8*I)*b^2*d^7*x^{(7/2)} + a^2*d^8*x^4 - (32*I)*a*b*d^7*x^{(7/2)}*ArcTan[E^{(I*(c + d*Sqrt[x]))}] + 56*b^2*d^6*x^3*Log[1 + E^{((2*I)*(c + d*Sqrt[x]))}] + (112*I)*a*b*d^6*x^3*PolyLog[2, (-I)*E^{(I*(c + d*Sqrt[x]))}] - (112*I)*a*b*d^6*x^3*PolyLog[2, I*E^{(I*(c + d*Sqrt[x]))}] - (168*I)*b^2*d^5*x^{(5/2)}*PolyLog[2, -E^{((2*I)*(c + d*Sqrt[x]))}] - 672*a*b*d^5*x^{(5/2)}*PolyLog[3, (-I)*E^{(I*(c + d*Sqrt[x]))}] + 672*a*b*d^5*x^{(5/2)}*PolyLog[3, I*E^{(I*(c + d*Sqrt[x]))}] + 420*b^2*d^4*x^2*PolyLog[3, -E^{((2*I)*(c + d*Sqrt[x]))}] - (3360*I)*a*b*d^4*x^2*PolyLog[4, (-I)*E^{(I*(c + d*Sqrt[x]))}] + (3360*I)*a*b*d^4*x^2*PolyLog[4, I*E^{(I*(c + d*Sqrt[x]))}] + (840*I)*b^2*d^3*x^{(3/2)}*PolyLog[4, -E^{((2*I)*(c + d*Sqrt[x]))}] + 13440*a*b*d^3*x^{(3/2)}*PolyLog[5, (-I)*E^{(I*(c + d*Sqrt[x]))}] - 13440*a*b*d^3*x^{(3/2)}*PolyLog[5, I*E^{(I*(c + d*Sqrt[x]))}] - 1260*b^2*d^2*x*PolyLog[5, -E^{((2*I)*(c + d*Sqrt[x]))}] + (40320*I)*a*b*d^2*x*PolyLog[6, (-I)*E^{(I*(c + d*Sqrt[x]))}] - (40320*I)*a*b*d^2*x*PolyLog[6, I*E^{(I*(c + d*Sqrt[x]))}] - (1260*I)*b^2*d*Sqrt[x]*PolyLog[6, -E^{((2*I)*(c + d*Sqrt[x]))}] - 80640*a*b*d*Sqrt[x]*PolyLog[7, (-I)*E^{(I*(c + d*Sqrt[x]))}] + 80640*a*b*d*Sqrt[x]*PolyLog[7, I*E^{(I*(c + d*Sqrt[x]))}] + 630*b^2*PolyLog[7, -E^{((2*I)*(c + d*Sqrt[x]))}] - (80640*I)*a*b*PolyLog[8, (-I)*E^{(I*(c + d*Sqrt[x]))}] + (80640*I)*a*b*PolyLog[8, I*E^{(I*(c + d*Sqrt[x]))}] + 8*b^2*d^7*x^{(7/2)}*Tan[c + d*Sqrt[x]]/(4*d^8)$

**fricas** [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(b^2x^3 \sec(d\sqrt{x} + c)^2 + 2abx^3 \sec(d\sqrt{x} + c) + a^2x^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*sec(c+d\*x^(1/2)))^2,x, algorithm="fricas")

[Out] integral(b^2\*x^3\*sec(d\*sqrt(x) + c)^2 + 2\*a\*b\*x^3\*sec(d\*sqrt(x) + c) + a^2\*x^3, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sec(d\sqrt{x} + c) + a)^2 x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*sec(c+d\*x^(1/2)))^2,x, algorithm="giac")

[Out] integrate((b\*sec(d\*sqrt(x) + c) + a)^2\*x^3, x)

**maple** [F] time = 1.38, size = 0, normalized size = 0.00

$$\int x^3 (a + b \sec(c + d\sqrt{x}))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(x^3*(a+b*\sec(c+d*x^{(1/2)}))^2,x)$

[Out]  $\text{int}(x^3*(a+b*\sec(c+d*x^{(1/2)}))^2,x)$

**maxima** [B] time = 1.62, size = 6373, normalized size = 8.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(x^3*(a+b*\sec(c+d*x^{(1/2)}))^2,x, \text{algorithm}="maxima")$

[Out]  $\frac{1}{4}*((d*\sqrt{x} + c)^8*a^2 - 8*(d*\sqrt{x} + c)^7*a^2*c + 28*(d*\sqrt{x} + c)^6*a^2*c^2 - 56*(d*\sqrt{x} + c)^5*a^2*c^3 + 70*(d*\sqrt{x} + c)^4*a^2*c^4 - 56*(d*\sqrt{x} + c)^3*a^2*c^5 + 28*(d*\sqrt{x} + c)^2*a^2*c^6 - 8*(d*\sqrt{x} + c)*a^2*c^7 - 16*a*b*c^7*\log(\sec(d*\sqrt{x} + c) + \tan(d*\sqrt{x} + c)) - 8*(60*b^2*c^7 + (60*(d*\sqrt{x} + c)^7*a*b - 420*(d*\sqrt{x} + c)^6*a*b*c + 1260*(d*\sqrt{x} + c)^5*a*b*c^2 - 2100*(d*\sqrt{x} + c)^4*a*b*c^3 + 2100*(d*\sqrt{x} + c)^3*a*b*c^4 - 1260*(d*\sqrt{x} + c)^2*a*b*c^5 + 420*(d*\sqrt{x} + c)*a*b*c^6 + 60*((d*\sqrt{x} + c)^7*a*b - 7*(d*\sqrt{x} + c)^6*a*b*c + 21*(d*\sqrt{x} + c)^5*a*b*c^2 - 35*(d*\sqrt{x} + c)^4*a*b*c^3 + 35*(d*\sqrt{x} + c)^3*a*b*c^4 - 21*(d*\sqrt{x} + c)^2*a*b*c^5 + 7*(d*\sqrt{x} + c)*a*b*c^6)*\cos(2*d*\sqrt{x} + 2*c) - (-60*I*(d*\sqrt{x} + c)^7*a*b + 420*I*(d*\sqrt{x} + c)^6*a*b*c - 1260*I*(d*\sqrt{x} + c)^5*a*b*c^2 + 2100*I*(d*\sqrt{x} + c)^4*a*b*c^3 - 2100*I*(d*\sqrt{x} + c)^3*a*b*c^4 + 1260*I*(d*\sqrt{x} + c)^2*a*b*c^5 - 420*I*(d*\sqrt{x} + c)*a*b*c^6)*\sin(2*d*\sqrt{x} + 2*c))*\arctan2(\cos(d*\sqrt{x} + c), \sin(d*\sqrt{x} + c) + 1) + (60*(d*\sqrt{x} + c)^7*a*b - 420*(d*\sqrt{x} + c)^6*a*b*c + 1260*(d*\sqrt{x} + c)^5*a*b*c^2 - 2100*(d*\sqrt{x} + c)^4*a*b*c^3 + 2100*(d*\sqrt{x} + c)^3*a*b*c^4 - 1260*(d*\sqrt{x} + c)^2*a*b*c^5 + 420*(d*\sqrt{x} + c)*a*b*c^6 + 60*((d*\sqrt{x} + c)^7*a*b - 7*(d*\sqrt{x} + c)^6*a*b*c + 21*(d*\sqrt{x} + c)^5*a*b*c^2 - 35*(d*\sqrt{x} + c)^4*a*b*c^3 + 35*(d*\sqrt{x} + c)^3*a*b*c^4 - 21*(d*\sqrt{x} + c)^2*a*b*c^5 + 7*(d*\sqrt{x} + c)*a*b*c^6)*\cos(2*d*\sqrt{x} + 2*c) - (-60*I*(d*\sqrt{x} + c)^7*a*b + 420*I*(d*\sqrt{x} + c)^6*a*b*c - 1260*I*(d*\sqrt{x} + c)^5*a*b*c^2 + 2100*I*(d*\sqrt{x} + c)^4*a*b*c^3 - 2100*I*(d*\sqrt{x} + c)^3*a*b*c^4 + 1260*I*(d*\sqrt{x} + c)^2*a*b*c^5 - 420*I*(d*\sqrt{x} + c)*a*b*c^6)*\sin(2*d*\sqrt{x} + 2*c))*\arctan2(\cos(d*\sqrt{x} + c), -\sin(d*\sqrt{x} + c) + 1) - (1120*(d*\sqrt{x} + c)^6*b^2 - 4032*(d*\sqrt{x} + c)^5*b^2*c + 6300*(d*\sqrt{x} + c)^4*b^2*c^2 - 5600*(d*\sqrt{x} + c)^3*b^2*c^3 + 3150*(d*\sqrt{x} + c)^2*b^2*c^4 - 1260*(d*\sqrt{x} + c)*b^2*c^5 + 210*b^2*c^6 + 14*(80*(d*\sqrt{x} + c)^6*b^2 - 288*(d*\sqrt{x} + c)^5*b^2*c + 450*(d*\sqrt{x} + c)^4*b^2*c^2 - 400*(d*\sqrt{x} + c)^3*b^2*c^3 + 225*(d*\sqrt{x} + c)^2*b^2*c^4 - 90*(d*\sqrt{x} + c)*b^2*c^5 + 15*b^2*c^6)*\cos(2*d*\sqrt{x} + 2*c) + (1120*I*(d*\sqrt{x} + c)^6*b^2 - 4032*I*(d*\sqrt{x} + c)^5*b^2*c + 6300*I*(d*\sqrt{x} + c)^4*b^2*c^2 - 5600*I*(d*\sqrt{x} + c)^3*b^2*c$

$$\begin{aligned}
& c^3 + 3150*I*(d*\sqrt{x} + c)^2*b^2*c^4 - 1260*I*(d*\sqrt{x} + c)*b^2*c^5 + 2 \\
& 10*I*b^2*c^6)*\sin(2*d*\sqrt{x} + 2*c))*\arctan2(\sin(2*d*\sqrt{x} + 2*c), \cos(2 \\
& *d*\sqrt{x} + 2*c) + 1) + 60*((d*\sqrt{x} + c)^7*b^2 - 7*(d*\sqrt{x} + c)^6*b^ \\
& 2*c + 21*(d*\sqrt{x} + c)^5*b^2*c^2 - 35*(d*\sqrt{x} + c)^4*b^2*c^3 + 35*(d*s \\
& qrt(x) + c)^3*b^2*c^4 - 21*(d*\sqrt{x} + c)^2*b^2*c^5 + 7*(d*\sqrt{x} + c)*b^ \\
& 2*c^6)*\cos(2*d*\sqrt{x} + 2*c) + (3360*(d*\sqrt{x} + c)^5*b^2 - 10080*(d*\sqrt{ \\
& x) + c)^4*b^2*c + 12600*(d*\sqrt{x} + c)^3*b^2*c^2 - 8400*(d*\sqrt{x} + c)^2 \\
& *b^2*c^3 + 3150*(d*\sqrt{x} + c)*b^2*c^4 - 630*b^2*c^5 + 210*(16*(d*\sqrt{x} \\
& + c)^5*b^2 - 48*(d*\sqrt{x} + c)^4*b^2*c + 60*(d*\sqrt{x} + c)^3*b^2*c^2 - 40 \\
& *(d*\sqrt{x} + c)^2*b^2*c^3 + 15*(d*\sqrt{x} + c)*b^2*c^4 - 3*b^2*c^5)*\cos(2* \\
& d*\sqrt{x} + 2*c) - (-3360*I*(d*\sqrt{x} + c)^5*b^2 + 10080*I*(d*\sqrt{x} + c) \\
& ^4*b^2*c - 12600*I*(d*\sqrt{x} + c)^3*b^2*c^2 + 8400*I*(d*\sqrt{x} + c)^2*b^2 \\
& *c^3 - 3150*I*(d*\sqrt{x} + c)*b^2*c^4 + 630*I*b^2*c^5)*\sin(2*d*\sqrt{x} + 2* \\
& c))*\operatorname{dilog}(-e^{(2*I*d*\sqrt{x} + 2*I*c)}) + (420*(d*\sqrt{x} + c)^6*a*b - 2520*( \\
& d*\sqrt{x} + c)^5*a*b*c + 6300*(d*\sqrt{x} + c)^4*a*b*c^2 - 8400*(d*\sqrt{x} + \\
& c)^3*a*b*c^3 + 6300*(d*\sqrt{x} + c)^2*a*b*c^4 - 2520*(d*\sqrt{x} + c)*a*b*c \\
& ^5 + 420*a*b*c^6 + 420*((d*\sqrt{x} + c)^6*a*b - 6*(d*\sqrt{x} + c)^5*a*b*c + \\
& 15*(d*\sqrt{x} + c)^4*a*b*c^2 - 20*(d*\sqrt{x} + c)^3*a*b*c^3 + 15*(d*\sqrt{x} \\
& ) + c)^2*a*b*c^4 - 6*(d*\sqrt{x} + c)*a*b*c^5 + a*b*c^6)*\cos(2*d*\sqrt{x} + 2 \\
& *c) - (-420*I*(d*\sqrt{x} + c)^6*a*b + 2520*I*(d*\sqrt{x} + c)^5*a*b*c - 6300 \\
& *I*(d*\sqrt{x} + c)^4*a*b*c^2 + 8400*I*(d*\sqrt{x} + c)^3*a*b*c^3 - 6300*I*(d \\
& *\sqrt{x} + c)^2*a*b*c^4 + 2520*I*(d*\sqrt{x} + c)*a*b*c^5 - 420*I*a*b*c^6)*\sin \\
& (2*d*\sqrt{x} + 2*c))*\operatorname{dilog}(I*e^{(I*d*\sqrt{x} + I*c)}) - (420*(d*\sqrt{x} + c) \\
& )^6*a*b - 2520*(d*\sqrt{x} + c)^5*a*b*c + 6300*(d*\sqrt{x} + c)^4*a*b*c^2 - 8 \\
& 400*(d*\sqrt{x} + c)^3*a*b*c^3 + 6300*(d*\sqrt{x} + c)^2*a*b*c^4 - 2520*(d*\sqrt{ \\
& x) + c)*a*b*c^5 + 420*a*b*c^6 + 420*((d*\sqrt{x} + c)^6*a*b - 6*(d*\sqrt{x} \\
& ) + c)^5*a*b*c + 15*(d*\sqrt{x} + c)^4*a*b*c^2 - 20*(d*\sqrt{x} + c)^3*a*b*c^ \\
& 3 + 15*(d*\sqrt{x} + c)^2*a*b*c^4 - 6*(d*\sqrt{x} + c)*a*b*c^5 + a*b*c^6)*\cos \\
& (2*d*\sqrt{x} + 2*c) + (420*I*(d*\sqrt{x} + c)^6*a*b - 2520*I*(d*\sqrt{x} + c) \\
& ^5*a*b*c + 6300*I*(d*\sqrt{x} + c)^4*a*b*c^2 - 8400*I*(d*\sqrt{x} + c)^3*a*b* \\
& c^3 + 6300*I*(d*\sqrt{x} + c)^2*a*b*c^4 - 2520*I*(d*\sqrt{x} + c)*a*b*c^5 + 4 \\
& 20*I*a*b*c^6)*\sin(2*d*\sqrt{x} + 2*c))*\operatorname{dilog}(-I*e^{(I*d*\sqrt{x} + I*c)}) - (-5 \\
& 60*I*(d*\sqrt{x} + c)^6*b^2 + 2016*I*(d*\sqrt{x} + c)^5*b^2*c - 3150*I*(d*\sqrt{ \\
& x) + c)^4*b^2*c^2 + 2800*I*(d*\sqrt{x} + c)^3*b^2*c^3 - 1575*I*(d*\sqrt{x} \\
& + c)^2*b^2*c^4 + 630*I*(d*\sqrt{x} + c)*b^2*c^5 - 105*I*b^2*c^6 + (-560*I*(d \\
& *\sqrt{x} + c)^6*b^2 + 2016*I*(d*\sqrt{x} + c)^5*b^2*c - 3150*I*(d*\sqrt{x} + \\
& c)^4*b^2*c^2 + 2800*I*(d*\sqrt{x} + c)^3*b^2*c^3 - 1575*I*(d*\sqrt{x} + c)^2* \\
& b^2*c^4 + 630*I*(d*\sqrt{x} + c)*b^2*c^5 - 105*I*b^2*c^6)*\cos(2*d*\sqrt{x} + \\
& 2*c) + 7*(80*(d*\sqrt{x} + c)^6*b^2 - 288*(d*\sqrt{x} + c)^5*b^2*c + 450*(d*s \\
& qrt(x) + c)^4*b^2*c^2 - 400*(d*\sqrt{x} + c)^3*b^2*c^3 + 225*(d*\sqrt{x} + c) \\
& ^2*b^2*c^4 - 90*(d*\sqrt{x} + c)*b^2*c^5 + 15*b^2*c^6)*\sin(2*d*\sqrt{x} + 2*c) \\
& ))*\log(\cos(2*d*\sqrt{x} + 2*c)^2 + \sin(2*d*\sqrt{x} + 2*c)^2 + 2*\cos(2*d*\sqrt{ \\
& x) + 2*c) + 1) - (-30*I*(d*\sqrt{x} + c)^7*a*b + 210*I*(d*\sqrt{x} + c)^6*a* \\
& b*c - 630*I*(d*\sqrt{x} + c)^5*a*b*c^2 + 1050*I*(d*\sqrt{x} + c)^4*a*b*c^3 - \\
& 1050*I*(d*\sqrt{x} + c)^3*a*b*c^4 + 630*I*(d*\sqrt{x} + c)^2*a*b*c^5 - 210*I*
\end{aligned}$$

$$\begin{aligned}
& (d\sqrt{x} + c)abc^6 + (-30I(d\sqrt{x} + c)^7ab + 210I(d\sqrt{x} + c)^6abc - 630I(d\sqrt{x} + c)^5abc^2 + 1050I(d\sqrt{x} + c)^4abc^3 - 1050I(d\sqrt{x} + c)^3abc^4 + 630I(d\sqrt{x} + c)^2abc^5 - 210I(d\sqrt{x} + c)abc^6) \cos(2d\sqrt{x} + 2c) + 30((d\sqrt{x} + c)^7ab - 7(d\sqrt{x} + c)^6abc + 21(d\sqrt{x} + c)^5abc^2 - 35(d\sqrt{x} + c)^4abc^3 + 35(d\sqrt{x} + c)^3abc^4 - 21(d\sqrt{x} + c)^2abc^5 + 7(d\sqrt{x} + c)abc^6) \sin(2d\sqrt{x} + 2c) \log(\cos(d\sqrt{x} + c)^2 + \sin(d\sqrt{x} + c)^2 + 2\sin(d\sqrt{x} + c) + 1) - (30I(d\sqrt{x} + c)^7ab - 210I(d\sqrt{x} + c)^6abc + 630I(d\sqrt{x} + c)^5abc^2 - 1050I(d\sqrt{x} + c)^4abc^3 + 1050I(d\sqrt{x} + c)^3abc^4 - 630I(d\sqrt{x} + c)^2abc^5 + 210I(d\sqrt{x} + c)abc^6 + (30I(d\sqrt{x} + c)^7ab - 210I(d\sqrt{x} + c)^6abc + 630I(d\sqrt{x} + c)^5abc^2 - 1050I(d\sqrt{x} + c)^4abc^3 + 1050I(d\sqrt{x} + c)^3abc^4 - 630I(d\sqrt{x} + c)^2abc^5 + 210I(d\sqrt{x} + c)abc^6) \cos(2d\sqrt{x} + 2c) - 30((d\sqrt{x} + c)^7ab - 7(d\sqrt{x} + c)^6abc + 21(d\sqrt{x} + c)^5abc^2 - 35(d\sqrt{x} + c)^4abc^3 + 35(d\sqrt{x} + c)^3abc^4 - 21(d\sqrt{x} + c)^2abc^5 + 7(d\sqrt{x} + c)abc^6) \sin(2d\sqrt{x} + 2c) \log(\cos(d\sqrt{x} + c)^2 + \sin(d\sqrt{x} + c)^2 - 2\sin(d\sqrt{x} + c) + 1) - 302400(ab\cos(2d\sqrt{x} + 2c) + Iab\sin(2d\sqrt{x} + 2c) + ab) \operatorname{polylog}(8, Ie^{(I d\sqrt{x} + I c)}) + 302400(ab\cos(2d\sqrt{x} + 2c) + Iab\sin(2d\sqrt{x} + 2c) + ab) \operatorname{polylog}(8, -Ie^{(I d\sqrt{x} + I c)}) - (-12600Ib^2\cos(2d\sqrt{x} + 2c) + 12600b^2\sin(2d\sqrt{x} + 2c) - 12600Ib^2) \operatorname{polylog}(7, -e^{(2I d\sqrt{x} + 2I c)}) - (-302400I(d\sqrt{x} + c)ab + 302400Iaabc + (-302400I(d\sqrt{x} + c)ab + 302400Iaabc) \cos(2d\sqrt{x} + 2c) + 302400((d\sqrt{x} + c)ab - abc) \sin(2d\sqrt{x} + 2c)) \operatorname{polylog}(7, Ie^{(I d\sqrt{x} + I c)}) - (302400I(d\sqrt{x} + c)ab - 302400Iaabc + (302400I(d\sqrt{x} + c)ab - 302400Iaabc) \cos(2d\sqrt{x} + 2c) - 302400((d\sqrt{x} + c)ab - abc) \sin(2d\sqrt{x} + 2c)) \operatorname{polylog}(7, -Ie^{(I d\sqrt{x} + I c)}) + (25200(d\sqrt{x} + c)b^2 - 15120b^2c + 5040(5(d\sqrt{x} + c)b^2 - 3b^2c) \cos(2d\sqrt{x} + 2c) - (-25200I(d\sqrt{x} + c)b^2 + 15120Ib^2c) \sin(2d\sqrt{x} + 2c)) \operatorname{polylog}(6, -e^{(2I d\sqrt{x} + 2I c)}) + (151200(d\sqrt{x} + c)^2ab - 302400(d\sqrt{x} + c)abc + 151200aabc^2 + 151200((d\sqrt{x} + c)^2ab - 2(d\sqrt{x} + c)abc + abc^2) \cos(2d\sqrt{x} + 2c) - (-151200I(d\sqrt{x} + c)^2ab + 302400I(d\sqrt{x} + c)abc - 151200Iaabc^2) \sin(2d\sqrt{x} + 2c)) \operatorname{polylog}(6, Ie^{(I d\sqrt{x} + I c)}) - (151200(d\sqrt{x} + c)^2ab - 302400(d\sqrt{x} + c)abc + 151200aabc^2 + 151200((d\sqrt{x} + c)^2ab - 2(d\sqrt{x} + c)abc + abc^2) \cos(2d\sqrt{x} + 2c) + (151200I(d\sqrt{x} + c)^2ab - 302400I(d\sqrt{x} + c)abc + 151200Iaabc^2) \sin(2d\sqrt{x} + 2c)) \operatorname{polylog}(6, -Ie^{(I d\sqrt{x} + I c)}) - (25200I(d\sqrt{x} + c)^2b^2 - 30240I(d\sqrt{x} + c)b^2c + 9450Ib^2c^2 + (25200I(d\sqrt{x} + c)^2b^2 - 30240I(d\sqrt{x} + c)b^2c + 9450Ib^2c^2) \cos(2d\sqrt{x} + 2c) - 630(40(d\sqrt{x} + c)^2b^2 - 48(d\sqrt{x} + c)b^2c + 15b^2c^2) \sin(2d\sqrt{x} + 2c)) \operatorname{polylog}(5, -e^{(2I d\sqrt{x} + 2I c)}) - (50400*
\end{aligned}$$

$$\begin{aligned}
& I*(d*\sqrt{x} + c)^3*a*b - 151200*I*(d*\sqrt{x} + c)^2*a*b*c + 151200*I*(d*\sqrt{x} + c)*a*b*c^2 - 50400*I*a*b*c^3 + (50400*I*(d*\sqrt{x} + c)^3*a*b - 151200*I*(d*\sqrt{x} + c)^2*a*b*c + 151200*I*(d*\sqrt{x} + c)*a*b*c^2 - 50400*I*a*b*c^3)*\cos(2*d*\sqrt{x} + 2*c) - 50400*((d*\sqrt{x} + c)^3*a*b - 3*(d*\sqrt{x} + c)^2*a*b*c + 3*(d*\sqrt{x} + c)*a*b*c^2 - a*b*c^3)*\sin(2*d*\sqrt{x} + 2*c))*\text{polylog}(5, I*e^{(I*d*\sqrt{x} + I*c)}) - (-50400*I*(d*\sqrt{x} + c)^3*a*b + 151200*I*(d*\sqrt{x} + c)^2*a*b*c - 151200*I*(d*\sqrt{x} + c)*a*b*c^2 + 50400*I*a*b*c^3 + (-50400*I*(d*\sqrt{x} + c)^3*a*b + 151200*I*(d*\sqrt{x} + c)^2*a*b*c - 151200*I*(d*\sqrt{x} + c)*a*b*c^2 + 50400*I*a*b*c^3)*\cos(2*d*\sqrt{x} + 2*c) + 50400*((d*\sqrt{x} + c)^3*a*b - 3*(d*\sqrt{x} + c)^2*a*b*c + 3*(d*\sqrt{x} + c)*a*b*c^2 - a*b*c^3)*\sin(2*d*\sqrt{x} + 2*c))*\text{polylog}(5, -I*e^{(I*d*\sqrt{x} + I*c)}) - (16800*(d*\sqrt{x} + c)^3*b^2 - 30240*(d*\sqrt{x} + c)^2*b^2*c + 18900*(d*\sqrt{x} + c)*b^2*c^2 - 4200*b^2*c^3 + 420*(40*(d*\sqrt{x} + c)^3*b^2 - 72*(d*\sqrt{x} + c)^2*b^2*c + 45*(d*\sqrt{x} + c)*b^2*c^2 - 10*b^2*c^3)*\cos(2*d*\sqrt{x} + 2*c) + (16800*I*(d*\sqrt{x} + c)^3*b^2 - 30240*I*(d*\sqrt{x} + c)^2*b^2*c + 18900*I*(d*\sqrt{x} + c)*b^2*c^2 - 4200*I*b^2*c^3)*\sin(2*d*\sqrt{x} + 2*c))*\text{polylog}(4, -e^{(2*I*d*\sqrt{x} + 2*I*c)}) - (12600*(d*\sqrt{x} + c)^4*a*b - 50400*(d*\sqrt{x} + c)^3*a*b*c + 75600*(d*\sqrt{x} + c)^2*a*b*c^2 - 50400*(d*\sqrt{x} + c)*a*b*c^3 + 12600*a*b*c^4 + 12600*((d*\sqrt{x} + c)^4*a*b - 4*(d*\sqrt{x} + c)^3*a*b*c + 6*(d*\sqrt{x} + c)^2*a*b*c^2 - 4*(d*\sqrt{x} + c)*a*b*c^3 + a*b*c^4)*\cos(2*d*\sqrt{x} + 2*c) + (12600*I*(d*\sqrt{x} + c)^4*a*b - 50400*I*(d*\sqrt{x} + c)^3*a*b*c + 75600*I*(d*\sqrt{x} + c)^2*a*b*c^2 - 50400*I*(d*\sqrt{x} + c)*a*b*c^3 + 12600*I*a*b*c^4)*\sin(2*d*\sqrt{x} + 2*c))*\text{polylog}(4, I*e^{(I*d*\sqrt{x} + I*c)}) + (12600*(d*\sqrt{x} + c)^4*a*b - 50400*(d*\sqrt{x} + c)^3*a*b*c + 75600*(d*\sqrt{x} + c)^2*a*b*c^2 - 50400*(d*\sqrt{x} + c)*a*b*c^3 + 12600*a*b*c^4 + 12600*((d*\sqrt{x} + c)^4*a*b - 4*(d*\sqrt{x} + c)^3*a*b*c + 6*(d*\sqrt{x} + c)^2*a*b*c^2 - 4*(d*\sqrt{x} + c)*a*b*c^3 + a*b*c^4)*\cos(2*d*\sqrt{x} + 2*c) - (-12600*I*(d*\sqrt{x} + c)^4*a*b + 50400*I*(d*\sqrt{x} + c)^3*a*b*c - 75600*I*(d*\sqrt{x} + c)^2*a*b*c^2 + 50400*I*(d*\sqrt{x} + c)*a*b*c^3 - 12600*I*a*b*c^4)*\sin(2*d*\sqrt{x} + 2*c))*\text{polylog}(4, -I*e^{(I*d*\sqrt{x} + I*c)}) - (-8400*I*(d*\sqrt{x} + c)^4*b^2 + 20160*I*(d*\sqrt{x} + c)^3*b^2*c - 18900*I*(d*\sqrt{x} + c)^2*b^2*c^2 + 8400*I*(d*\sqrt{x} + c)*b^2*c^3 - 1575*I*b^2*c^4 + (-8400*I*(d*\sqrt{x} + c)^4*b^2 + 20160*I*(d*\sqrt{x} + c)^3*b^2*c - 18900*I*(d*\sqrt{x} + c)^2*b^2*c^2 + 8400*I*(d*\sqrt{x} + c)*b^2*c^3 - 1575*I*b^2*c^4)*\cos(2*d*\sqrt{x} + 2*c) + 105*(80*(d*\sqrt{x} + c)^4*b^2 - 192*(d*\sqrt{x} + c)^3*b^2*c + 180*(d*\sqrt{x} + c)^2*b^2*c^2 - 80*(d*\sqrt{x} + c)*b^2*c^3 + 15*b^2*c^4)*\sin(2*d*\sqrt{x} + 2*c))*\text{polylog}(3, -e^{(2*I*d*\sqrt{x} + 2*I*c)}) - (-2520*I*(d*\sqrt{x} + c)^5*a*b + 12600*I*(d*\sqrt{x} + c)^4*a*b*c - 25200*I*(d*\sqrt{x} + c)^3*a*b*c^2 + 25200*I*(d*\sqrt{x} + c)^2*a*b*c^3 - 12600*I*(d*\sqrt{x} + c)*a*b*c^4 + 2520*I*a*b*c^5 + (-2520*I*(d*\sqrt{x} + c)^5*a*b + 12600*I*(d*\sqrt{x} + c)^4*a*b*c - 25200*I*(d*\sqrt{x} + c)^3*a*b*c^2 + 25200*I*(d*\sqrt{x} + c)^2*a*b*c^3 - 12600*I*(d*\sqrt{x} + c)*a*b*c^4 + 2520*I*a*b*c^5)*\cos(2*d*\sqrt{x} + 2*c) + 2520*((d*\sqrt{x} + c)^5*a*b - 5*(d*\sqrt{x} + c)^4*a*b*c + 10*(d*\sqrt{x} + c)^3*a*b*c^2 - 10*(d*\sqrt{x} + c)^2*a*b*c^3 + 5*(d*\sqrt{x} + c)*a*b*c^4 - a*b*
\end{aligned}$$



$c^5 \sin(2d\sqrt{x} + 2c) \operatorname{polylog}(3, Ie^{(I d \sqrt{x} + I c)}) - (2520 I (d\sqrt{x} + c)^5 a b - 12600 I (d\sqrt{x} + c)^4 a b c + 25200 I (d\sqrt{x} + c)^3 a b c^2 - 25200 I (d\sqrt{x} + c)^2 a b c^3 + 12600 I (d\sqrt{x} + c) a b c^4 - 2520 I a b c^5 + (2520 I (d\sqrt{x} + c)^5 a b - 12600 I (d\sqrt{x} + c)^4 a b c + 25200 I (d\sqrt{x} + c)^3 a b c^2 - 25200 I (d\sqrt{x} + c)^2 a b c^3 + 12600 I (d\sqrt{x} + c) a b c^4 - 2520 I a b c^5) \cos(2d\sqrt{x} + 2c) - 2520 ((d\sqrt{x} + c)^5 a b - 5 (d\sqrt{x} + c)^4 a b c + 10 (d\sqrt{x} + c)^3 a b c^2 - 10 (d\sqrt{x} + c)^2 a b c^3 + 5 (d\sqrt{x} + c) a b c^4 - a b c^5) \sin(2d\sqrt{x} + 2c) \operatorname{polylog}(3, -Ie^{(I d \sqrt{x} + I c)}) - (-60 I (d\sqrt{x} + c)^7 b^2 + 420 I (d\sqrt{x} + c)^6 b^2 c - 1260 I (d\sqrt{x} + c)^5 b^2 c^2 + 2100 I (d\sqrt{x} + c)^4 b^2 c^3 - 2100 I (d\sqrt{x} + c)^3 b^2 c^4 + 1260 I (d\sqrt{x} + c)^2 b^2 c^5 - 420 I (d\sqrt{x} + c) b^2 c^6) \sin(2d\sqrt{x} + 2c) / (-30 I \cos(2d\sqrt{x} + 2c) + 30 \sin(2d\sqrt{x} + 2c) - 30 I) / d^8$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \left( a + \frac{b}{\cos(c + d\sqrt{x})} \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(a + b/cos(c + d*x^(1/2)))^2,x)`

[Out] `int(x^3*(a + b/cos(c + d*x^(1/2)))^2, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 (a + b \sec(c + d\sqrt{x}))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(a+b*sec(c+d*x**(1/2)))**2,x)`

[Out] `Integral(x**3*(a + b*sec(c + d*sqrt(x)))**2, x)`

### 3.37 $\int x^2 \left( a + b \sec \left( c + d\sqrt{x} \right) \right)^2 dx$

**Optimal.** Leaf size=551

$$\frac{a^2 x^3}{3} + \frac{480iab \operatorname{Li}_6 \left( -ie^{i(c+d\sqrt{x})} \right)}{d^6} - \frac{480iab \operatorname{Li}_6 \left( ie^{i(c+d\sqrt{x})} \right)}{d^6} + \frac{480ab\sqrt{x} \operatorname{Li}_5 \left( -ie^{i(c+d\sqrt{x})} \right)}{d^5} - \frac{480ab\sqrt{x} \operatorname{Li}_5 \left( ie^{i(c+d\sqrt{x})} \right)}{d^5} + \dots$$

[Out]  $-480I*a*b*\operatorname{polylog}(6, I*\exp(I*(c+d*x^{(1/2)})))/d^6 + 1/3*a^2*x^3 - 2*I*b^2*x^{(5/2)}/d + 10*b^2*x^2*\ln(1+\exp(2*I*(c+d*x^{(1/2)})))/d^2 + 20*I*a*b*x^2*\operatorname{polylog}(2, -I*\exp(I*(c+d*x^{(1/2)})))/d^2 + 480*I*a*b*\operatorname{polylog}(6, -I*\exp(I*(c+d*x^{(1/2)})))/d^6 - 8*I*a*b*x^{(5/2)}*\arctan(\exp(I*(c+d*x^{(1/2)})))/d - 80*a*b*x^{(3/2)}*\operatorname{polylog}(3, -I*\exp(I*(c+d*x^{(1/2)})))/d^3 + 80*a*b*x^{(3/2)}*\operatorname{polylog}(3, I*\exp(I*(c+d*x^{(1/2)})))/d^3 + 30*b^2*x*\operatorname{polylog}(3, -\exp(2*I*(c+d*x^{(1/2)})))/d^4 - 20*I*a*b*x^2*\operatorname{polylog}(2, I*\exp(I*(c+d*x^{(1/2)})))/d^2 - 20*I*b^2*x^{(3/2)}*\operatorname{polylog}(2, -\exp(2*I*(c+d*x^{(1/2)})))/d^3 - 15*b^2*\operatorname{polylog}(5, -\exp(2*I*(c+d*x^{(1/2)})))/d^6 + 240*I*a*b*x*\operatorname{polylog}(4, I*\exp(I*(c+d*x^{(1/2)})))/d^4 - 240*I*a*b*x*\operatorname{polylog}(4, -I*\exp(I*(c+d*x^{(1/2)})))/d^4 + 30*I*b^2*\operatorname{polylog}(4, -\exp(2*I*(c+d*x^{(1/2)})))*x^{(1/2)}/d^5 + 480*a*b*\operatorname{polylog}(5, -I*\exp(I*(c+d*x^{(1/2)})))*x^{(1/2)}/d^5 - 480*a*b*\operatorname{polylog}(5, I*\exp(I*(c+d*x^{(1/2)})))*x^{(1/2)}/d^5 + 2*b^2*x^{(5/2)}*\tan(c+d*x^{(1/2)})/d$

**Rubi [A]** time = 0.63, antiderivative size = 551, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 10, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {4204, 4190, 4181, 2531, 6609, 2282, 6589, 4184, 3719, 2190}

$$\frac{20iabx^2 \operatorname{PolyLog} \left( 2, -ie^{i(c+d\sqrt{x})} \right)}{d^2} - \frac{20iabx^2 \operatorname{PolyLog} \left( 2, ie^{i(c+d\sqrt{x})} \right)}{d^2} - \frac{80abx^{3/2} \operatorname{PolyLog} \left( 3, -ie^{i(c+d\sqrt{x})} \right)}{d^3} + \dots$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^2*(a + b*\operatorname{Sec}[c + d*\operatorname{Sqrt}[x]])^2, x]$

[Out]  $((-2*I)*b^2*x^{(5/2)})/d + (a^2*x^3)/3 - ((8*I)*a*b*x^{(5/2)}*\operatorname{ArcTan}[E^{(I*(c + d*\operatorname{Sqrt}[x])})])/d + (10*b^2*x^2*\operatorname{Log}[1 + E^{((2*I)*(c + d*\operatorname{Sqrt}[x])})}])/d^2 + ((20*I)*a*b*x^2*\operatorname{PolyLog}[2, (-I)*E^{(I*(c + d*\operatorname{Sqrt}[x])})}])/d^2 - ((20*I)*a*b*x^2*\operatorname{PolyLog}[2, I*E^{(I*(c + d*\operatorname{Sqrt}[x])})}])/d^2 - ((20*I)*b^2*x^{(3/2)}*\operatorname{PolyLog}[2, -E^{((2*I)*(c + d*\operatorname{Sqrt}[x])})}])/d^3 - (80*a*b*x^{(3/2)}*\operatorname{PolyLog}[3, (-I)*E^{(I*(c + d*\operatorname{Sqrt}[x])})}])/d^3 + (80*a*b*x^{(3/2)}*\operatorname{PolyLog}[3, I*E^{(I*(c + d*\operatorname{Sqrt}[x])})}])/d^3 + (30*b^2*x*\operatorname{PolyLog}[3, -E^{((2*I)*(c + d*\operatorname{Sqrt}[x])})}])/d^4 - ((240*I)*a*b*x*\operatorname{PolyLog}[4, (-I)*E^{(I*(c + d*\operatorname{Sqrt}[x])})}])/d^4 + ((240*I)*a*b*x*\operatorname{PolyLog}[4, I*E^{(I*(c + d*\operatorname{Sqrt}[x])})}])/d^4 + ((30*I)*b^2*\operatorname{Sqrt}[x]*\operatorname{PolyLog}[4, -E^{((2*I)*(c + d*\operatorname{Sqrt}[x])})}])/d^5 + (480*a*b*\operatorname{Sqrt}[x]*\operatorname{PolyLog}[5, (-I)*E^{(I*(c + d*\operatorname{Sqrt}[x])})}])/d^5 - (480*a*b*\operatorname{Sqrt}[x]*\operatorname{PolyLog}[5, I*E^{(I*(c + d*\operatorname{Sqrt}[x])})}])/d^5 - (15*b^2*x^{(5/2)}*\tan(c+d*\operatorname{Sqrt}[x]))/d$

$2*\text{PolyLog}[5, -E^{((2*I)*(c + d*\text{Sqrt}[x]))}]/d^6 + ((480*I)*a*b*\text{PolyLog}[6, (-I)*E^{(I*(c + d*\text{Sqrt}[x]))}]/d^6 - ((480*I)*a*b*\text{PolyLog}[6, I*E^{(I*(c + d*\text{Sqrt}[x]))}]/d^6 + (2*b^2*x^{(5/2)}*\text{Tan}[c + d*\text{Sqrt}[x]])/d$

### Rule 2190

$\text{Int}[(((F\_)^{(g\_)*(e\_)} + (f\_)*(x\_)))^{(n\_)*((c\_)} + (d\_)*(x\_))^{(m\_)}]/((a\_)} + (b\_)*((F\_)^{(g\_)*(e\_)} + (f\_)*(x\_)))^{(n\_)}], x\_Symbol] \text{:> Simp} [((c + d*x)^m*\text{Log}[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*\text{Log}[F]), x] - \text{Dist}[(d*m)/(b*f*g*n*\text{Log}[F]), \text{Int}[(c + d*x)^{(m - 1)}*\text{Log}[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x] \&\& \text{IGtQ}[m, 0]$

### Rule 2282

$\text{Int}[u_, x\_Symbol] \text{:> With}\{v = \text{FunctionOfExponential}[u, x]\}, \text{Dist}[v/D[v, x], \text{Subst}[\text{Int}[\text{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x] /; \text{FunctionOfExponentialQ}[u, x] \&\& !\text{MatchQ}[u, (w\_)*((a\_)*(v\_)^{(n\_))^{(m\_)} /; \text{FreeQ}\{a, m, n\}, x] \&\& \text{IntegerQ}[m*n] \&\& !\text{MatchQ}[u, E^{((c\_)*(a\_)} + (b\_)*x))* (F\_)[v_] /; \text{FreeQ}\{a, b, c\}, x] \&\& \text{InverseFunctionQ}[F[x]]]$

### Rule 2531

$\text{Int}[\text{Log}[1 + (e\_)*((F\_)^{(c\_)*(a\_)} + (b\_)*(x\_)))^{(n\_)}]*(f\_)} + (g\_)*(x\_)]^{(m\_)}], x\_Symbol] \text{:> -Simp} [((f + g*x)^m*\text{PolyLog}[2, -(e*(F^(c*(a + b*x)))^n)])/(b*c*n*\text{Log}[F]), x] + \text{Dist}[(g*m)/(b*c*n*\text{Log}[F]), \text{Int}[(f + g*x)^{(m - 1)}*\text{PolyLog}[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; \text{FreeQ}\{F, a, b, c, e, f, g, n\}, x] \&\& \text{GtQ}[m, 0]$

### Rule 3719

$\text{Int}[((c\_)} + (d\_)*(x\_))^{(m\_)}*\text{tan}[(e\_)} + (f\_)*(x\_)], x\_Symbol] \text{:> Simp} [((I*(c + d*x)^{(m + 1)})/(d*(m + 1)), x] - \text{Dist}[2*I, \text{Int}[(c + d*x)^m*E^{(2*I*(e + f*x))}]/(1 + E^{(2*I*(e + f*x))}), x], x] /; \text{FreeQ}\{c, d, e, f\}, x] \&\& \text{IGtQ}[m, 0]$

### Rule 4181

$\text{Int}[\text{csc}[(e\_)} + \text{Pi}*(k\_)} + (f\_)*(x_)]*((c\_)} + (d\_)*(x_))^{(m\_)}], x\_Symbol] \text{:> Simp} [(-2*(c + d*x)^m*\text{ArcTanh}[E^{(I*k*Pi)}*E^{(I*(e + f*x))}])/f, x] + (-\text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m - 1)}*\text{Log}[1 - E^{(I*k*Pi)}*E^{(I*(e + f*x))}], x], x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m - 1)}*\text{Log}[1 + E^{(I*k*Pi)}*E^{(I*(e + f*x))}], x], x]) /; \text{FreeQ}\{c, d, e, f\}, x] \&\& \text{IntegerQ}[2*k] \&\& \text{IGtQ}[m, 0]$

### Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := -Simp[
((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

#### Rule 4190

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

#### Rule 4204

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

#### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

#### Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

#### Rubi steps

$$\begin{aligned}
\int x^2 (a + b \sec(c + d\sqrt{x}))^2 dx &= 2 \text{Subst} \left( \int x^5 (a + b \sec(c + dx))^2 dx, x, \sqrt{x} \right) \\
&= 2 \text{Subst} \left( \int (a^2 x^5 + 2abx^5 \sec(c + dx) + b^2 x^5 \sec^2(c + dx)) dx, x, \sqrt{x} \right) \\
&= \frac{a^2 x^3}{3} + (4ab) \text{Subst} \left( \int x^5 \sec(c + dx) dx, x, \sqrt{x} \right) + (2b^2) \text{Subst} \left( \int x^5 \sec^2(c + dx) dx, x, \sqrt{x} \right) \\
&= \frac{a^2 x^3}{3} - \frac{8iabx^{5/2} \tan^{-1} \left( e^{i(c+d\sqrt{x})} \right)}{d} + \frac{2b^2 x^{5/2} \tan(c + d\sqrt{x})}{d} - \frac{(20ab) \text{Subst} \left( \int x^5 \sec(c + dx) dx, x, \sqrt{x} \right)}{d} \\
&= -\frac{2ib^2 x^{5/2}}{d} + \frac{a^2 x^3}{3} - \frac{8iabx^{5/2} \tan^{-1} \left( e^{i(c+d\sqrt{x})} \right)}{d} + \frac{20iabx^2 \text{Li}_2 \left( -ie^{i(c+d\sqrt{x})} \right)}{d^2} \\
&= -\frac{2ib^2 x^{5/2}}{d} + \frac{a^2 x^3}{3} - \frac{8iabx^{5/2} \tan^{-1} \left( e^{i(c+d\sqrt{x})} \right)}{d} + \frac{10b^2 x^2 \log \left( 1 + e^{2i(c+d\sqrt{x})} \right)}{d^2} \\
&= -\frac{2ib^2 x^{5/2}}{d} + \frac{a^2 x^3}{3} - \frac{8iabx^{5/2} \tan^{-1} \left( e^{i(c+d\sqrt{x})} \right)}{d} + \frac{10b^2 x^2 \log \left( 1 + e^{2i(c+d\sqrt{x})} \right)}{d^2} \\
&= -\frac{2ib^2 x^{5/2}}{d} + \frac{a^2 x^3}{3} - \frac{8iabx^{5/2} \tan^{-1} \left( e^{i(c+d\sqrt{x})} \right)}{d} + \frac{10b^2 x^2 \log \left( 1 + e^{2i(c+d\sqrt{x})} \right)}{d^2} \\
&= -\frac{2ib^2 x^{5/2}}{d} + \frac{a^2 x^3}{3} - \frac{8iabx^{5/2} \tan^{-1} \left( e^{i(c+d\sqrt{x})} \right)}{d} + \frac{10b^2 x^2 \log \left( 1 + e^{2i(c+d\sqrt{x})} \right)}{d^2} \\
&= -\frac{2ib^2 x^{5/2}}{d} + \frac{a^2 x^3}{3} - \frac{8iabx^{5/2} \tan^{-1} \left( e^{i(c+d\sqrt{x})} \right)}{d} + \frac{10b^2 x^2 \log \left( 1 + e^{2i(c+d\sqrt{x})} \right)}{d^2} \\
&= -\frac{2ib^2 x^{5/2}}{d} + \frac{a^2 x^3}{3} - \frac{8iabx^{5/2} \tan^{-1} \left( e^{i(c+d\sqrt{x})} \right)}{d} + \frac{10b^2 x^2 \log \left( 1 + e^{2i(c+d\sqrt{x})} \right)}{d^2} \\
&= -\frac{2ib^2 x^{5/2}}{d} + \frac{a^2 x^3}{3} - \frac{8iabx^{5/2} \tan^{-1} \left( e^{i(c+d\sqrt{x})} \right)}{d} + \frac{10b^2 x^2 \log \left( 1 + e^{2i(c+d\sqrt{x})} \right)}{d^2}
\end{aligned}$$

**Mathematica [A]** time = 1.26, size = 543, normalized size = 0.99

$$\frac{a^2 d^6 x^3 - 24iab d^5 x^{5/2} \tan^{-1} \left( e^{i(c+d\sqrt{x})} \right) + 60iab d^4 x^2 \text{Li}_2 \left( -ie^{i(c+d\sqrt{x})} \right) - 60iab d^4 x^2 \text{Li}_2 \left( ie^{i(c+d\sqrt{x})} \right) - 240abd^3 x^{3/2} \log \left( 1 + e^{2i(c+d\sqrt{x})} \right)}{d^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*(a + b\*Sec[c + d\*Sqrt[x]])^2,x]

[Out] ((-6\*I)\*b^2\*d^5\*x^(5/2) + a^2\*d^6\*x^3 - (24\*I)\*a\*b\*d^5\*x^(5/2)\*ArcTan[E^(I\*(c + d\*Sqrt[x]))] + 30\*b^2\*d^4\*x^2\*Log[1 + E^((2\*I)\*(c + d\*Sqrt[x]))] + (60

$$\begin{aligned}
 & *I)*a*b*d^4*x^2*PolyLog[2, (-I)*E^(I*(c + d*Sqrt[x]))] - (60*I)*a*b*d^4*x^2 \\
 & *PolyLog[2, I*E^(I*(c + d*Sqrt[x]))] - (60*I)*b^2*d^3*x^(3/2)*PolyLog[2, -E \\
 & ^((2*I)*(c + d*Sqrt[x]))] - 240*a*b*d^3*x^(3/2)*PolyLog[3, (-I)*E^(I*(c + d \\
 & *Sqrt[x]))] + 240*a*b*d^3*x^(3/2)*PolyLog[3, I*E^(I*(c + d*Sqrt[x]))] + 90* \\
 & b^2*d^2*x*PolyLog[3, -E^((2*I)*(c + d*Sqrt[x]))] - (720*I)*a*b*d^2*x*PolyLo \\
 & g[4, (-I)*E^(I*(c + d*Sqrt[x]))] + (720*I)*a*b*d^2*x*PolyLog[4, I*E^(I*(c + \\
 & d*Sqrt[x]))] + (90*I)*b^2*d*Sqrt[x]*PolyLog[4, -E^((2*I)*(c + d*Sqrt[x]))] \\
 & + 1440*a*b*d*Sqrt[x]*PolyLog[5, (-I)*E^(I*(c + d*Sqrt[x]))] - 1440*a*b*d*S \\
 & qrt[x]*PolyLog[5, I*E^(I*(c + d*Sqrt[x]))] - 45*b^2*PolyLog[5, -E^((2*I)*(c \\
 & + d*Sqrt[x]))] + (1440*I)*a*b*PolyLog[6, (-I)*E^(I*(c + d*Sqrt[x]))] - (14 \\
 & 40*I)*a*b*PolyLog[6, I*E^(I*(c + d*Sqrt[x]))] + 6*b^2*d^5*x^(5/2)*Tan[c + d \\
 & *Sqrt[x]]/(3*d^6)
 \end{aligned}$$

**fricas** [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(b^2x^2 \sec(d\sqrt{x} + c)^2 + 2abx^2 \sec(d\sqrt{x} + c) + a^2x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*sec(c+d\*x^(1/2)))^2,x, algorithm="fricas")

[Out] integral(b^2\*x^2\*sec(d\*sqrt(x) + c)^2 + 2\*a\*b\*x^2\*sec(d\*sqrt(x) + c) + a^2\*x^2, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sec(d\sqrt{x} + c) + a)^2 x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*sec(c+d\*x^(1/2)))^2,x, algorithm="giac")

[Out] integrate((b\*sec(d\*sqrt(x) + c) + a)^2\*x^2, x)

**maple** [F] time = 1.38, size = 0, normalized size = 0.00

$$\int x^2 (a + b \sec(c + d\sqrt{x}))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a+b\*sec(c+d\*x^(1/2)))^2,x)

[Out] int(x^2\*(a+b\*sec(c+d\*x^(1/2)))^2,x)

**maxima** [B] time = 1.43, size = 3898, normalized size = 7.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*sec(c+d\*x^(1/2)))^2,x, algorithm="maxima")

[Out]  $\frac{1}{3}((d\sqrt{x} + c)^6 a^2 - 6(d\sqrt{x} + c)^5 a^2 c + 15(d\sqrt{x} + c)^4 a^2 c^2 - 20(d\sqrt{x} + c)^3 a^2 c^3 + 15(d\sqrt{x} + c)^2 a^2 c^4 - 6(d\sqrt{x} + c) a^2 c^5 - 12 a b c^5 \log(\sec(d\sqrt{x} + c) + \tan(d\sqrt{x} + c)) - 6(12 b^2 c^5 + (12(d\sqrt{x} + c)^5 a b - 60(d\sqrt{x} + c)^4 a b c + 120(d\sqrt{x} + c)^3 a b c^2 - 120(d\sqrt{x} + c)^2 a b c^3 + 60(d\sqrt{x} + c) a b c^4 + 12((d\sqrt{x} + c)^5 a b - 5(d\sqrt{x} + c)^4 a b c + 10(d\sqrt{x} + c)^3 a b c^2 - 10(d\sqrt{x} + c)^2 a b c^3 + 5(d\sqrt{x} + c) a b c^4) \cos(2d\sqrt{x} + 2c) - (-12 I (d\sqrt{x} + c)^5 a b + 60 I (d\sqrt{x} + c)^4 a b c - 120 I (d\sqrt{x} + c)^3 a b c^2 + 120 I (d\sqrt{x} + c)^2 a b c^3 - 60 I (d\sqrt{x} + c) a b c^4) \sin(2d\sqrt{x} + 2c)) \arctan2(\cos(d\sqrt{x} + c), \sin(d\sqrt{x} + c) + 1) + (12(d\sqrt{x} + c)^5 a b - 60(d\sqrt{x} + c)^4 a b c + 120(d\sqrt{x} + c)^3 a b c^2 - 120(d\sqrt{x} + c)^2 a b c^3 + 60(d\sqrt{x} + c) a b c^4 + 12((d\sqrt{x} + c)^5 a b - 5(d\sqrt{x} + c)^4 a b c + 10(d\sqrt{x} + c)^3 a b c^2 - 10(d\sqrt{x} + c)^2 a b c^3 + 5(d\sqrt{x} + c) a b c^4) \cos(2d\sqrt{x} + 2c) - (-12 I (d\sqrt{x} + c)^5 a b + 60 I (d\sqrt{x} + c)^4 a b c - 120 I (d\sqrt{x} + c)^3 a b c^2 + 120 I (d\sqrt{x} + c)^2 a b c^3 - 60 I (d\sqrt{x} + c) a b c^4) \sin(2d\sqrt{x} + 2c)) \arctan2(\cos(d\sqrt{x} + c), -\sin(d\sqrt{x} + c) + 1) - (60(d\sqrt{x} + c)^4 b^2 - 160(d\sqrt{x} + c)^3 b^2 c + 180(d\sqrt{x} + c)^2 b^2 c^2 - 120(d\sqrt{x} + c) b^2 c^3 + 30 b^2 c^4 + 10(6(d\sqrt{x} + c)^4 b^2 - 16(d\sqrt{x} + c)^3 b^2 c + 18(d\sqrt{x} + c)^2 b^2 c^2 - 12(d\sqrt{x} + c) b^2 c^3 + 3 b^2 c^4) \cos(2d\sqrt{x} + 2c) + (60 I (d\sqrt{x} + c)^4 b^2 - 160 I (d\sqrt{x} + c)^3 b^2 c + 180 I (d\sqrt{x} + c)^2 b^2 c^2 - 120 I (d\sqrt{x} + c) b^2 c^3 + 30 I b^2 c^4) \sin(2d\sqrt{x} + 2c)) \arctan2(\sin(2d\sqrt{x} + 2c), \cos(2d\sqrt{x} + 2c) + 1) + 12((d\sqrt{x} + c)^5 b^2 - 5(d\sqrt{x} + c)^4 b^2 c + 10(d\sqrt{x} + c)^3 b^2 c^2 - 10(d\sqrt{x} + c)^2 b^2 c^3 + 5(d\sqrt{x} + c) b^2 c^4) \cos(2d\sqrt{x} + 2c) + (120(d\sqrt{x} + c)^3 b^2 - 240(d\sqrt{x} + c)^2 b^2 c + 180(d\sqrt{x} + c) b^2 c^2 - 60 b^2 c^3 + 60(2(d\sqrt{x} + c)^3 b^2 - 4(d\sqrt{x} + c)^2 b^2 c + 3(d\sqrt{x} + c) b^2 c^2 - b^2 c^3) \cos(2d\sqrt{x} + 2c) - (-120 I (d\sqrt{x} + c)^3 b^2 + 240 I (d\sqrt{x} + c)^2 b^2 c - 180 I (d\sqrt{x} + c) b^2 c^2 + 60 I b^2 c^3) \sin(2d\sqrt{x} + 2c)) \operatorname{dilog}(-e^{(2 I d\sqrt{x} + 2 I c)}) + (60(d\sqrt{x} + c)^4 a b - 240(d\sqrt{x} + c)^3 a b c + 360(d\sqrt{x} + c)^2 a b c^2 - 240(d\sqrt{x} + c) a b c^3 + 60 a b c^4 + 60((d\sqrt{x} + c)^4 a b - 4(d\sqrt{x} + c)^3 a b c + 6(d\sqrt{x} + c)^2 a b c^2 - 4(d\sqrt{x} + c) a b c^3 + a b c^4) \cos(2d\sqrt{x} + 2c) - (-60 I (d\sqrt{x} + c)^4 a b + 240 I (d\sqrt{x} + c)^3 a b c - 360 I (d\sqrt{x} + c)^2 a b c^2 + 240 I (d\sqrt{x} + c) a b c^3 - 60 I a b c^4) \sin(2d\sqrt{x} + 2c)) \operatorname{dilog}(I e^{(I d\sqrt{x} + I c)}) - (60(d\sqrt{x} + c)^4 a b - 240(d\sqrt{x} + c)^3 a b c + 360(d\sqrt{x} + c)^2 a b c^2 - 240(d\sqrt{x} + c) a b c^3 + 60 a b c^4 + 60((d\sqrt{x} + c)^4 a b - 4(d\sqrt{x} + c)^3 a b c + 6(d\sqrt{x} + c)^2 a b c^2 - 4(d\sqrt{x} + c) a b c^3 + a b c^4) \cos(2d\sqrt{x} + 2c) - (-60 I (d\sqrt{x} + c)^4 a b + 240 I (d\sqrt{x} + c)^3 a b c - 360 I (d\sqrt{x} + c)^2 a b c^2 + 240 I (d\sqrt{x} + c) a b c^3 - 60 I a b c^4) \sin(2d\sqrt{x} + 2c)) \operatorname{dilog}(I e^{(I d\sqrt{x} + I c)}) - (60(d\sqrt{x} + c)^4 a b - 240(d\sqrt{x} + c)^3 a b c + 360(d\sqrt{x} + c)^2 a b c^2 - 240(d\sqrt{x} + c) a b c^3 + 60 a b c^4 + 60((d\sqrt{x} + c)^4 a b - 4(d\sqrt{x} + c)^3 a b c + 6(d\sqrt{x} + c)^2 a b c^2 - 4(d\sqrt{x} + c) a b c^3 + a b c^4) \cos(2d\sqrt{x} + 2c) - (-60 I (d\sqrt{x} + c)^4 a b + 240 I (d\sqrt{x} + c)^3 a b c - 360 I (d\sqrt{x} + c)^2 a b c^2 + 240 I (d\sqrt{x} + c) a b c^3 - 60 I a b c^4) \sin(2d\sqrt{x} + 2c)) \operatorname{dilog}(I e^{(I d\sqrt{x} + I c)})$

$$\begin{aligned}
& + c)^4 a^2 b - 4(d\sqrt{x} + c)^3 a^2 b^2 c + 6(d\sqrt{x} + c)^2 a^2 b^2 c^2 - 4(d\sqrt{x} + c) a^2 b^2 c^3 + a^2 b^2 c^4) \cos(2d\sqrt{x} + 2c) + (60I(d\sqrt{x} + c)^4 a^2 b - 240I(d\sqrt{x} + c)^3 a^2 b^2 c + 360I(d\sqrt{x} + c)^2 a^2 b^2 c^2 - 240I(d\sqrt{x} + c) a^2 b^2 c^3 + 60I a^2 b^2 c^4) \sin(2d\sqrt{x} + 2c)) * \\
& \operatorname{dilog}(-Ie^{(I d\sqrt{x} + I c)}) - (-30I(d\sqrt{x} + c)^4 b^2 + 80I(d\sqrt{x} + c)^3 b^2 c - 90I(d\sqrt{x} + c)^2 b^2 c^2 + 60I(d\sqrt{x} + c) b^2 c^3 - 15I b^2 c^4 + (-30I(d\sqrt{x} + c)^4 b^2 + 80I(d\sqrt{x} + c)^3 b^2 c - 90I(d\sqrt{x} + c)^2 b^2 c^2 + 60I(d\sqrt{x} + c) b^2 c^3 - 15I b^2 c^4) \cos(2d\sqrt{x} + 2c) + 5(6(d\sqrt{x} + c)^4 b^2 - 16(d\sqrt{x} + c)^3 b^2 c + 18(d\sqrt{x} + c)^2 b^2 c^2 - 12(d\sqrt{x} + c) b^2 c^3 + 3b^2 c^4) \sin(2d\sqrt{x} + 2c)) * \log(\cos(2d\sqrt{x} + 2c)^2 + \sin(2d\sqrt{x} + 2c)^2 + 2\cos(2d\sqrt{x} + 2c) + 1) - (-6I(d\sqrt{x} + c)^5 a^2 b + 30I(d\sqrt{x} + c)^4 a^2 b^2 c - 60I(d\sqrt{x} + c)^3 a^2 b^2 c^2 + 60I(d\sqrt{x} + c)^2 a^2 b^2 c^3 - 30I(d\sqrt{x} + c) a^2 b^2 c^4 + (-6I(d\sqrt{x} + c)^5 a^2 b + 30I(d\sqrt{x} + c)^4 a^2 b^2 c - 60I(d\sqrt{x} + c)^3 a^2 b^2 c^2 + 60I(d\sqrt{x} + c)^2 a^2 b^2 c^3 - 30I(d\sqrt{x} + c) a^2 b^2 c^4) \cos(2d\sqrt{x} + 2c) + 6((d\sqrt{x} + c)^5 a^2 b - 5(d\sqrt{x} + c)^4 a^2 b^2 c + 10(d\sqrt{x} + c)^3 a^2 b^2 c^2 - 10(d\sqrt{x} + c)^2 a^2 b^2 c^3 + 5(d\sqrt{x} + c) a^2 b^2 c^4) \sin(2d\sqrt{x} + 2c)) * \log(\cos(d\sqrt{x} + c)^2 + \sin(d\sqrt{x} + c)^2 + 2\sin(d\sqrt{x} + c) + 1) - (6I(d\sqrt{x} + c)^5 a^2 b - 30I(d\sqrt{x} + c)^4 a^2 b^2 c + 60I(d\sqrt{x} + c)^3 a^2 b^2 c^2 - 60I(d\sqrt{x} + c)^2 a^2 b^2 c^3 + 30I(d\sqrt{x} + c) a^2 b^2 c^4 + (6I(d\sqrt{x} + c)^5 a^2 b - 30I(d\sqrt{x} + c)^4 a^2 b^2 c + 60I(d\sqrt{x} + c)^3 a^2 b^2 c^2 - 60I(d\sqrt{x} + c)^2 a^2 b^2 c^3 + 30I(d\sqrt{x} + c) a^2 b^2 c^4) \cos(2d\sqrt{x} + 2c) - 6((d\sqrt{x} + c)^5 a^2 b - 5(d\sqrt{x} + c)^4 a^2 b^2 c + 10(d\sqrt{x} + c)^3 a^2 b^2 c^2 - 10(d\sqrt{x} + c)^2 a^2 b^2 c^3 + 5(d\sqrt{x} + c) a^2 b^2 c^4) \sin(2d\sqrt{x} + 2c)) * \log(\cos(d\sqrt{x} + c)^2 + \sin(d\sqrt{x} + c)^2 - 2\sin(d\sqrt{x} + c) + 1) + 1440(a^2 b \cos(2d\sqrt{x} + 2c) + I a^2 b \sin(2d\sqrt{x} + 2c) + a^2 b) \operatorname{polylog}(6, Ie^{(I d\sqrt{x} + I c)}) - 1440(a^2 b \cos(2d\sqrt{x} + 2c) + I a^2 b \sin(2d\sqrt{x} + 2c) + a^2 b) \operatorname{polylog}(6, -Ie^{(I d\sqrt{x} + I c)}) - (90I b^2 \cos(2d\sqrt{x} + 2c) - 90I b^2 \sin(2d\sqrt{x} + 2c) + 90I b^2) \operatorname{polylog}(5, -e^{(2I d\sqrt{x} + 2I c)}) - (1440I(d\sqrt{x} + c) a^2 b - 1440I a^2 b^2 c + (1440I(d\sqrt{x} + c) a^2 b - 1440I a^2 b^2 c) \cos(2d\sqrt{x} + 2c) - 1440((d\sqrt{x} + c) a^2 b - a^2 b^2 c) \sin(2d\sqrt{x} + 2c)) \operatorname{polylog}(5, Ie^{(I d\sqrt{x} + I c)}) - (-1440I(d\sqrt{x} + c) a^2 b + 1440I a^2 b^2 c + (-1440I(d\sqrt{x} + c) a^2 b + 1440I a^2 b^2 c) \cos(2d\sqrt{x} + 2c) + 1440((d\sqrt{x} + c) a^2 b - a^2 b^2 c) \sin(2d\sqrt{x} + 2c)) \operatorname{polylog}(5, -Ie^{(I d\sqrt{x} + I c)}) - (180(d\sqrt{x} + c) b^2 - 120b^2 c + 60(3(d\sqrt{x} + c) b^2 - 2b^2 c) \cos(2d\sqrt{x} + 2c) + (180I(d\sqrt{x} + c) b^2 - 120I b^2 c) \sin(2d\sqrt{x} + 2c)) \operatorname{polylog}(4, -e^{(2I d\sqrt{x} + 2I c)}) - (720(d\sqrt{x} + c)^2 a^2 b - 1440(d\sqrt{x} + c) a^2 b^2 c + 720a^2 b^2 c^2 + 720((d\sqrt{x} + c)^2 a^2 b - 2(d\sqrt{x} + c) a^2 b^2 c + a^2 b^2 c^2) \cos(2d\sqrt{x} + 2c) + (720I(d\sqrt{x} + c)^2 a^2 b - 1440I(d\sqrt{x} + c) a^2 b^2 c + 720I a^2 b^2 c^2) \sin(2d\sqrt{x} + 2c)) \operatorname{polylog}(4, Ie^{(I d\sqrt{x} + I c)}) + (720(d\sqrt{x} + c)^2 a^2 b - 1440(d\sqrt{x} + c) a^2
\end{aligned}$$



$b*c + 720*a*b*c^2 + 720*((d*\sqrt{x} + c)^2*a*b - 2*(d*\sqrt{x} + c)*a*b*c + a*b*c^2)*\cos(2*d*\sqrt{x} + 2*c) - (-720*I*(d*\sqrt{x} + c)^2*a*b + 1440*I*(d*\sqrt{x} + c)*a*b*c - 720*I*a*b*c^2)*\sin(2*d*\sqrt{x} + 2*c))*\text{polylog}(4, -I*e^{(I*d*\sqrt{x} + I*c)}) - (-180*I*(d*\sqrt{x} + c)^2*b^2 + 240*I*(d*\sqrt{x} + c)*b^2*c - 90*I*b^2*c^2 + (-180*I*(d*\sqrt{x} + c)^2*b^2 + 240*I*(d*\sqrt{x} + c)*b^2*c - 90*I*b^2*c^2)*\cos(2*d*\sqrt{x} + 2*c) + 30*(6*(d*\sqrt{x} + c)^2*b^2 - 8*(d*\sqrt{x} + c)*b^2*c + 3*b^2*c^2)*\sin(2*d*\sqrt{x} + 2*c))*\text{polylog}(3, -e^{(2*I*d*\sqrt{x} + 2*I*c)}) - (-240*I*(d*\sqrt{x} + c)^3*a*b + 720*I*(d*\sqrt{x} + c)^2*a*b*c - 720*I*(d*\sqrt{x} + c)*a*b*c^2 + 240*I*a*b*c^3 + (-240*I*(d*\sqrt{x} + c)^3*a*b + 720*I*(d*\sqrt{x} + c)^2*a*b*c - 720*I*(d*\sqrt{x} + c)*a*b*c^2 + 240*I*a*b*c^3)*\cos(2*d*\sqrt{x} + 2*c) + 240*((d*\sqrt{x} + c)^3*a*b - 3*(d*\sqrt{x} + c)^2*a*b*c + 3*(d*\sqrt{x} + c)*a*b*c^2 - a*b*c^3)*\sin(2*d*\sqrt{x} + 2*c))*\text{polylog}(3, I*e^{(I*d*\sqrt{x} + I*c)}) - (240*I*(d*\sqrt{x} + c)^3*a*b - 720*I*(d*\sqrt{x} + c)^2*a*b*c + 720*I*(d*\sqrt{x} + c)*a*b*c^2 - 240*I*a*b*c^3 + (240*I*(d*\sqrt{x} + c)^3*a*b - 720*I*(d*\sqrt{x} + c)^2*a*b*c + 720*I*(d*\sqrt{x} + c)*a*b*c^2 - 240*I*a*b*c^3)*\cos(2*d*\sqrt{x} + 2*c) - 240*((d*\sqrt{x} + c)^3*a*b - 3*(d*\sqrt{x} + c)^2*a*b*c + 3*(d*\sqrt{x} + c)*a*b*c^2 - a*b*c^3)*\sin(2*d*\sqrt{x} + 2*c))*\text{polylog}(3, -I*e^{(I*d*\sqrt{x} + I*c)}) - (-12*I*(d*\sqrt{x} + c)^5*b^2 + 60*I*(d*\sqrt{x} + c)^4*b^2*c - 120*I*(d*\sqrt{x} + c)^3*b^2*c^2 + 120*I*(d*\sqrt{x} + c)^2*b^2*c^3 - 60*I*(d*\sqrt{x} + c)*b^2*c^4)*\sin(2*d*\sqrt{x} + 2*c))/(-6*I*\cos(2*d*\sqrt{x} + 2*c) + 6*\sin(2*d*\sqrt{x} + 2*c) - 6*I))/d^6$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \left( a + \frac{b}{\cos(c + d\sqrt{x})} \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(a + b/cos(c + d*x^(1/2)))^2,x)`

[Out] `int(x^2*(a + b/cos(c + d*x^(1/2)))^2, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 (a + b \sec(c + d\sqrt{x}))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(a+b*sec(c+d*x**(1/2)))**2,x)`

[Out] `Integral(x**2*(a + b*sec(c + d*sqrt(x)))**2, x)`

### 3.38 $\int x \left( a + b \sec \left( c + d\sqrt{x} \right) \right)^2 dx$

**Optimal.** Leaf size=355

$$\frac{a^2 x^2}{2} - \frac{24iab \operatorname{Li}_4 \left( -ie^{i(c+d\sqrt{x})} \right)}{d^4} + \frac{24iab \operatorname{Li}_4 \left( ie^{i(c+d\sqrt{x})} \right)}{d^4} - \frac{24ab\sqrt{x} \operatorname{Li}_3 \left( -ie^{i(c+d\sqrt{x})} \right)}{d^3} + \frac{24ab\sqrt{x} \operatorname{Li}_3 \left( ie^{i(c+d\sqrt{x})} \right)}{d^3} + \frac{12iabx}{d^2}$$

[Out]  $-2*I*b^2*x^{(3/2)}/d+1/2*a^2*x^2-8*I*a*b*x^{(3/2)}*\arctan(\exp(I*(c+d*x^{(1/2))}))$   
 $/d+6*b^2*x*\ln(1+\exp(2*I*(c+d*x^{(1/2))}))/d^2+12*I*a*b*x*\operatorname{polylog}(2,-I*\exp(I*(c+d*x^{(1/2))}))/d^2-12*I*a*b*x*\operatorname{polylog}(2,I*\exp(I*(c+d*x^{(1/2))}))/d^2+3*b^2*$   
 $\operatorname{polylog}(3,-\exp(2*I*(c+d*x^{(1/2))}))/d^4-24*I*a*b*\operatorname{polylog}(4,-I*\exp(I*(c+d*x^{(1/2))}))/d^4+24*I*a*b*$   
 $\operatorname{polylog}(4,I*\exp(I*(c+d*x^{(1/2))}))/d^4-6*I*b^2*\operatorname{polylog}(2,-\exp(2*I*(c+d*x^{(1/2))}))*x^{(1/2)}/d^3-24*a*b*\operatorname{polylog}(3,-I*\exp(I*(c+d*x^{(1/2))}))*x^{(1/2)}/d^3+24*a*b*$   
 $\operatorname{polylog}(3,I*\exp(I*(c+d*x^{(1/2))}))*x^{(1/2)}/d^3+2*b^2*x^{(3/2)}*\tan(c+d*x^{(1/2)})/d$

**Rubi [A]** time = 0.45, antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$ , Rules used = {4204, 4190, 4181, 2531, 6609, 2282, 6589, 4184, 3719, 2190}

$$\frac{12iabx \operatorname{PolyLog} \left( 2, -ie^{i(c+d\sqrt{x})} \right)}{d^2} - \frac{12iabx \operatorname{PolyLog} \left( 2, ie^{i(c+d\sqrt{x})} \right)}{d^2} - \frac{24ab\sqrt{x} \operatorname{PolyLog} \left( 3, -ie^{i(c+d\sqrt{x})} \right)}{d^3} + \frac{24ab\sqrt{x} \operatorname{PolyLog} \left( 3, ie^{i(c+d\sqrt{x})} \right)}{d^3} + \frac{12iabx}{d^2}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x*(a + b*\operatorname{Sec}[c + d*\operatorname{Sqrt}[x]])^2, x]$

[Out]  $((-2*I)*b^2*x^{(3/2)})/d + (a^2*x^2)/2 - ((8*I)*a*b*x^{(3/2)}*\operatorname{ArcTan}[E^{(I*(c + d*\operatorname{Sqrt}[x])})])/d + (6*b^2*x*\operatorname{Log}[1 + E^{((2*I)*(c + d*\operatorname{Sqrt}[x])})}])/d^2 + ((12*I)*a*b*x*\operatorname{PolyLog}[2, (-I)*E^{(I*(c + d*\operatorname{Sqrt}[x])})}])/d^2 - ((12*I)*a*b*x*\operatorname{PolyLog}[2, I*E^{(I*(c + d*\operatorname{Sqrt}[x])})}])/d^2 - ((6*I)*b^2*\operatorname{Sqrt}[x]*\operatorname{PolyLog}[2, -E^{((2*I)*(c + d*\operatorname{Sqrt}[x])})}])/d^3 - (24*a*b*\operatorname{Sqrt}[x]*\operatorname{PolyLog}[3, (-I)*E^{(I*(c + d*\operatorname{Sqrt}[x])})}])/d^3 + (24*a*b*\operatorname{Sqrt}[x]*\operatorname{PolyLog}[3, I*E^{(I*(c + d*\operatorname{Sqrt}[x])})}])/d^3 + (3*b^2*\operatorname{PolyLog}[3, -E^{((2*I)*(c + d*\operatorname{Sqrt}[x])})}])/d^4 - ((24*I)*a*b*\operatorname{PolyLog}[4, (-I)*E^{(I*(c + d*\operatorname{Sqrt}[x])})}])/d^4 + ((24*I)*a*b*\operatorname{PolyLog}[4, I*E^{(I*(c + d*\operatorname{Sqrt}[x])})}])/d^4 + (2*b^2*x^{(3/2)}*\operatorname{Tan}[c + d*\operatorname{Sqrt}[x]])/d$

**Rule 2190**

$\operatorname{Int}[(((F_)^\wedge((g_)*(e_) + (f_)*(x_)))^\wedge(n_))*((c_) + (d_)*(x_))^\wedge(m_)]/((a_) + (b_)*((F_)^\wedge((g_)*(e_) + (f_)*(x_)))^\wedge(n_)), x\_Symbol] \rightarrow \operatorname{Simp}[(c + d*x)^\wedge m * \operatorname{Log}[1 + (b*(F^\wedge(g*(e + f*x)))^\wedge n)/a]]/(b*f*g*n*\operatorname{Log}[F]), x] - \operatorname{Dist}[(d*m)/(b*f*g*n*\operatorname{Log}[F]), \operatorname{Int}[(c + d*x)^\wedge(m - 1)*\operatorname{Log}[1 + (b*(F^\wedge(g*(e + f*x)))^\wedge n)]]]$

))<sup>n</sup>)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2282

Int[u\_, x\_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_.)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_.)\*((a\_.) + (b\_.)\*x))\*(F\_) [v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

### Rule 2531

Int[Log[1 + (e\_.)\*((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(n\_.)]\*((f\_.) + (g\_.)\*(x\_)^(m\_.), x\_Symbol] := -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n)])/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m - 1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

### Rule 3719

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*tan[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := Simp[(I\*(c + d\*x)^(m + 1))/(d\*(m + 1)), x] - Dist[2\*I, Int[((c + d\*x)^m\*E^(2\*I\*(e + f\*x)))/(1 + E^(2\*I\*(e + f\*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]

### Rule 4181

Int[csc[(e\_.) + Pi\*(k\_.) + (f\_.)\*(x\_)]\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[(-2\*(c + d\*x)^m\*ArcTanh[E^(I\*k\*Pi)\*E^(I\*(e + f\*x))]/f, x] + (-Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Log[1 - E^(I\*k\*Pi)\*E^(I\*(e + f\*x))], x], x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Log[1 + E^(I\*k\*Pi)\*E^(I\*(e + f\*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2\*k] && IGtQ[m, 0]

### Rule 4184

Int[csc[(e\_.) + (f\_.)\*(x\_)]^2\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[((c + d\*x)^m\*Cot[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cot[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

### Rule 4190

Int[(csc[(e\_.) + (f\_.)\*(x\_)]\*(b\_.) + (a\_.))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Int[ExpandIntegrand[(c + d\*x)^m, (a + b\*Csc[e + f\*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]

Rule 4204

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol]
:= Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p, x], x, x^n], x]
/; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol]
:= Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x]
&& EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_)))^(p_.)], x_Symbol]
:= Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p])/(b*c*p*Log[F]), x]
- Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x]
/; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int x(a + b \sec(c + d\sqrt{x}))^2 dx &= 2 \operatorname{Subst}\left(\int x^3(a + b \sec(c + dx))^2 dx, x, \sqrt{x}\right) \\
&= 2 \operatorname{Subst}\left(\int (a^2x^3 + 2abx^3 \sec(c + dx) + b^2x^3 \sec^2(c + dx)) dx, x, \sqrt{x}\right) \\
&= \frac{a^2x^2}{2} + (4ab) \operatorname{Subst}\left(\int x^3 \sec(c + dx) dx, x, \sqrt{x}\right) + (2b^2) \operatorname{Subst}\left(\int x^3 \sec^2(c + dx) dx, x, \sqrt{x}\right) \\
&= \frac{a^2x^2}{2} - \frac{8iabx^{3/2} \tan^{-1}\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{2b^2x^{3/2} \tan(c + d\sqrt{x})}{d} - \frac{(12ab) \operatorname{Subst}\left(\int x^3 \sec(c + dx) dx, x, \sqrt{x}\right)}{d} \\
&= -\frac{2ib^2x^{3/2}}{d} + \frac{a^2x^2}{2} - \frac{8iabx^{3/2} \tan^{-1}\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{12iabx \operatorname{Li}_2\left(-ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{12ab \operatorname{Subst}\left(\int x^3 \sec(c + dx) dx, x, \sqrt{x}\right)}{d} \\
&= -\frac{2ib^2x^{3/2}}{d} + \frac{a^2x^2}{2} - \frac{8iabx^{3/2} \tan^{-1}\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{6b^2x \log\left(1 + e^{2i(c+d\sqrt{x})}\right)}{d^2} + \frac{12iabx \operatorname{Li}_2\left(-ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{12ab \operatorname{Subst}\left(\int x^3 \sec(c + dx) dx, x, \sqrt{x}\right)}{d} \\
&= -\frac{2ib^2x^{3/2}}{d} + \frac{a^2x^2}{2} - \frac{8iabx^{3/2} \tan^{-1}\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{6b^2x \log\left(1 + e^{2i(c+d\sqrt{x})}\right)}{d^2} + \frac{12iabx \operatorname{Li}_2\left(-ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{12ab \operatorname{Subst}\left(\int x^3 \sec(c + dx) dx, x, \sqrt{x}\right)}{d} \\
&= -\frac{2ib^2x^{3/2}}{d} + \frac{a^2x^2}{2} - \frac{8iabx^{3/2} \tan^{-1}\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{6b^2x \log\left(1 + e^{2i(c+d\sqrt{x})}\right)}{d^2} + \frac{12iabx \operatorname{Li}_2\left(-ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{12ab \operatorname{Subst}\left(\int x^3 \sec(c + dx) dx, x, \sqrt{x}\right)}{d} \\
&= -\frac{2ib^2x^{3/2}}{d} + \frac{a^2x^2}{2} - \frac{8iabx^{3/2} \tan^{-1}\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{6b^2x \log\left(1 + e^{2i(c+d\sqrt{x})}\right)}{d^2} + \frac{12iabx \operatorname{Li}_2\left(-ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{12ab \operatorname{Subst}\left(\int x^3 \sec(c + dx) dx, x, \sqrt{x}\right)}{d}
\end{aligned}$$

**Mathematica [A]** time = 0.71, size = 347, normalized size = 0.98

$$\frac{a^2d^4x^2 - 16iabd^3x^{3/2} \tan^{-1}\left(e^{i(c+d\sqrt{x})}\right) + 24iabd^2x \operatorname{Li}_2\left(-ie^{i(c+d\sqrt{x})}\right) - 24iabd^2x \operatorname{Li}_2\left(ie^{i(c+d\sqrt{x})}\right) - 48abd\sqrt{x} \operatorname{Li}_3\left(-ie^{i(c+d\sqrt{x})}\right) + 48abd\sqrt{x} \operatorname{Li}_3\left(ie^{i(c+d\sqrt{x})}\right)}{2d^4}$$

Antiderivative was successfully verified.

[In] Integrate[x\*(a + b\*Sec[c + d\*Sqrt[x]])^2,x]

[Out]  $((-4*I)*b^2*d^3*x^{(3/2)} + a^2*d^4*x^2 - (16*I)*a*b*d^3*x^{(3/2)}*ArcTan[E^{(I*(c + d*Sqrt[x]))}] + 12*b^2*d^2*x*Log[1 + E^{((2*I)*(c + d*Sqrt[x]))}] + (24*I)*a*b*d^2*x*PolyLog[2, (-I)*E^{(I*(c + d*Sqrt[x]))}] - (24*I)*a*b*d^2*x*PolyLog[2, I*E^{(I*(c + d*Sqrt[x]))}] - (12*I)*b^2*d*Sqrt[x]*PolyLog[2, -E^{((2*I)*(c + d*Sqrt[x]))}] - 48*a*b*d*Sqrt[x]*PolyLog[3, (-I)*E^{(I*(c + d*Sqrt[x]))}] + 48*a*b*d*Sqrt[x]*PolyLog[3, I*E^{(I*(c + d*Sqrt[x]))}] + 6*b^2*PolyLog[3, -E^{((2*I)*(c + d*Sqrt[x]))}] - (48*I)*a*b*PolyLog[4, (-I)*E^{(I*(c + d*Sqrt[x]))}] + (48*I)*a*b*PolyLog[4, I*E^{(I*(c + d*Sqrt[x]))}] + 4*b^2*d^3*x^{(3/2)}*Tan[c + d*Sqrt[x]])/(2*d^4)$

**fricas** [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(b^2x \sec(d\sqrt{x} + c)^2 + 2abx \sec(d\sqrt{x} + c) + a^2x, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*sec(c+d\*x^(1/2)))^2,x, algorithm="fricas")

[Out] integral(b^2\*x\*sec(d\*sqrt(x) + c)^2 + 2\*a\*b\*x\*sec(d\*sqrt(x) + c) + a^2\*x, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sec(d\sqrt{x} + c) + a)^2 x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*sec(c+d\*x^(1/2)))^2,x, algorithm="giac")

[Out] integrate((b\*sec(d\*sqrt(x) + c) + a)^2\*x, x)

**maple** [F] time = 1.38, size = 0, normalized size = 0.00

$$\int x (a + b \sec(c + d\sqrt{x}))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a+b\*sec(c+d\*x^(1/2)))^2,x)

[Out] int(x\*(a+b\*sec(c+d\*x^(1/2)))^2,x)

**maxima** [B] time = 0.92, size = 2006, normalized size = 5.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*sec(c+d\*x^(1/2)))^2,x, algorithm="maxima")

[Out] 1/2\*((d\*sqrt(x) + c)^4\*a^2 - 4\*(d\*sqrt(x) + c)^3\*a^2\*c + 6\*(d\*sqrt(x) + c)^2\*a^2\*c^2 - 4\*(d\*sqrt(x) + c)\*a^2\*c^3 - 8\*a\*b\*c^3\*log(sec(d\*sqrt(x) + c) + tan(d\*sqrt(x) + c)) - 4\*(4\*b^2\*c^3 + (4\*(d\*sqrt(x) + c)^3\*a\*b - 12\*(d\*sqrt(x) + c)^2\*a\*b\*c + 12\*(d\*sqrt(x) + c)\*a\*b\*c^2 + 4\*((d\*sqrt(x) + c)^3\*a\*b - 3\*(d\*sqrt(x) + c)^2\*a\*b\*c + 3\*(d\*sqrt(x) + c)\*a\*b\*c^2)\*cos(2\*d\*sqrt(x) + 2\*c)) - (-4\*I\*(d\*sqrt(x) + c)^3\*a\*b + 12\*I\*(d\*sqrt(x) + c)^2\*a\*b\*c - 12\*I\*(d\*sqrt(x) + c)\*a\*b\*c^2)\*sin(2\*d\*sqrt(x) + 2\*c))\*arctan2(cos(d\*sqrt(x) + c), sin

$$\begin{aligned}
& (d\sqrt{x} + c) + 1) + (4*(d\sqrt{x} + c)^3*a*b - 12*(d\sqrt{x} + c)^2*a*b*c + 12*(d\sqrt{x} + c)*a*b*c^2 + 4*((d\sqrt{x} + c)^3*a*b - 3*(d\sqrt{x} + c)^2*a*b*c + 3*(d\sqrt{x} + c)*a*b*c^2)*\cos(2*d\sqrt{x} + 2*c) - (-4*I*(d\sqrt{x} + c)^3*a*b + 12*I*(d\sqrt{x} + c)^2*a*b*c - 12*I*(d\sqrt{x} + c)*a*b*c^2)*\sin(2*d\sqrt{x} + 2*c))*\arctan2(\cos(d\sqrt{x} + c), -\sin(d\sqrt{x} + c) + 1) - (6*(d\sqrt{x} + c)^2*b^2 - 12*(d\sqrt{x} + c)*b^2*c + 6*b^2*c^2 + 6*((d\sqrt{x} + c)^2*b^2 - 2*(d\sqrt{x} + c)*b^2*c + b^2*c^2)*\cos(2*d\sqrt{x} + 2*c) + (6*I*(d\sqrt{x} + c)^2*b^2 - 12*I*(d\sqrt{x} + c)*b^2*c + 6*I*b^2*c^2)*\sin(2*d\sqrt{x} + 2*c))*\arctan2(\sin(2*d\sqrt{x} + 2*c), \cos(2*d\sqrt{x} + 2*c) + 1) + 4*((d\sqrt{x} + c)^3*b^2 - 3*(d\sqrt{x} + c)^2*b^2*c + 3*(d\sqrt{x} + c)*b^2*c^2)*\cos(2*d\sqrt{x} + 2*c) + (6*(d\sqrt{x} + c)*b^2 - 6*b^2*c + 6*((d\sqrt{x} + c)*b^2 - b^2*c)*\cos(2*d\sqrt{x} + 2*c) - (-6*I*(d\sqrt{x} + c)*b^2 + 6*I*b^2*c)*\sin(2*d\sqrt{x} + 2*c))*\operatorname{dilog}(-e^{(2*I*d\sqrt{x} + 2*I*c)}) + (12*(d\sqrt{x} + c)^2*a*b - 24*(d\sqrt{x} + c)*a*b*c + 12*a*b*c^2 + 12*((d\sqrt{x} + c)^2*a*b - 2*(d\sqrt{x} + c)*a*b*c + a*b*c^2)*\cos(2*d\sqrt{x} + 2*c) - (-12*I*(d\sqrt{x} + c)^2*a*b + 24*I*(d\sqrt{x} + c)*a*b*c - 12*I*a*b*c^2)*\sin(2*d\sqrt{x} + 2*c))*\operatorname{dilog}(I*e^{(I*d\sqrt{x} + I*c)}) - (12*(d\sqrt{x} + c)^2*a*b - 24*(d\sqrt{x} + c)*a*b*c + 12*a*b*c^2 + 12*((d\sqrt{x} + c)^2*a*b - 2*(d\sqrt{x} + c)*a*b*c + a*b*c^2)*\cos(2*d\sqrt{x} + 2*c) + (12*I*(d\sqrt{x} + c)^2*a*b - 24*I*(d\sqrt{x} + c)*a*b*c + 12*I*a*b*c^2)*\sin(2*d\sqrt{x} + 2*c))*\operatorname{dilog}(-I*e^{(I*d\sqrt{x} + I*c)}) - (-3*I*(d\sqrt{x} + c)^2*b^2 + 6*I*(d\sqrt{x} + c)*b^2*c - 3*I*b^2*c^2 + (-3*I*(d\sqrt{x} + c)^2*b^2 + 6*I*(d\sqrt{x} + c)*b^2*c - 3*I*b^2*c^2)*\cos(2*d\sqrt{x} + 2*c) + 3*((d\sqrt{x} + c)^2*b^2 - 2*(d\sqrt{x} + c)*b^2*c + b^2*c^2)*\sin(2*d\sqrt{x} + 2*c))*\log(\cos(2*d\sqrt{x} + 2*c)^2 + \sin(2*d\sqrt{x} + 2*c)^2 + 2*\cos(2*d\sqrt{x} + 2*c) + 1) - (-2*I*(d\sqrt{x} + c)^3*a*b + 6*I*(d\sqrt{x} + c)^2*a*b*c - 6*I*(d\sqrt{x} + c)*a*b*c^2 + (-2*I*(d\sqrt{x} + c)^3*a*b + 6*I*(d\sqrt{x} + c)^2*a*b*c - 6*I*(d\sqrt{x} + c)*a*b*c^2)*\cos(2*d\sqrt{x} + 2*c) + 2*((d\sqrt{x} + c)^3*a*b - 3*(d\sqrt{x} + c)^2*a*b*c + 3*(d\sqrt{x} + c)*a*b*c^2)*\sin(2*d\sqrt{x} + 2*c))*\log(\cos(d\sqrt{x} + c)^2 + \sin(d\sqrt{x} + c)^2 + 2*\sin(d\sqrt{x} + c) + 1) - (2*I*(d\sqrt{x} + c)^3*a*b - 6*I*(d\sqrt{x} + c)^2*a*b*c + 6*I*(d\sqrt{x} + c)*a*b*c^2 + (2*I*(d\sqrt{x} + c)^3*a*b - 6*I*(d\sqrt{x} + c)^2*a*b*c + 6*I*(d\sqrt{x} + c)*a*b*c^2)*\cos(2*d\sqrt{x} + 2*c) - 2*((d\sqrt{x} + c)^3*a*b - 3*(d\sqrt{x} + c)^2*a*b*c + 3*(d\sqrt{x} + c)*a*b*c^2)*\sin(2*d\sqrt{x} + 2*c))*\log(\cos(d\sqrt{x} + c)^2 + \sin(d\sqrt{x} + c)^2 - 2*\sin(d\sqrt{x} + c) + 1) - 24*(a*b*\cos(2*d\sqrt{x} + 2*c) + I*a*b*\sin(2*d\sqrt{x} + 2*c) + a*b)*\operatorname{polylog}(4, I*e^{(I*d\sqrt{x} + I*c)}) + 24*(a*b*\cos(2*d\sqrt{x} + 2*c) + I*a*b*\sin(2*d\sqrt{x} + 2*c) + a*b)*\operatorname{polylog}(4, -I*e^{(I*d\sqrt{x} + I*c)}) - (-3*I*b^2*\cos(2*d\sqrt{x} + 2*c) + 3*b^2*\sin(2*d\sqrt{x} + 2*c) - 3*I*b^2)*\operatorname{polylog}(3, -e^{(2*I*d\sqrt{x} + 2*I*c)}) - (-24*I*(d\sqrt{x} + c)*a*b + 24*I*a*b*c + (-24*I*(d\sqrt{x} + c)*a*b + 24*I*a*b*c)*\cos(2*d\sqrt{x} + 2*c) + 24*((d\sqrt{x} + c)*a*b - a*b*c)*\sin(2*d\sqrt{x} + 2*c))*\operatorname{polylog}(3, I*e^{(I*d\sqrt{x} + I*c)}) - (24*I*(d\sqrt{x} + c)*a*b - 24*I*a*b*c + (24*I*(d\sqrt{x} + c)*a*b - 24*I*a*b*c)*\cos(2*d\sqrt{x} + 2*c) - 24*((d\sqrt{x} + c)*a*b - a*b*c)*\sin(2*d\sqrt{x} + 2*c) +
\end{aligned}$$

```
2*c))*polylog(3, -I*e^(I*d*sqrt(x) + I*c)) - (-4*I*(d*sqrt(x) + c)^3*b^2 +
12*I*(d*sqrt(x) + c)^2*b^2*c - 12*I*(d*sqrt(x) + c)*b^2*c^2)*sin(2*d*sqrt(
x) + 2*c))/(-2*I*cos(2*d*sqrt(x) + 2*c) + 2*sin(2*d*sqrt(x) + 2*c) - 2*I))/
d^4
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x \left( a + \frac{b}{\cos(c + d\sqrt{x})} \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b/cos(c + d*x^(1/2)))^2,x)
```

```
[Out] int(x*(a + b/cos(c + d*x^(1/2)))^2, x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x (a + b \sec(c + d\sqrt{x}))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*sec(c+d*x**(1/2)))**2,x)
```

```
[Out] Integral(x*(a + b*sec(c + d*sqrt(x)))**2, x)
```



$$3.39 \quad \int \frac{(a+b \sec(c+d\sqrt{x}))^2}{x} dx$$

Optimal. Leaf size=23

$$\text{Int} \left( \frac{(a+b \sec(c+d\sqrt{x}))^2}{x}, x \right)$$

[Out] Unintegrable((a+b\*sec(c+d\*x^(1/2)))^2/x,x)

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{(a+b \sec(c+d\sqrt{x}))^2}{x} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*Sec[c + d\*Sqrt[x]])^2/x,x]

[Out] Defer[Int] [(a + b\*Sec[c + d\*Sqrt[x]])^2/x, x]

Rubi steps

$$\int \frac{(a+b \sec(c+d\sqrt{x}))^2}{x} dx = \int \frac{(a+b \sec(c+d\sqrt{x}))^2}{x} dx$$

Mathematica [A] time = 40.08, size = 0, normalized size = 0.00

$$\int \frac{(a+b \sec(c+d\sqrt{x}))^2}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*Sec[c + d\*Sqrt[x]])^2/x,x]

[Out] Integrate[(a + b\*Sec[c + d\*Sqrt[x]])^2/x, x]

fricas [A] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{b^2 \sec(d\sqrt{x} + c)^2 + 2ab \sec(d\sqrt{x} + c) + a^2}{x}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sec(c+d\*x^(1/2)))^2/x,x, algorithm="fricas")

[Out] integral((b^2\*sec(d\*sqrt(x) + c)^2 + 2\*a\*b\*sec(d\*sqrt(x) + c) + a^2)/x, x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \sec(d\sqrt{x} + c) + a)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sec(c+d\*x^(1/2)))^2/x,x, algorithm="giac")

[Out] integrate((b\*sec(d\*sqrt(x) + c) + a)^2/x, x)

maple [A] time = 1.40, size = 0, normalized size = 0.00

$$\int \frac{(a + b \sec(c + d\sqrt{x}))^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*sec(c+d\*x^(1/2)))^2/x,x)

[Out] int((a+b\*sec(c+d\*x^(1/2)))^2/x,x)

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$4b^2\sqrt{x} \sin(2d\sqrt{x} + 2c) + 2(d \cos(2d\sqrt{x} + 2c)^2 + d \sin(2d\sqrt{x} + 2c)^2 + 2d \cos(2d\sqrt{x} + 2c) + d)x \int \frac{b^2\sqrt{x}}{d \cos(2d\sqrt{x} + 2c)}$$

(d co

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sec(c+d\*x^(1/2)))^2/x,x, algorithm="maxima")

[Out] (4\*b^2\*sqrt(x)\*sin(2\*d\*sqrt(x) + 2\*c) + (d\*cos(2\*d\*sqrt(x) + 2\*c)^2 + d\*sin(2\*d\*sqrt(x) + 2\*c)^2 + 2\*d\*cos(2\*d\*sqrt(x) + 2\*c) + d)\*x\*integrate(2\*(b^2\*sqrt(x)\*sin(2\*d\*sqrt(x) + 2\*c) + 2\*(a\*b\*d\*cos(2\*d\*sqrt(x) + 2\*c)\*cos(d\*sqrt(x) + c) + a\*b\*d\*sin(2\*d\*sqrt(x) + 2\*c)\*sin(d\*sqrt(x) + c) + a\*b\*d\*cos(d\*sqrt(x) + c))\*x)/((d\*cos(2\*d\*sqrt(x) + 2\*c)^2 + d\*sin(2\*d\*sqrt(x) + 2\*c)^2 + 2\*d\*cos(2\*d\*sqrt(x) + 2\*c) + d)\*x^2), x) + (a^2\*d\*cos(2\*d\*sqrt(x) + 2\*c)^2 + a^2\*d\*sin(2\*d\*sqrt(x) + 2\*c)^2 + 2\*a^2\*d\*cos(2\*d\*sqrt(x) + 2\*c) + a^2\*d)\*

$x \cdot \log(x) / ((d \cdot \cos(2d \cdot \sqrt{x}) + 2c)^2 + d \cdot \sin(2d \cdot \sqrt{x}) + 2c)^2 + 2d \cdot \cos(2d \cdot \sqrt{x}) + 2c) + d) \cdot x$

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a + \frac{b}{\cos(c+d\sqrt{x})}\right)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cos(c + d\*x^(1/2)))^2/x, x)

[Out] int((a + b/cos(c + d\*x^(1/2)))^2/x, x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \sec(c + d\sqrt{x})\right)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sec(c+d\*x\*\*(1/2)))\*\*2/x, x)

[Out] Integral((a + b\*sec(c + d\*sqrt(x)))\*\*2/x, x)

$$3.40 \quad \int \frac{(a+b \sec(c+d\sqrt{x}))^2}{x^2} dx$$

Optimal. Leaf size=23

$$\text{Int} \left( \frac{(a+b \sec(c+d\sqrt{x}))^2}{x^2}, x \right)$$

[Out] Unintegrable((a+b\*sec(c+d\*x^(1/2)))^2/x^2,x)

**Rubi [A]** time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{(a+b \sec(c+d\sqrt{x}))^2}{x^2} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*Sec[c + d\*Sqrt[x]])^2/x^2,x]

[Out] Defer[Int] [(a + b\*Sec[c + d\*Sqrt[x]])^2/x^2, x]

Rubi steps

$$\int \frac{(a+b \sec(c+d\sqrt{x}))^2}{x^2} dx = \int \frac{(a+b \sec(c+d\sqrt{x}))^2}{x^2} dx$$

**Mathematica [A]** time = 21.03, size = 0, normalized size = 0.00

$$\int \frac{(a+b \sec(c+d\sqrt{x}))^2}{x^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*Sec[c + d\*Sqrt[x]])^2/x^2,x]

[Out] Integrate[(a + b\*Sec[c + d\*Sqrt[x]])^2/x^2, x]

**fricas [A]** time = 0.74, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{b^2 \sec(d\sqrt{x} + c)^2 + 2ab \sec(d\sqrt{x} + c) + a^2}{x^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sec(c+d\*x^(1/2)))^2/x^2,x, algorithm="fricas")

[Out] integral((b^2\*sec(d\*sqrt(x) + c)^2 + 2\*a\*b\*sec(d\*sqrt(x) + c) + a^2)/x^2, x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \sec(d\sqrt{x} + c) + a)^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sec(c+d\*x^(1/2)))^2/x^2,x, algorithm="giac")

[Out] integrate((b\*sec(d\*sqrt(x) + c) + a)^2/x^2, x)

**maple** [A] time = 1.40, size = 0, normalized size = 0.00

$$\int \frac{(a + b \sec(c + d\sqrt{x}))^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*sec(c+d\*x^(1/2)))^2/x^2,x)

[Out] int((a+b\*sec(c+d\*x^(1/2)))^2/x^2,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2 \left( d \cos(2d\sqrt{x} + 2c) \right)^2 + d \sin(2d\sqrt{x} + 2c)^2 + 2d \cos(2d\sqrt{x} + 2c) + d}{x^2} \int \frac{3b^2\sqrt{x} \sin(2d\sqrt{x} + 2c) + 2(abd \cos(2d\sqrt{x} + 2c))}{(d \cos(2d\sqrt{x} + 2c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sec(c+d\*x^(1/2)))^2/x^2,x, algorithm="maxima")

[Out] ((d\*cos(2\*d\*sqrt(x) + 2\*c)^2 + d\*sin(2\*d\*sqrt(x) + 2\*c)^2 + 2\*d\*cos(2\*d\*sqrt(x) + 2\*c) + d)\*x^2\*integrate(2\*(3\*b^2\*sqrt(x)\*sin(2\*d\*sqrt(x) + 2\*c) + 2\*(a\*b\*d\*cos(2\*d\*sqrt(x) + 2\*c)\*cos(d\*sqrt(x) + c) + a\*b\*d\*sin(2\*d\*sqrt(x) + 2\*c)\*sin(d\*sqrt(x) + c) + a\*b\*d\*cos(d\*sqrt(x) + c))\*x)/((d\*cos(2\*d\*sqrt(x) + 2\*c)^2 + d\*sin(2\*d\*sqrt(x) + 2\*c)^2 + 2\*d\*cos(2\*d\*sqrt(x) + 2\*c) + d)\*x^3), x) + 4\*b^2\*sqrt(x)\*sin(2\*d\*sqrt(x) + 2\*c) - (a^2\*d\*cos(2\*d\*sqrt(x) + 2\*c)^2 + a^2\*d\*sin(2\*d\*sqrt(x) + 2\*c)^2 + 2\*a^2\*d\*cos(2\*d\*sqrt(x) + 2\*c) + a^2

$*d)*x)/((d*\cos(2*d*\sqrt{x}) + 2*c)^2 + d*\sin(2*d*\sqrt{x}) + 2*c)^2 + 2*d*\cos(2*d*\sqrt{x}) + 2*c) + d)*x^2)$

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a + \frac{b}{\cos(c+d\sqrt{x})}\right)^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b/cos(c + d*x^(1/2)))^2/x^2,x)`

[Out] `int((a + b/cos(c + d*x^(1/2)))^2/x^2, x)`

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \sec(c + d\sqrt{x}))^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sec(c+d*x**(1/2)))**2/x**2,x)`

[Out] `Integral((a + b*sec(c + d*sqrt(x)))**2/x**2, x)`

$$3.41 \quad \int \frac{x^3}{a+b \sec(c+d\sqrt{x})} dx$$

Optimal. Leaf size=1041

$$\frac{x^4}{4a} + \frac{2ib \log\left(\frac{e^{i(c+d\sqrt{x})} a}{b-\sqrt{b^2-a^2}} + 1\right) x^{7/2}}{a\sqrt{b^2-a^2} d} - \frac{2ib \log\left(\frac{e^{i(c+d\sqrt{x})} a}{b+\sqrt{b^2-a^2}} + 1\right) x^{7/2}}{a\sqrt{b^2-a^2} d} + \frac{14b \operatorname{Li}_2\left(-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right) x^3}{a\sqrt{b^2-a^2} d^2} - \frac{14b \operatorname{Li}_2\left(-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right) x^3}{a\sqrt{b^2-a^2} d^2} + \dots$$

[Out]  $\frac{1}{4}x^4/a - 84I*b*x^{(5/2)}*polylog(3, -a*\exp(I*(c+d*x^{(1/2)})))/(b+(-a^2+b^2)^{(1/2)})/a/d^3/(-a^2+b^2)^{(1/2)} + 1680I*b*x^{(3/2)}*polylog(5, -a*\exp(I*(c+d*x^{(1/2)})))/(b+(-a^2+b^2)^{(1/2)})/a/d^5/(-a^2+b^2)^{(1/2)} + 14*b*x^3*polylog(2, -a*\exp(I*(c+d*x^{(1/2)})))/(b-(-a^2+b^2)^{(1/2)})/a/d^2/(-a^2+b^2)^{(1/2)} - 14*b*x^3*polylog(2, -a*\exp(I*(c+d*x^{(1/2)})))/(b+(-a^2+b^2)^{(1/2)})/a/d^2/(-a^2+b^2)^{(1/2)} - 2I*b*x^{(7/2)}*\ln(1+a*\exp(I*(c+d*x^{(1/2)})))/(b+(-a^2+b^2)^{(1/2)})/a/d/(-a^2+b^2)^{(1/2)} + 2I*b*x^{(7/2)}*\ln(1+a*\exp(I*(c+d*x^{(1/2)})))/(b-(-a^2+b^2)^{(1/2)})/a/d/(-a^2+b^2)^{(1/2)} - 420*b*x^2*polylog(4, -a*\exp(I*(c+d*x^{(1/2)})))/(b-(-a^2+b^2)^{(1/2)})/a/d^4/(-a^2+b^2)^{(1/2)} + 420*b*x^2*polylog(4, -a*\exp(I*(c+d*x^{(1/2)})))/(b+(-a^2+b^2)^{(1/2)})/a/d^4/(-a^2+b^2)^{(1/2)} - 10080I*b*polylog(7, -a*\exp(I*(c+d*x^{(1/2)})))/(b+(-a^2+b^2)^{(1/2)})*x^{(1/2)}/a/d^7/(-a^2+b^2)^{(1/2)} - 1680I*b*x^{(3/2)}*polylog(5, -a*\exp(I*(c+d*x^{(1/2)})))/(b-(-a^2+b^2)^{(1/2)})/a/d^5/(-a^2+b^2)^{(1/2)} + 5040*b*x*polylog(6, -a*\exp(I*(c+d*x^{(1/2)})))/(b-(-a^2+b^2)^{(1/2)})/a/d^6/(-a^2+b^2)^{(1/2)} - 5040*b*x*polylog(6, -a*\exp(I*(c+d*x^{(1/2)})))/(b+(-a^2+b^2)^{(1/2)})/a/d^6/(-a^2+b^2)^{(1/2)} - 10080*b*polylog(8, -a*\exp(I*(c+d*x^{(1/2)})))/(b-(-a^2+b^2)^{(1/2)})/a/d^8/(-a^2+b^2)^{(1/2)} + 10080*b*polylog(8, -a*\exp(I*(c+d*x^{(1/2)})))/(b+(-a^2+b^2)^{(1/2)})/a/d^8/(-a^2+b^2)^{(1/2)} + 84I*b*x^{(5/2)}*polylog(3, -a*\exp(I*(c+d*x^{(1/2)})))/(b-(-a^2+b^2)^{(1/2)})/a/d^3/(-a^2+b^2)^{(1/2)} + 10080I*b*polylog(7, -a*\exp(I*(c+d*x^{(1/2)})))/(b-(-a^2+b^2)^{(1/2)})*x^{(1/2)}/a/d^7/(-a^2+b^2)^{(1/2)}$

**Rubi [A]** time = 1.48, antiderivative size = 1041, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 9, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$ , Rules used = {4204, 4191, 3321, 2264, 2190, 2531, 6609, 2282, 6589}

$$\frac{x^4}{4a} + \frac{2ib \log\left(\frac{e^{i(c+d\sqrt{x})} a}{b-\sqrt{b^2-a^2}} + 1\right) x^{7/2}}{a\sqrt{b^2-a^2} d} - \frac{2ib \log\left(\frac{e^{i(c+d\sqrt{x})} a}{b+\sqrt{b^2-a^2}} + 1\right) x^{7/2}}{a\sqrt{b^2-a^2} d} + \frac{14b \operatorname{PolyLog}\left(2, -\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right) x^3}{a\sqrt{b^2-a^2} d^2} - \frac{14b \operatorname{PolyLog}\left(2, -\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right) x^3}{a\sqrt{b^2-a^2} d^2} + \dots$$

Antiderivative was successfully verified.

[In] Int[x^3/(a + b\*Sec[c + d\*Sqrt[x]]), x]

[Out]  $x^4/(4*a) + ((2*I)*b*x^{(7/2)}*\log[1 + (a*E^{(I*(c + d*Sqrt[x]))})]/(b - \sqrt{-a^2 + b^2}))/a*\sqrt{-a^2 + b^2}*d - ((2*I)*b*x^{(7/2)}*\log[1 + (a*E^{(I*(c +$

```

d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))/(a*Sqrt[-a^2 + b^2]*d) + (14*b*x^3*PolyLog[2, -((a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])))/(a*Sqrt[-a^2 + b^2]*d^2) - (14*b*x^3*PolyLog[2, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))))/(a*Sqrt[-a^2 + b^2]*d^2) + ((84*I)*b*x^(5/2)*PolyLog[3, -((a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])))/(a*Sqrt[-a^2 + b^2]*d^3) - ((84*I)*b*x^(5/2)*PolyLog[3, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))))/(a*Sqrt[-a^2 + b^2]*d^3) - (420*b*x^2*PolyLog[4, -((a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])))/(a*Sqrt[-a^2 + b^2]*d^4) + (420*b*x^2*PolyLog[4, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])))/(a*Sqrt[-a^2 + b^2]*d^4) - ((1680*I)*b*x^(3/2)*PolyLog[5, -((a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])))/(a*Sqrt[-a^2 + b^2]*d^5) + ((1680*I)*b*x^(3/2)*PolyLog[5, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])))/(a*Sqrt[-a^2 + b^2]*d^5) + (5040*b*x*PolyLog[6, -((a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])))/(a*Sqrt[-a^2 + b^2]*d^6) - (5040*b*x*PolyLog[6, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])))/(a*Sqrt[-a^2 + b^2]*d^6) + ((10080*I)*b*Sqrt[x]*PolyLog[7, -((a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])))/(a*Sqrt[-a^2 + b^2]*d^7) - ((10080*I)*b*Sqrt[x]*PolyLog[7, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])))/(a*Sqrt[-a^2 + b^2]*d^7) - (10080*b*PolyLog[8, -((a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])))/(a*Sqrt[-a^2 + b^2]*d^8) + (10080*b*PolyLog[8, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])))/(a*Sqrt[-a^2 + b^2]*d^8)

```

### Rule 2190

```

Int[(((F_)^((g_)*((e_) + (f_)*(x_)))^(n_)*((c_) + (d_)*(x_))^(m_)))/((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp[(((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

```

### Rule 2264

```

Int[((F_)^(u_)*((f_) + (g_)*(x_))^(m_))/((a_) + (b_)*(F_)^(u_) + (c_)*(F_)^(v_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[(f + g*x)^m*F^u/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[(f + g*x)^m*F^u/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]

```

### Rule 2282

```

Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

```



Rule 2531

Int[Log[1 + (e\_.)\*((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_)))^(n\_.)]\*(f\_.) + (g\_.)\*(x\_)^(m\_.), x\_Symbol] := -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n)])/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m - 1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

Rule 3321

Int[((c\_.) + (d\_.)\*(x\_)^(m\_.))/((a\_.) + (b\_.)\*sin[(e\_.) + Pi\*(k\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Dist[2, Int[((c + d\*x)^m\*E^(I\*Pi\*(k - 1/2))\*E^(I\*(e + f\*x)))/(b + 2\*a\*E^(I\*Pi\*(k - 1/2))\*E^(I\*(e + f\*x)) - b\*E^(2\*I\*k\*Pi)\*E^(2\*I\*(e + f\*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2\*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]

Rule 4191

Int[(csc[(e\_.) + (f\_.)\*(x\_)]\*(b\_.) + (a\_.))^(n\_.)\*((c\_.) + (d\_.)\*(x\_)^(m\_.)), x\_Symbol] := Int[ExpandIntegrand[(c + d\*x)^m, 1/(Sin[e + f\*x]^n/(b + a\*Sin[e + f\*x]^n)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGtQ[m, 0]

Rule 4204

Int[(x\_)^(m\_.)\*((a\_.) + (b\_.)\*Sec[(c\_.) + (d\_.)\*(x\_)^(n\_)])^(p\_.), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*Sec[c + d\*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]

Rule 6589

Int[PolyLog[n, (c\_.)\*((a\_.) + (b\_.)\*(x\_)^(p\_.))]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)^p/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

Rule 6609

Int[((e\_.) + (f\_.)\*(x\_)^(m\_.))\*PolyLog[n, (d\_.)\*((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_)))^(p\_.)], x\_Symbol] := Simp[((e + f\*x)^m\*PolyLog[n + 1, d\*(F^(c\*(a + b\*x)))^p]/(b\*c\*p\*Log[F]), x] - Dist[(f\*m)/(b\*c\*p\*Log[F]), Int[(e + f\*x)^(m - 1)\*PolyLog[n + 1, d\*(F^(c\*(a + b\*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]

Rubi steps



**Mathematica [A]** time = 2.67, size = 1122, normalized size = 1.08

$$(b + a \cos(c + d\sqrt{x})) \left( x^4 + \frac{8be^{ic} \left( 7x^3 \operatorname{Li}_2 \left( -\frac{ae^{i(2c+d\sqrt{x})}}{be^{ic} - \sqrt{(b^2-a^2)e^{2ic}}} \right) \right) d^6 - 7x^3 \operatorname{Li}_2 \left( -\frac{ae^{i(2c+d\sqrt{x})}}{e^{ic}b + \sqrt{(b^2-a^2)e^{2ic}}} \right) d^6 + i \left( x^{7/2} \log \left( \frac{e^{i(2c+d\sqrt{x})} a}{be^{ic} - \sqrt{(b^2-a^2)e^{2ic}}} + 1 \right) \right) d^7 - x^{7/2} \log \left( \frac{e^{i(2c+d\sqrt{x})} a}{be^{ic} - \sqrt{(b^2-a^2)e^{2ic}}} + 1 \right) d^7}{d^7 - x^{7/2} \log \left( \frac{e^{i(2c+d\sqrt{x})} a}{be^{ic} - \sqrt{(b^2-a^2)e^{2ic}}} + 1 \right) d^7 - x^{7/2} \log \left( \frac{e^{i(2c+d\sqrt{x})} a}{be^{ic} - \sqrt{(b^2-a^2)e^{2ic}}} + 1 \right) d^7} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(a + b\*Sec[c + d\*Sqrt[x]]),x]

[Out] ((b + a\*Cos[c + d\*Sqrt[x]])\*(x^4 + (8\*b\*E^(I\*c)\*(7\*d^6\*x^3\*PolyLog[2, -((a\*E^(I\*(2\*c + d\*Sqrt[x])))/(b\*E^(I\*c) - Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)])))] - 7\*d^6\*x^3\*PolyLog[2, -((a\*E^(I\*(2\*c + d\*Sqrt[x])))/(b\*E^(I\*c) + Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)])))] + I\*(d^7\*x^(7/2)\*Log[1 + (a\*E^(I\*(2\*c + d\*Sqrt[x])))/(b\*E^(I\*c) - Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)]]) - d^7\*x^(7/2)\*Log[1 + (a\*E^(I\*(2\*c + d\*Sqrt[x])))/(b\*E^(I\*c) + Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)]]) + 42\*d^5\*x^(5/2)\*PolyLog[3, -((a\*E^(I\*(2\*c + d\*Sqrt[x])))/(b\*E^(I\*c) - Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)])))] - 42\*d^5\*x^(5/2)\*PolyLog[3, -((a\*E^(I\*(2\*c + d\*Sqrt[x])))/(b\*E^(I\*c) + Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)])))] + (210\*I)\*d^4\*x^2\*PolyLog[4, -((a\*E^(I\*(2\*c + d\*Sqrt[x])))/(b\*E^(I\*c) - Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)])))] - (210\*I)\*d^4\*x^2\*PolyLog[4, -((a\*E^(I\*(2\*c + d\*Sqrt[x])))/(b\*E^(I\*c) + Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)])))] - 840\*d^3\*x^(3/2)\*PolyLog[5, -((a\*E^(I\*(2\*c + d\*Sqrt[x])))/(b\*E^(I\*c) - Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)])))] + 840\*d^3\*x^(3/2)\*PolyLog[5, -((a\*E^(I\*(2\*c + d\*Sqrt[x])))/(b\*E^(I\*c) + Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)])))] - (2520\*I)\*d^2\*x\*PolyLog[6, -((a\*E^(I\*(2\*c + d\*Sqrt[x])))/(b\*E^(I\*c) - Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)])))] + (2520\*I)\*d^2\*x\*PolyLog[6, -((a\*E^(I\*(2\*c + d\*Sqrt[x])))/(b\*E^(I\*c) + Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)])))] + 5040\*d\*Sqrt[x]\*PolyLog[7, -((a\*E^(I\*(2\*c + d\*Sqrt[x])))/(b\*E^(I\*c) - Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)])))] - 5040\*d\*Sqrt[x]\*PolyLog[7, -((a\*E^(I\*(2\*c + d\*Sqrt[x])))/(b\*E^(I\*c) + Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)])))] + (5040\*I)\*PolyLog[8, -((a\*E^(I\*(2\*c + d\*Sqrt[x])))/(b\*E^(I\*c) - Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)])))] - (5040\*I)\*PolyLog[8, -((a\*E^(I\*(2\*c + d\*Sqrt[x])))/(b\*E^(I\*c) + Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)])))])))/(d^8\*Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)])\*Sec[c + d\*Sqrt[x]]/(4\*a\*(a + b\*Sec[c + d\*Sqrt[x]]))

**fricas [F]** time = 1.04, size = 0, normalized size = 0.00

$$\operatorname{integral} \left( \frac{x^3}{b \sec(d\sqrt{x} + c) + a}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+b\*sec(c+d\*x^(1/2))),x, algorithm="fricas")

[Out] `integral(x^3/(b*sec(d*sqrt(x) + c) + a), x)`

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{b \sec(d\sqrt{x} + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(a+b*sec(c+d*x^(1/2))),x, algorithm="giac")`

[Out] `integrate(x^3/(b*sec(d*sqrt(x) + c) + a), x)`

**maple** [F] time = 1.13, size = 0, normalized size = 0.00

$$\int \frac{x^3}{a + b \sec(c + d\sqrt{x})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/(a+b*sec(c+d*x^(1/2))),x)`

[Out] `int(x^3/(a+b*sec(c+d*x^(1/2))),x)`

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(a+b*sec(c+d*x^(1/2))),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a^2-4\*b^2>0)', see `assume?` for more details)Is 4\*a^2-4\*b^2 positive or negative?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{a + \frac{b}{\cos(c+d\sqrt{x})}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/(a + b/cos(c + d*x^(1/2))),x)`

```
[Out] int(x^3/(a + b/cos(c + d*x^(1/2))), x)
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{x^3}{a + b \sec(c + d\sqrt{x})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*sec(c+d*x**(1/2))),x)
```

```
[Out] Integral(x**3/(a + b*sec(c + d*sqrt(x))), x)
```

$$3.42 \quad \int \frac{x^2}{a+b \sec(c+d\sqrt{x})} dx$$

**Optimal.** Leaf size=781

$$\frac{240b\text{Li}_6\left(-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)}{ad^6\sqrt{b^2-a^2}} - \frac{240b\text{Li}_6\left(-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)}{ad^6\sqrt{b^2-a^2}} - \frac{240ib\sqrt{x}\text{Li}_5\left(-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)}{ad^5\sqrt{b^2-a^2}} + \frac{240ib\sqrt{x}\text{Li}_5\left(-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)}{ad^5\sqrt{b^2-a^2}} - \frac{120bx\text{Li}_4\left(-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)}{ad^4\sqrt{b^2-a^2}} - \frac{120bx\text{Li}_4\left(-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)}{ad^4\sqrt{b^2-a^2}}$$

[Out]  $\frac{1}{3}x^3/a+2*I*b*x^{(5/2)}*\ln(1+a*\exp(I*(c+d*x^{(1/2)})))/(b-(-a^2+b^2)^{(1/2)})/a/d/(-a^2+b^2)^{(1/2)}-2*I*b*x^{(5/2)}*\ln(1+a*\exp(I*(c+d*x^{(1/2)})))/(b+(-a^2+b^2)^{(1/2)})/a/d/(-a^2+b^2)^{(1/2)}+10*b*x^2*polylog(2,-a*\exp(I*(c+d*x^{(1/2)})))/(b-(-a^2+b^2)^{(1/2)})/a/d^2/(-a^2+b^2)^{(1/2)}-10*b*x^2*polylog(2,-a*\exp(I*(c+d*x^{(1/2)})))/(b+(-a^2+b^2)^{(1/2)})/a/d^2/(-a^2+b^2)^{(1/2)}+40*I*b*x^{(3/2)}*polylog(3,-a*\exp(I*(c+d*x^{(1/2)})))/(b-(-a^2+b^2)^{(1/2)})/a/d^3/(-a^2+b^2)^{(1/2)}-40*I*b*x^{(3/2)}*polylog(3,-a*\exp(I*(c+d*x^{(1/2)})))/(b+(-a^2+b^2)^{(1/2)})/a/d^3/(-a^2+b^2)^{(1/2)}-120*b*x*polylog(4,-a*\exp(I*(c+d*x^{(1/2)})))/(b-(-a^2+b^2)^{(1/2)})/a/d^4/(-a^2+b^2)^{(1/2)}+120*b*x*polylog(4,-a*\exp(I*(c+d*x^{(1/2)})))/(b+(-a^2+b^2)^{(1/2)})/a/d^4/(-a^2+b^2)^{(1/2)}+240*b*polylog(6,-a*\exp(I*(c+d*x^{(1/2)})))/(b-(-a^2+b^2)^{(1/2)})/a/d^6/(-a^2+b^2)^{(1/2)}-240*b*polylog(6,-a*\exp(I*(c+d*x^{(1/2)})))/(b+(-a^2+b^2)^{(1/2)})/a/d^6/(-a^2+b^2)^{(1/2)}-240*I*b*polylog(5,-a*\exp(I*(c+d*x^{(1/2)})))/(b-(-a^2+b^2)^{(1/2)})*x^{(1/2)}/a/d^5/(-a^2+b^2)^{(1/2)}+240*I*b*polylog(5,-a*\exp(I*(c+d*x^{(1/2)})))/(b+(-a^2+b^2)^{(1/2)})*x^{(1/2)}/a/d^5/(-a^2+b^2)^{(1/2)}$

**Rubi [A]** time = 1.18, antiderivative size = 781, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 9, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$ , Rules used = {4204, 4191, 3321, 2264, 2190, 2531, 6609, 2282, 6589}

$$\frac{10bx^2\text{PolyLog}\left(2, -\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}} - \frac{10bx^2\text{PolyLog}\left(2, -\frac{ae^{i(c+d\sqrt{x})}}{\sqrt{b^2-a^2}+b}\right)}{ad^2\sqrt{b^2-a^2}} + \frac{40ibx^{3/2}\text{PolyLog}\left(3, -\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}} - \frac{40ibx^{3/2}\text{PolyLog}\left(3, -\frac{ae^{i(c+d\sqrt{x})}}{\sqrt{b^2-a^2}+b}\right)}{ad^3\sqrt{b^2-a^2}}$$

Antiderivative was successfully verified.

[In] Int[x^2/(a + b\*Sec[c + d\*Sqrt[x]]), x]

[Out]  $x^3/(3*a) + ((2*I)*b*x^{(5/2)}*\text{Log}[1 + (a*E^{(I*(c + d*Sqrt[x]))})]/(b - \text{Sqrt}[-a^2 + b^2]))/(a*\text{Sqrt}[-a^2 + b^2]*d) - ((2*I)*b*x^{(5/2)}*\text{Log}[1 + (a*E^{(I*(c + d*Sqrt[x]))})]/(b + \text{Sqrt}[-a^2 + b^2]))/(a*\text{Sqrt}[-a^2 + b^2]*d) + (10*b*x^2*\text{PolyLog}[2, -((a*E^{(I*(c + d*Sqrt[x]))})/(b - \text{Sqrt}[-a^2 + b^2]))]/(a*\text{Sqrt}[-a^2 + b^2]*d^2) - (10*b*x^2*\text{PolyLog}[2, -((a*E^{(I*(c + d*Sqrt[x]))})/(b + \text{Sqrt}[-a^2 + b^2]))]/(a*\text{Sqrt}[-a^2 + b^2]*d^2) + ((40*I)*b*x^{(3/2)}*\text{PolyLog}[3, -((a*E^{(I*(c + d*Sqrt[x]))})/(b - \text{Sqrt}[-a^2 + b^2]))]/(a*\text{Sqrt}[-a^2 + b^2]*d^3) - (40*I)*b*x^{(3/2)}*\text{PolyLog}[3, -((a*E^{(I*(c + d*Sqrt[x]))})/(b + \text{Sqrt}[-a^2 + b^2]))]/(a*\text{Sqrt}[-a^2 + b^2]*d^3))$

```

- ((40*I)*b*x^(3/2)*PolyLog[3, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2
+ b^2]))]/(a*Sqrt[-a^2 + b^2]*d^3) - (120*b*x*PolyLog[4, -((a*E^(I*(c + d*
Sqrt[x])))/(b - Sqrt[-a^2 + b^2]))]/(a*Sqrt[-a^2 + b^2]*d^4) + (120*b*x*Po
lyLog[4, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))]/(a*Sqrt[-a^2
+ b^2]*d^4) - ((240*I)*b*Sqrt[x]*PolyLog[5, -((a*E^(I*(c + d*Sqrt[x])))/(b
- Sqrt[-a^2 + b^2]))]/(a*Sqrt[-a^2 + b^2]*d^5) + ((240*I)*b*Sqrt[x]*PolyL
og[5, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))]/(a*Sqrt[-a^2 +
b^2]*d^5) + (240*b*PolyLog[6, -((a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 +
b^2]))]/(a*Sqrt[-a^2 + b^2]*d^6) - (240*b*PolyLog[6, -((a*E^(I*(c + d*Sqrt
[x])))/(b + Sqrt[-a^2 + b^2]))]/(a*Sqrt[-a^2 + b^2]*d^6)

```

### Rule 2190

```

Int[(((F_)^((g_)*((e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_))/
((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_)))^(n_))), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

```

### Rule 2264

```

Int[((F_)^(u_)*((f_) + (g_)*(x_))^(m_))/((a_) + (b_)*(F_)^(u_) + (c_)
*(F_)^(v_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]

```

### Rule 2282

```

Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

```

### Rule 2531

```

Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_)))^(n_))]*((f_) + (g_)
*(x_))^(m_), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]

```

### Rule 3321

```

Int[((c_) + (d_)*(x_))^(m_)/((a_) + (b_)*sin[(e_) + Pi*(k_) + (f_)*(

```



```
x_]], x_Symbol] := Dist[2, Int[((c + d*x)^m*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

### Rule 4191

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGtQ[m, 0]
```

### Rule 4204

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{x^2}{a + b \sec(c + d\sqrt{x})} dx &= 2 \text{Subst} \left( \int \frac{x^5}{a + b \sec(c + dx)} dx, x, \sqrt{x} \right) \\
&= 2 \text{Subst} \left( \int \left( \frac{x^5}{a} - \frac{bx^5}{a(b + a \cos(c + dx))} \right) dx, x, \sqrt{x} \right) \\
&= \frac{x^3}{3a} - \frac{(2b) \text{Subst} \left( \int \frac{x^5}{b + a \cos(c + dx)} dx, x, \sqrt{x} \right)}{a} \\
&= \frac{x^3}{3a} - \frac{(4b) \text{Subst} \left( \int \frac{e^{i(c+dx)} x^5}{a + 2be^{i(c+dx)} + ae^{2i(c+dx)}} dx, x, \sqrt{x} \right)}{a} \\
&= \frac{x^3}{3a} - \frac{(4b) \text{Subst} \left( \int \frac{e^{i(c+dx)} x^5}{2b - 2\sqrt{-a^2 + b^2} + 2ae^{i(c+dx)}} dx, x, \sqrt{x} \right)}{a\sqrt{-a^2 + b^2}} + \frac{(4b) \text{Subst} \left( \int \frac{e^{i(c+dx)} x^5}{2b + 2\sqrt{-a^2 + b^2} + 2ae^{i(c+dx)}} dx, x, \sqrt{x} \right)}{a\sqrt{-a^2 + b^2}} \\
&= \frac{x^3}{3a} + \frac{2ibx^{5/2} \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} - \frac{2ibx^{5/2} \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b + \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} - \frac{(10ib) \text{Subst} \left( \int x \right)}{a\sqrt{-a^2 + b^2} d} \\
&= \frac{x^3}{3a} + \frac{2ibx^{5/2} \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} - \frac{2ibx^{5/2} \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b + \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} + \frac{10bx^2 \text{Li}_2 \left( -\frac{ae^{i(c+d\sqrt{x})}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} \\
&= \frac{x^3}{3a} + \frac{2ibx^{5/2} \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} - \frac{2ibx^{5/2} \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b + \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} + \frac{10bx^2 \text{Li}_2 \left( -\frac{ae^{i(c+d\sqrt{x})}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} \\
&= \frac{x^3}{3a} + \frac{2ibx^{5/2} \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} - \frac{2ibx^{5/2} \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b + \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} + \frac{10bx^2 \text{Li}_2 \left( -\frac{ae^{i(c+d\sqrt{x})}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} \\
&= \frac{x^3}{3a} + \frac{2ibx^{5/2} \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} - \frac{2ibx^{5/2} \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b + \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} + \frac{10bx^2 \text{Li}_2 \left( -\frac{ae^{i(c+d\sqrt{x})}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} \\
&= \frac{x^3}{3a} + \frac{2ibx^{5/2} \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} - \frac{2ibx^{5/2} \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b + \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} + \frac{10bx^2 \text{Li}_2 \left( -\frac{ae^{i(c+d\sqrt{x})}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} \\
&= \frac{x^3}{3a} + \frac{2ibx^{5/2} \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} - \frac{2ibx^{5/2} \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b + \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} + \frac{10bx^2 \text{Li}_2 \left( -\frac{ae^{i(c+d\sqrt{x})}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} \\
&= \frac{x^3}{3a} + \frac{2ibx^{5/2} \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} - \frac{2ibx^{5/2} \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b + \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} + \frac{10bx^2 \text{Li}_2 \left( -\frac{ae^{i(c+d\sqrt{x})}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d}
\end{aligned}$$

**Mathematica [A]** time = 2.13, size = 858, normalized size = 1.10

$$(b + a \cos(c + d\sqrt{x})) \left( x^3 + \frac{6be^{ic} \left( 5x^2 \operatorname{Li}_2 \left( -\frac{ae^{i(2c+d\sqrt{x})}}{be^{ic} - \sqrt{(b^2-a^2)e^{2ic}}} \right) d^4 - 5x^2 \operatorname{Li}_2 \left( -\frac{ae^{i(2c+d\sqrt{x})}}{e^{ic}b + \sqrt{(b^2-a^2)e^{2ic}}} \right) d^4 + i \left( x^{5/2} \log \left( \frac{e^{i(2c+d\sqrt{x})} a}{be^{ic} - \sqrt{(b^2-a^2)e^{2ic}}} + 1 \right) d^5 - x^{5/2} \right)}{d^5 - x^{5/2}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(a + b\*Sec[c + d\*Sqrt[x]]), x]

[Out] ((b + a\*Cos[c + d\*Sqrt[x]])\*(x^3 + (6\*b\*E^(I\*c)\*(5\*d^4\*x^2\*PolyLog[2, -(a\*E^(I\*(2\*c + d\*Sqrt[x])))/(b\*E^(I\*c) - Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)])] - 5\*d^4\*x^2\*PolyLog[2, -(a\*E^(I\*(2\*c + d\*Sqrt[x])))/(b\*E^(I\*c) + Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)])] + I\*(d^5\*x^(5/2)\*Log[1 + (a\*E^(I\*(2\*c + d\*Sqrt[x])))/(b\*E^(I\*c) - Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)])] - d^5\*x^(5/2)\*Log[1 + (a\*E^(I\*(2\*c + d\*Sqrt[x])))/(b\*E^(I\*c) + Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)])] + 20\*d^3\*x^(3/2)\*PolyLog[3, -(a\*E^(I\*(2\*c + d\*Sqrt[x])))/(b\*E^(I\*c) - Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)])] - 20\*d^3\*x^(3/2)\*PolyLog[3, -(a\*E^(I\*(2\*c + d\*Sqrt[x])))/(b\*E^(I\*c) + Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)])] + (60\*I)\*d^2\*x\*PolyLog[4, -(a\*E^(I\*(2\*c + d\*Sqrt[x])))/(b\*E^(I\*c) - Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)])] - (60\*I)\*d^2\*x\*PolyLog[4, -(a\*E^(I\*(2\*c + d\*Sqrt[x])))/(b\*E^(I\*c) + Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)])] - 120\*d\*Sqrt[x]\*PolyLog[5, -(a\*E^(I\*(2\*c + d\*Sqrt[x])))/(b\*E^(I\*c) - Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)])] + 120\*d\*Sqrt[x]\*PolyLog[5, -(a\*E^(I\*(2\*c + d\*Sqrt[x])))/(b\*E^(I\*c) + Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)])] - (120\*I)\*PolyLog[6, -(a\*E^(I\*(2\*c + d\*Sqrt[x])))/(b\*E^(I\*c) - Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)])] + (120\*I)\*PolyLog[6, -(a\*E^(I\*(2\*c + d\*Sqrt[x])))/(b\*E^(I\*c) + Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)])])/(d^6\*Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)])\*Sec[c + d\*Sqrt[x]]/(3\*a\*(a + b\*Sec[c + d\*Sqrt[x]])))

**fricas [F]** time = 0.85, size = 0, normalized size = 0.00

$$\operatorname{integral} \left( \frac{x^2}{b \sec(d\sqrt{x} + c) + a}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b\*sec(c+d\*x^(1/2))), x, algorithm="fricas")

[Out] integral(x^2/(b\*sec(d\*sqrt(x) + c) + a), x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{b \sec(d\sqrt{x} + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b\*sec(c+d\*x^(1/2))),x, algorithm="giac")

[Out] integrate(x^2/(b\*sec(d\*sqrt(x) + c) + a), x)

maple [F] time = 1.13, size = 0, normalized size = 0.00

$$\int \frac{x^2}{a + b \sec(c + d\sqrt{x})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a+b\*sec(c+d\*x^(1/2))),x)

[Out] int(x^2/(a+b\*sec(c+d\*x^(1/2))),x)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b\*sec(c+d\*x^(1/2))),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a^2-4\*b^2>0)', see `assume?` for more details)Is 4\*a^2-4\*b^2 positive or negative?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{a + \frac{b}{\cos(c+d\sqrt{x})}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a + b/cos(c + d\*x^(1/2))),x)

[Out] int(x^2/(a + b/cos(c + d\*x^(1/2))), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{a + b \sec(c + d\sqrt{x})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*sec(c+d*x**(1/2))),x)
```

```
[Out] Integral(x**2/(a + b*sec(c + d*sqrt(x))), x)
```

$$3.43 \quad \int \frac{x}{a+b \sec(c+d\sqrt{x})} dx$$

**Optimal.** Leaf size=521

$$-\frac{12b\text{Li}_4\left(-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)}{ad^4\sqrt{b^2-a^2}} + \frac{12b\text{Li}_4\left(-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)}{ad^4\sqrt{b^2-a^2}} + \frac{12ib\sqrt{x}\text{Li}_3\left(-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}} - \frac{12ib\sqrt{x}\text{Li}_3\left(-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}} + \frac{6bx\text{Li}_2\left(-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}} - \frac{6bx\text{Li}_2\left(-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}}$$

[Out]  $\frac{1}{2}x^2/a + 2Ibx^{3/2} \ln(1+a \exp(I(c+d\sqrt{x}))) / (b - (-a^2+b^2)^{1/2}) / a / d / (-a^2+b^2)^{1/2} - 2Ibx^{3/2} \ln(1+a \exp(I(c+d\sqrt{x}))) / (b + (-a^2+b^2)^{1/2}) / a / d / (-a^2+b^2)^{1/2} + 6bxx \text{polylog}(2, -a \exp(I(c+d\sqrt{x}))) / (b - (-a^2+b^2)^{1/2}) / a / d^2 / (-a^2+b^2)^{1/2} - 6bxx \text{polylog}(2, -a \exp(I(c+d\sqrt{x}))) / (b + (-a^2+b^2)^{1/2}) / a / d^2 / (-a^2+b^2)^{1/2} - 12b \text{polylog}(4, -a \exp(I(c+d\sqrt{x}))) / (b - (-a^2+b^2)^{1/2}) / a / d^4 / (-a^2+b^2)^{1/2} + 12b \text{polylog}(4, -a \exp(I(c+d\sqrt{x}))) / (b + (-a^2+b^2)^{1/2}) / a / d^4 / (-a^2+b^2)^{1/2} + 12Ib \text{polylog}(3, -a \exp(I(c+d\sqrt{x}))) / (b - (-a^2+b^2)^{1/2}) * x^{1/2} / a / d^3 / (-a^2+b^2)^{1/2} - 12Ib \text{polylog}(3, -a \exp(I(c+d\sqrt{x}))) / (b + (-a^2+b^2)^{1/2}) * x^{1/2} / a / d^3 / (-a^2+b^2)^{1/2}$

**Rubi [A]** time = 0.97, antiderivative size = 521, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {4204, 4191, 3321, 2264, 2190, 2531, 6609, 2282, 6589}

$$\frac{6bx\text{PolyLog}\left(2, -\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}} - \frac{6bx\text{PolyLog}\left(2, -\frac{ae^{i(c+d\sqrt{x})}}{\sqrt{b^2-a^2}+b}\right)}{ad^2\sqrt{b^2-a^2}} + \frac{12ib\sqrt{x}\text{PolyLog}\left(3, -\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}} - \frac{12ib\sqrt{x}\text{PolyLog}\left(3, -\frac{ae^{i(c+d\sqrt{x})}}{\sqrt{b^2-a^2}+b}\right)}{ad^3\sqrt{b^2-a^2}}$$

Antiderivative was successfully verified.

[In] Int[x/(a + b\*Sec[c + d\*Sqrt[x]]), x]

[Out]  $x^2/(2*a) + ((2*I)*b*x^{3/2}*\text{Log}[1 + (a*E^{(I*(c + d*Sqrt[x]))})/(b - \text{Sqrt}[-a^2 + b^2])]) / (a*\text{Sqrt}[-a^2 + b^2]*d) - ((2*I)*b*x^{3/2}*\text{Log}[1 + (a*E^{(I*(c + d*Sqrt[x]))})/(b + \text{Sqrt}[-a^2 + b^2])]) / (a*\text{Sqrt}[-a^2 + b^2]*d) + (6*b*x*\text{PolyLog}[2, -((a*E^{(I*(c + d*Sqrt[x]))})/(b - \text{Sqrt}[-a^2 + b^2]))] / (a*\text{Sqrt}[-a^2 + b^2]*d^2) - (6*b*x*\text{PolyLog}[2, -((a*E^{(I*(c + d*Sqrt[x]))})/(b + \text{Sqrt}[-a^2 + b^2]))] / (a*\text{Sqrt}[-a^2 + b^2]*d^2) + ((12*I)*b*\text{Sqrt}[x]*\text{PolyLog}[3, -((a*E^{(I*(c + d*Sqrt[x]))})/(b - \text{Sqrt}[-a^2 + b^2]))] / (a*\text{Sqrt}[-a^2 + b^2]*d^3) - ((12*I)*b*\text{Sqrt}[x]*\text{PolyLog}[3, -((a*E^{(I*(c + d*Sqrt[x]))})/(b + \text{Sqrt}[-a^2 + b^2]))] / (a*\text{Sqrt}[-a^2 + b^2]*d^3) - (12*b*\text{PolyLog}[4, -((a*E^{(I*(c + d*Sqrt[x]))})/(b - \text{Sqrt}[-a^2 + b^2]))] / (a*\text{Sqrt}[-a^2 + b^2]*d^4) + (12*b*\text{PolyLog}[4, -((a*E^{(I*(c + d*Sqrt[x]))})/(b + \text{Sqrt}[-a^2 + b^2]))] / (a*\text{Sqrt}[-a^2 + b^2]*d^4)$

Rule 2190

```
Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2264

```
Int[((F_)^(u_)*((f_) + (g_)*(x_))^(m_))/((a_) + (b_)*(F_)^(u_) + (c_)
*(F_)^(v_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 2531

```
Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_)))^(n_))]*((f_) + (g_)
*(x_))^(m_), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 3321

```
Int[((c_) + (d_)*(x_))^(m_)/((a_) + (b_)*sin[(e_) + Pi*(k_) + (f_)*(
x_)]), x_Symbol] := Dist[2, Int[((c + d*x)^m*E^(I*Pi*(k - 1/2))*E^(I*(e + f
*x)))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(
e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

### Rule 4191

```
Int[(csc[(e_) + (f_)*(x_)]*(b_) + (a_))^(n_)*((c_) + (d_)*(x_))^(m_)
, x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x]^n)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 4204

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol]
  := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p, x], x, x^n], x]
  /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol]
  := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x]
  /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_)))^(p_.)], x_Symbol]
  := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p])/(b*c*p*Log[F]), x]
  - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x]
  /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps



$$\begin{aligned}
\int \frac{x}{a + b \sec(c + d\sqrt{x})} dx &= 2 \text{Subst} \left( \int \frac{x^3}{a + b \sec(c + dx)} dx, x, \sqrt{x} \right) \\
&= 2 \text{Subst} \left( \int \left( \frac{x^3}{a} - \frac{bx^3}{a(b + a \cos(c + dx))} \right) dx, x, \sqrt{x} \right) \\
&= \frac{x^2}{2a} - \frac{(2b) \text{Subst} \left( \int \frac{x^3}{b + a \cos(c + dx)} dx, x, \sqrt{x} \right)}{a} \\
&= \frac{x^2}{2a} - \frac{(4b) \text{Subst} \left( \int \frac{e^{i(c+dx)} x^3}{a + 2be^{i(c+dx)} + ae^{2i(c+dx)}} dx, x, \sqrt{x} \right)}{a} \\
&= \frac{x^2}{2a} - \frac{(4b) \text{Subst} \left( \int \frac{e^{i(c+dx)} x^3}{2b - 2\sqrt{-a^2 + b^2} + 2ae^{i(c+dx)}} dx, x, \sqrt{x} \right)}{\sqrt{-a^2 + b^2}} + \frac{(4b) \text{Subst} \left( \int \frac{e^{i(c+dx)} x^3}{2b + 2\sqrt{-a^2 + b^2} + 2ae^{i(c+dx)}} dx, x, \sqrt{x} \right)}{\sqrt{-a^2 + b^2}} \\
&= \frac{x^2}{2a} + \frac{2ibx^{3/2} \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} - \frac{2ibx^{3/2} \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b + \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} - \frac{(6ib) \text{Subst} \left( \int \frac{e^{i(c+dx)} x^3}{a + 2be^{i(c+dx)} + ae^{2i(c+dx)}} dx, x, \sqrt{x} \right)}{a\sqrt{-a^2 + b^2}} \\
&= \frac{x^2}{2a} + \frac{2ibx^{3/2} \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} - \frac{2ibx^{3/2} \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b + \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} + \frac{6bx \text{Li}_2 \left( -\frac{ae^{i(c+d\sqrt{x})}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2}} \\
&= \frac{x^2}{2a} + \frac{2ibx^{3/2} \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} - \frac{2ibx^{3/2} \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b + \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} + \frac{6bx \text{Li}_2 \left( -\frac{ae^{i(c+d\sqrt{x})}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2}} \\
&= \frac{x^2}{2a} + \frac{2ibx^{3/2} \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} - \frac{2ibx^{3/2} \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b + \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} + \frac{6bx \text{Li}_2 \left( -\frac{ae^{i(c+d\sqrt{x})}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2}} \\
&= \frac{x^2}{2a} + \frac{2ibx^{3/2} \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} - \frac{2ibx^{3/2} \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b + \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} + \frac{6bx \text{Li}_2 \left( -\frac{ae^{i(c+d\sqrt{x})}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2}}
\end{aligned}$$

**Mathematica [A]** time = 20.44, size = 632, normalized size = 1.21

$$d^4 x^2 \sqrt{e^{2ic} (b^2 - a^2)} + 4ibe^{ic} d^3 x^{3/2} \log \left( 1 + \frac{ae^{i(2c+d\sqrt{x})}}{be^{ic} - \sqrt{e^{2ic} (b^2 - a^2)}} \right) - 4ibe^{ic} d^3 x^{3/2} \log \left( 1 + \frac{ae^{i(2c+d\sqrt{x})}}{\sqrt{e^{2ic} (b^2 - a^2)} + be^{ic}} \right) + 12be^{ic} d^2 x \text{Li}_2 \left( -\frac{ae^{i(c+d\sqrt{x})}}{b - \sqrt{-a^2 + b^2}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x/(a + b\*Sec[c + d\*Sqrt[x]]),x]

[Out]  $(d^4 \sqrt{-a^2 + b^2} E^{(2I)c}) x^2 + (4I) b d^3 E^{Ic} x^{3/2} \text{Log}[1 + (a E^{I(2c + d\sqrt{x})}) / (b E^{Ic} - \sqrt{-a^2 + b^2} E^{(2I)c})] - (4I) b d^3 E^{Ic} x^{3/2} \text{Log}[1 + (a E^{I(2c + d\sqrt{x})}) / (b E^{Ic} + \sqrt{-a^2 + b^2} E^{(2I)c})] + 12 b d^2 E^{Ic} x \text{PolyLog}[2, -((a E^{I(2c + d\sqrt{x})}) / (b E^{Ic} - \sqrt{-a^2 + b^2} E^{(2I)c}))] - 12 b d^2 E^{Ic} x \text{PolyLog}[2, -((a E^{I(2c + d\sqrt{x})}) / (b E^{Ic} + \sqrt{-a^2 + b^2} E^{(2I)c}))] + (24I) b d E^{Ic} \sqrt{x} \text{PolyLog}[3, -((a E^{I(2c + d\sqrt{x})}) / (b E^{Ic} - \sqrt{-a^2 + b^2} E^{(2I)c}))] - (24I) b d E^{Ic} \sqrt{x} \text{PolyLog}[3, -((a E^{I(2c + d\sqrt{x})}) / (b E^{Ic} + \sqrt{-a^2 + b^2} E^{(2I)c}))] - 24 b E^{Ic} \text{PolyLog}[4, -((a E^{I(2c + d\sqrt{x})}) / (b E^{Ic} - \sqrt{-a^2 + b^2} E^{(2I)c}))] + 24 b E^{Ic} \text{PolyLog}[4, -((a E^{I(2c + d\sqrt{x})}) / (b E^{Ic} + \sqrt{-a^2 + b^2} E^{(2I)c}))] / (2 a d^4 \sqrt{-a^2 + b^2} E^{(2I)c})]$

**fricas** [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x}{b \sec(d\sqrt{x} + c) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*sec(c+d\*x^(1/2))),x, algorithm="fricas")

[Out] integral(x/(b\*sec(d\*sqrt(x) + c) + a), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{b \sec(d\sqrt{x} + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*sec(c+d\*x^(1/2))),x, algorithm="giac")

[Out] integrate(x/(b\*sec(d\*sqrt(x) + c) + a), x)

**maple** [F] time = 1.36, size = 0, normalized size = 0.00

$$\int \frac{x}{a + b \sec(c + d\sqrt{x})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a+b\*sec(c+d\*x^(1/2))),x)

[Out] `int(x/(a+b*sec(c+d*x^(1/2))),x)`

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(a+b*sec(c+d*x^(1/2))),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a^2-4\*b^2>0)', see 'assume?' for more details)Is 4\*a^2-4\*b^2 positive or negative?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{a + \frac{b}{\cos(c+d\sqrt{x})}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(a + b/cos(c + d*x^(1/2))),x)`

[Out] `int(x/(a + b/cos(c + d*x^(1/2))), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{a + b \sec(c + d\sqrt{x})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(a+b*sec(c+d*x**(1/2))),x)`

[Out] `Integral(x/(a + b*sec(c + d*sqrt(x))), x)`

$$3.44 \quad \int \frac{1}{x(a+b \sec(c+d\sqrt{x}))} dx$$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{1}{x(a+b \sec(c+d\sqrt{x}))}, x\right)$$

[Out] Unintegrable(1/x/(a+b\*sec(c+d\*x^(1/2))), x)

**Rubi [A]** time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{x(a+b \sec(c+d\sqrt{x}))} dx$$

Verification is Not applicable to the result.

[In] Int[1/(x\*(a + b\*Sec[c + d\*Sqrt[x]])), x]

[Out] Defer[Int][1/(x\*(a + b\*Sec[c + d\*Sqrt[x]])), x]

Rubi steps

$$\int \frac{1}{x(a+b \sec(c+d\sqrt{x}))} dx = \int \frac{1}{x(a+b \sec(c+d\sqrt{x}))} dx$$

**Mathematica [A]** time = 2.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a+b \sec(c+d\sqrt{x}))} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x\*(a + b\*Sec[c + d\*Sqrt[x]])), x]

[Out] Integrate[1/(x\*(a + b\*Sec[c + d\*Sqrt[x]])), x]

**fricas [A]** time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{bx \sec(d\sqrt{x} + c) + ax}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*sec(c+d\*x^(1/2))),x, algorithm="fricas")

[Out] integral(1/(b\*x\*sec(d\*sqrt(x) + c) + a\*x), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \sec(d\sqrt{x} + c) + a)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*sec(c+d\*x^(1/2))),x, algorithm="giac")

[Out] integrate(1/((b\*sec(d\*sqrt(x) + c) + a)\*x), x)

**maple** [A] time = 1.23, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a + b \sec(c + d\sqrt{x}))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(a+b\*sec(c+d\*x^(1/2))),x)

[Out] int(1/x/(a+b\*sec(c+d\*x^(1/2))),x)

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*sec(c+d\*x^(1/2))),x, algorithm="maxima")

[Out] Timed out

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x \left( a + \frac{b}{\cos(c + d\sqrt{x})} \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x\*(a + b/cos(c + d\*x^(1/2))))),x)

[Out] int(1/(x\*(a + b/cos(c + d\*x^(1/2))))), x)

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a + b \sec(c + d\sqrt{x}))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*sec(c+d\*x\*\*(1/2))),x)

[Out] Integral(1/(x\*(a + b\*sec(c + d\*sqrt(x)))), x)

$$3.45 \quad \int \frac{a+b \sec(c+d\sqrt{x})}{x^2} dx$$

Optimal. Leaf size=26

$$b \operatorname{Int} \left( \frac{\sec(c+d\sqrt{x})}{x^2}, x \right) - \frac{a}{x}$$

[Out]  $-a/x + b \operatorname{Unintegrable}(\sec(c+d\sqrt{x})/x^2, x)$

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{a + b \sec(c + d\sqrt{x})}{x^2} dx$$

Verification is Not applicable to the result.

[In]  $\operatorname{Int}[(a + b \operatorname{Sec}[c + d \operatorname{Sqrt}[x]])/x^2, x]$

[Out]  $-(a/x) + b \operatorname{Defer}[\operatorname{Int}[\operatorname{Sec}[c + d \operatorname{Sqrt}[x]]/x^2, x]]$

Rubi steps

$$\begin{aligned} \int \frac{a + b \sec(c + d\sqrt{x})}{x^2} dx &= \int \left( \frac{a}{x^2} + \frac{b \sec(c + d\sqrt{x})}{x^2} \right) dx \\ &= -\frac{a}{x} + b \int \frac{\sec(c + d\sqrt{x})}{x^2} dx \end{aligned}$$

Mathematica [A] time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{a + b \sec(c + d\sqrt{x})}{x^2} dx$$

Verification is Not applicable to the result.

[In]  $\operatorname{Integrate}[(a + b \operatorname{Sec}[c + d \operatorname{Sqrt}[x]])/x^2, x]$

[Out]  $\operatorname{Integrate}[(a + b \operatorname{Sec}[c + d \operatorname{Sqrt}[x]])/x^2, x]$

fricas [A] time = 0.54, size = 0, normalized size = 0.00

$$\operatorname{integral} \left( \frac{b \sec(d\sqrt{x} + c) + a}{x^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sec(c+d\*x^(1/2)))/x^2,x, algorithm="fricas")

[Out] integral((b\*sec(d\*sqrt(x) + c) + a)/x^2, x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \sec(d\sqrt{x} + c) + a}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sec(c+d\*x^(1/2)))/x^2,x, algorithm="giac")

[Out] integrate((b\*sec(d\*sqrt(x) + c) + a)/x^2, x)

**maple** [A] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{a + b \sec(c + d\sqrt{x})}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*sec(c+d\*x^(1/2)))/x^2,x)

[Out] int((a+b\*sec(c+d\*x^(1/2)))/x^2,x)

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sec(c+d\*x^(1/2)))/x^2,x, algorithm="maxima")

[Out] Timed out

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a + \frac{b}{\cos(c + d\sqrt{x})}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cos(c + d\*x^(1/2)))/x^2,x)

[Out] int((a + b/cos(c + d\*x^(1/2)))/x^2, x)



sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \sec(c + d\sqrt{x})}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sec(c+d\*x\*\*(1/2)))/x\*\*2,x)

[Out] Integral((a + b\*sec(c + d\*sqrt(x)))/x\*\*2, x)

$$3.46 \quad \int \frac{x^3}{(a+b \sec(c+d\sqrt{x}))^2} dx$$

Optimal. Leaf size=3123

result too large to display

```
[Out] -2*I*b^3*x^(7/2)*ln(1+a*exp(I*(c+d*x^(1/2))))/(b-(-a^2+b^2)^(1/2))/a^2/(-a^2+b^2)^(3/2)/d-84*I*b^2*x^(5/2)*polylog(2,-a*exp(I*(c+d*x^(1/2))))/(b-I*(a^2-b^2)^(1/2))/a^2/(a^2-b^2)/d^3-84*I*b^2*x^(5/2)*polylog(2,-a*exp(I*(c+d*x^(1/2))))/(b+I*(a^2-b^2)^(1/2))/a^2/(a^2-b^2)/d^3-84*I*b^3*x^(5/2)*polylog(3,-a*exp(I*(c+d*x^(1/2))))/(b-(-a^2+b^2)^(1/2))/a^2/(-a^2+b^2)^(3/2)/d^3-1680*I*b^3*x^(3/2)*polylog(5,-a*exp(I*(c+d*x^(1/2))))/(b+(-a^2+b^2)^(1/2))/a^2/(-a^2+b^2)^(3/2)/d^5-4*I*b*x^(7/2)*ln(1+a*exp(I*(c+d*x^(1/2))))/(b+(-a^2+b^2)^(1/2))/a^2/d/(-a^2+b^2)^(1/2)-168*I*b*x^(5/2)*polylog(3,-a*exp(I*(c+d*x^(1/2))))/(b+(-a^2+b^2)^(1/2))/a^2/d^3/(-a^2+b^2)^(1/2)-3360*I*b*x^(3/2)*polylog(5,-a*exp(I*(c+d*x^(1/2))))/(b-(-a^2+b^2)^(1/2))/a^2/d^5/(-a^2+b^2)^(1/2)-10080*I*b^2*polylog(6,-a*exp(I*(c+d*x^(1/2))))/(b-I*(a^2-b^2)^(1/2))*x^(1/2)/a^2/(a^2-b^2)/d^7-10080*I*b^2*polylog(6,-a*exp(I*(c+d*x^(1/2))))/(b+I*(a^2-b^2)^(1/2))*x^(1/2)/a^2/(a^2-b^2)/d^7-10080*I*b^3*polylog(7,-a*exp(I*(c+d*x^(1/2))))/(b-(-a^2+b^2)^(1/2))*x^(1/2)/a^2/(-a^2+b^2)^(3/2)/d^7-20160*I*b*polylog(7,-a*exp(I*(c+d*x^(1/2))))/(b+(-a^2+b^2)^(1/2))*x^(1/2)/a^2/d^7/(-a^2+b^2)^(1/2)+2*b^2*x^(7/2)*sin(c+d*x^(1/2))/a/(a^2-b^2)/d/(b+a*cos(c+d*x^(1/2)))+84*I*b^3*x^(5/2)*polylog(3,-a*exp(I*(c+d*x^(1/2))))/(b+(-a^2+b^2)^(1/2))/a^2/(-a^2+b^2)^(3/2)/d^3+1680*I*b^2*x^(3/2)*polylog(4,-a*exp(I*(c+d*x^(1/2))))/(b-I*(a^2-b^2)^(1/2))/a^2/(a^2-b^2)/d^5+1680*I*b^2*x^(3/2)*polylog(4,-a*exp(I*(c+d*x^(1/2))))/(b+I*(a^2-b^2)^(1/2))/a^2/(a^2-b^2)/d^5+1680*I*b^3*x^(3/2)*polylog(5,-a*exp(I*(c+d*x^(1/2))))/(b-(-a^2+b^2)^(1/2))/a^2/(-a^2+b^2)^(3/2)/d^5+4*I*b*x^(7/2)*ln(1+a*exp(I*(c+d*x^(1/2))))/(b-(-a^2+b^2)^(1/2))/a^2/d/(-a^2+b^2)^(1/2)-2*I*b^2*x^(7/2)/a^2/(a^2-b^2)/d+28*b*x^3*polylog(2,-a*exp(I*(c+d*x^(1/2))))/(b-(-a^2+b^2)^(1/2))/a^2/d^2/(-a^2+b^2)^(1/2)-28*b*x^3*polylog(2,-a*exp(I*(c+d*x^(1/2))))/(b+(-a^2+b^2)^(1/2))/a^2/d^2/(-a^2+b^2)^(1/2)-840*b*x^2*polylog(4,-a*exp(I*(c+d*x^(1/2))))/(b-(-a^2+b^2)^(1/2))/a^2/d^4/(-a^2+b^2)^(1/2)+840*b*x^2*polylog(4,-a*exp(I*(c+d*x^(1/2))))/(b+(-a^2+b^2)^(1/2))/a^2/d^4/(-a^2+b^2)^(1/2)+10080*b*x*polylog(6,-a*exp(I*(c+d*x^(1/2))))/(b-(-a^2+b^2)^(1/2))/a^2/d^6/(-a^2+b^2)^(1/2)-10080*b*x*polylog(6,-a*exp(I*(c+d*x^(1/2))))/(b+(-a^2+b^2)^(1/2))/a^2/d^6/(-a^2+b^2)^(1/2)+14*b^2*x^3*ln(1+a*exp(I*(c+d*x^(1/2))))/(b-I*(a^2-b^2)^(1/2))/a^2/(a^2-b^2)/d^2+14*b^2*x^3*ln(1+a*exp(I*(c+d*x^(1/2))))/(b+I*(a^2-b^2)^(1/2))/a^2/(a^2-b^2)/d^2-14*b^3*x^3*polylog(2,-a*exp(I*(c+d*x^(1/2))))/(b-(-a^2+b^2)^(1/2))/a^2/(-a^2+b^2)^(3/2)/d^2+14*b^3*x^3*polylog(2,-a*exp(I*(c+d*x^(1/2))))/(b+(-a^2+b^2)^(1/2))/a^2/(-a^2+b^2)^(3/2)/d^2+420*b^2*x^2*polylog(3,-a*exp(I*(c+d*x^(1/2))))/(b-I*(a^2-b^2)^(1/2))/a^2/(a^2-b^2)/d^4+420*b^2*x^2*polylog(3,-a*exp(I*(c+d*x^(1/2))))/(b+I*(a^2-b^2)^(1/2))/a^2/(a^2-b^2)/d^4+420*b^3*x^2*polylog(4,-a*exp(I*(c+d*x^(1/2))))/(b-(-a^2+b^2)^(1/2))/a^2
```

$$\begin{aligned} & /(-a^2+b^2)^{(3/2)}/d^4-420*b^3*x^2*polylog(4,-a*\exp(I*(c+d*x^{(1/2)})))/(b+(-a^2+b^2)^{(1/2)})/a^2/(-a^2+b^2)^{(3/2)}/d^4-5040*b^2*x*polylog(5,-a*\exp(I*(c+d*x^{(1/2)})))/(b-I*(a^2-b^2)^{(1/2)})/a^2/(a^2-b^2)/d^6-5040*b^2*x*polylog(5,-a*\exp(I*(c+d*x^{(1/2)})))/(b+I*(a^2-b^2)^{(1/2)})/a^2/(a^2-b^2)/d^6-5040*b^3*x*polylog(6,-a*\exp(I*(c+d*x^{(1/2)})))/(b+(-a^2+b^2)^{(1/2)})/a^2/(-a^2+b^2)^{(3/2)}/d^6+5040*b^3*x*polylog(6,-a*\exp(I*(c+d*x^{(1/2)})))/(b+(-a^2+b^2)^{(1/2)})/a^2/(-a^2+b^2)^{(3/2)}/d^6+168*I*b*x^{(5/2)*polylog(3,-a*\exp(I*(c+d*x^{(1/2)})))/(b+(-a^2+b^2)^{(1/2)})/a^2/d^3/(-a^2+b^2)^{(1/2)+3360*I*b*x^{(3/2)*polylog(5,-a*\exp(I*(c+d*x^{(1/2)})))/(b+(-a^2+b^2)^{(1/2)})/a^2/d^5/(-a^2+b^2)^{(1/2)+10080*I*b^3*polylog(7,-a*\exp(I*(c+d*x^{(1/2)})))/(b+(-a^2+b^2)^{(1/2)})/a^2/d^7/(-a^2+b^2)^{(1/2)+2*I*b^3*x^{(7/2)*ln(1+a*\exp(I*(c+d*x^{(1/2)})))/(b+(-a^2+b^2)^{(1/2)})/a^2/(-a^2+b^2)^{(3/2)}/d^7+20160*I*b*polylog(7,-a*\exp(I*(c+d*x^{(1/2)})))/(b+(-a^2+b^2)^{(1/2)})/a^2/d^7/(-a^2+b^2)^{(1/2)+2*I*b^3*x^{(7/2)*ln(1+a*\exp(I*(c+d*x^{(1/2)})))/(b+(-a^2+b^2)^{(1/2)})/a^2/(-a^2+b^2)^{(3/2)}/d+1/4*x^4/a^2+10080*b^2*polylog(7,-a*\exp(I*(c+d*x^{(1/2)})))/(b-I*(a^2-b^2)^{(1/2)})/a^2/(a^2-b^2)/d^8+10080*b^2*polylog(7,-a*\exp(I*(c+d*x^{(1/2)})))/(b+I*(a^2-b^2)^{(1/2)})/a^2/(a^2-b^2)/d^8+10080*b^3*polylog(8,-a*\exp(I*(c+d*x^{(1/2)})))/(b+(-a^2+b^2)^{(1/2)})/a^2/(-a^2+b^2)^{(3/2)}/d^8-10080*b^3*polylog(8,-a*\exp(I*(c+d*x^{(1/2)})))/(b+(-a^2+b^2)^{(1/2)})/a^2/(-a^2+b^2)^{(3/2)}/d^8-20160*b*polylog(8,-a*\exp(I*(c+d*x^{(1/2)})))/(b+(-a^2+b^2)^{(1/2)})/a^2/d^8/(-a^2+b^2)^{(1/2)+20160*b*polylog(8,-a*\exp(I*(c+d*x^{(1/2)})))/(b+(-a^2+b^2)^{(1/2)})/a^2/d^8/(-a^2+b^2)^{(1/2)} \end{aligned}$$

**Rubi [A]** time = 4.12, antiderivative size = 3123, normalized size of antiderivative = 1.00, number of steps used = 61, number of rules used = 11, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$ , Rules used = {4204, 4191, 3324, 3321, 2264, 2190, 2531, 6609, 2282, 6589, 4522}

result too large to display

Antiderivative was successfully verified.

[In] Int[x^3/(a + b\*Sec[c + d\*Sqrt[x]])^2,x]

[Out] 
$$\begin{aligned} & ((-2*I)*b^2*x^{(7/2)})/(a^2*(a^2 - b^2)*d) + x^4/(4*a^2) + (14*b^2*x^3*Log[1 + (a*E^{(I*(c + d*Sqrt[x]))})/(b - I*Sqrt[a^2 - b^2])])/(a^2*(a^2 - b^2)*d^2) \\ & + (14*b^2*x^3*Log[1 + (a*E^{(I*(c + d*Sqrt[x]))})/(b + I*Sqrt[a^2 - b^2])])/(a^2*(a^2 - b^2)*d^2) - ((2*I)*b^3*x^{(7/2)*Log[1 + (a*E^{(I*(c + d*Sqrt[x]))})/(b - Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^{(3/2)*d) + ((4*I)*b*x^{(7/2)*Log[1 + (a*E^{(I*(c + d*Sqrt[x]))})/(b - Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d) + ((2*I)*b^3*x^{(7/2)*Log[1 + (a*E^{(I*(c + d*Sqrt[x]))})/(b + Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^{(3/2)*d) - ((4*I)*b*x^{(7/2)*Log[1 + (a*E^{(I*(c + d*Sqrt[x]))})/(b + Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d) - ((84*I)*b^2*x^{(5/2)*PolyLog[2, -((a*E^{(I*(c + d*Sqrt[x]))})/(b - I*Sqrt[a^2 - b^2])])])/(a^2*(a^2 - b^2)*d^3) - ((84*I)*b^2*x^{(5/2)*PolyLog[2, -((a*E^{(I*(c + d*Sqrt[x]))})/(b + I*Sqrt[a^2 - b^2])])])/(a^2*(a^2 - b^2)*d^3) - (14*b^3*x^3*PolyLog[2, -((a*E^{(I*(c + d*Sqrt[x]))})/(b - Sqrt[-a^2 + b^2])])])/(a^2*(-a^2 + b^2)^{(3/2)*d^2) + (28*b*x^3*PolyLog[2, -((a*E^{(I*(c + d*Sqrt[x]))})/(b - Sqrt[-a^2 + b^2])])])/(a^2*Sqrt[-a^2 + b^2]*d^2) + (14*b^3*x^3*PolyLog[2, - \end{aligned}$$

$$\begin{aligned}
& \left( \frac{(a^2 E^{I(c + d\sqrt{x})})}{(b + \sqrt{-a^2 + b^2})} \right) / (a^2 (-a^2 + b^2)^{3/2} d^2) - (28 b x^3 \text{PolyLog}[2, -\left( \frac{(a^2 E^{I(c + d\sqrt{x})})}{(b + \sqrt{-a^2 + b^2})} \right)] / (a^2 \sqrt{-a^2 + b^2} d^2) + (420 b^2 x^2 \text{PolyLog}[3, -\left( \frac{(a^2 E^{I(c + d\sqrt{x})})}{(b - I\sqrt{a^2 - b^2})} \right)] / (a^2 (a^2 - b^2) d^4) + (420 b^2 x^2 \text{PolyLog}[3, -\left( \frac{(a^2 E^{I(c + d\sqrt{x})})}{(b + I\sqrt{a^2 - b^2})} \right)] / (a^2 (a^2 - b^2) d^4) - ((84 I) b^3 x^{5/2} \text{PolyLog}[3, -\left( \frac{(a^2 E^{I(c + d\sqrt{x})})}{(b - \sqrt{-a^2 + b^2})} \right)] / (a^2 (-a^2 + b^2)^{3/2} d^3) + ((168 I) b x^{5/2} \text{PolyLog}[3, -\left( \frac{(a^2 E^{I(c + d\sqrt{x})})}{(b - \sqrt{-a^2 + b^2})} \right)] / (a^2 \sqrt{-a^2 + b^2} d^3) + ((84 I) b^3 x^{5/2} \text{PolyLog}[3, -\left( \frac{(a^2 E^{I(c + d\sqrt{x})})}{(b + \sqrt{-a^2 + b^2})} \right)] / (a^2 \sqrt{-a^2 + b^2} d^3) + ((1680 I) b^2 x^{3/2} \text{PolyLog}[4, -\left( \frac{(a^2 E^{I(c + d\sqrt{x})})}{(b - I\sqrt{a^2 - b^2})} \right)] / (a^2 (a^2 - b^2) d^5) + ((1680 I) b^2 x^{3/2} \text{PolyLog}[4, -\left( \frac{(a^2 E^{I(c + d\sqrt{x})})}{(b + I\sqrt{a^2 - b^2})} \right)] / (a^2 (a^2 - b^2) d^5) + (420 b^3 x^2 \text{PolyLog}[4, -\left( \frac{(a^2 E^{I(c + d\sqrt{x})})}{(b - \sqrt{-a^2 + b^2})} \right)] / (a^2 (-a^2 + b^2)^{3/2} d^4) - (840 b x^2 \text{PolyLog}[4, -\left( \frac{(a^2 E^{I(c + d\sqrt{x})})}{(b - \sqrt{-a^2 + b^2})} \right)] / (a^2 \sqrt{-a^2 + b^2} d^4) - (420 b^3 x^2 \text{PolyLog}[4, -\left( \frac{(a^2 E^{I(c + d\sqrt{x})})}{(b + \sqrt{-a^2 + b^2})} \right)] / (a^2 (-a^2 + b^2)^{3/2} d^4) + (840 b x^2 \text{PolyLog}[4, -\left( \frac{(a^2 E^{I(c + d\sqrt{x})})}{(b + \sqrt{-a^2 + b^2})} \right)] / (a^2 \sqrt{-a^2 + b^2} d^4) - (5040 b^2 x \text{PolyLog}[5, -\left( \frac{(a^2 E^{I(c + d\sqrt{x})})}{(b - I\sqrt{a^2 - b^2})} \right)] / (a^2 (a^2 - b^2) d^6) - (5040 b^2 x \text{PolyLog}[5, -\left( \frac{(a^2 E^{I(c + d\sqrt{x})})}{(b + I\sqrt{a^2 - b^2})} \right)] / (a^2 (a^2 - b^2) d^6) + ((1680 I) b^3 x^{3/2} \text{PolyLog}[5, -\left( \frac{(a^2 E^{I(c + d\sqrt{x})})}{(b - \sqrt{-a^2 + b^2})} \right)] / (a^2 (-a^2 + b^2)^{3/2} d^5) - ((3360 I) b x^{3/2} \text{PolyLog}[5, -\left( \frac{(a^2 E^{I(c + d\sqrt{x})})}{(b - \sqrt{-a^2 + b^2})} \right)] / (a^2 \sqrt{-a^2 + b^2} d^5) - ((1680 I) b^3 x^{3/2} \text{PolyLog}[5, -\left( \frac{(a^2 E^{I(c + d\sqrt{x})})}{(b + \sqrt{-a^2 + b^2})} \right)] / (a^2 (-a^2 + b^2)^{3/2} d^5) + ((3360 I) b x^{3/2} \text{PolyLog}[5, -\left( \frac{(a^2 E^{I(c + d\sqrt{x})})}{(b + \sqrt{-a^2 + b^2})} \right)] / (a^2 \sqrt{-a^2 + b^2} d^5) - ((10080 I) b^2 \sqrt{x} \text{PolyLog}[6, -\left( \frac{(a^2 E^{I(c + d\sqrt{x})})}{(b - I\sqrt{a^2 - b^2})} \right)] / (a^2 (a^2 - b^2) d^7) - ((10080 I) b^2 \sqrt{x} \text{PolyLog}[6, -\left( \frac{(a^2 E^{I(c + d\sqrt{x})})}{(b + I\sqrt{a^2 - b^2})} \right)] / (a^2 (a^2 - b^2) d^7) - (5040 b^3 x \text{PolyLog}[6, -\left( \frac{(a^2 E^{I(c + d\sqrt{x})})}{(b - \sqrt{-a^2 + b^2})} \right)] / (a^2 (-a^2 + b^2)^{3/2} d^6) + (10080 b x \text{PolyLog}[6, -\left( \frac{(a^2 E^{I(c + d\sqrt{x})})}{(b - \sqrt{-a^2 + b^2})} \right)] / (a^2 \sqrt{-a^2 + b^2} d^6) + (5040 b^3 x \text{PolyLog}[6, -\left( \frac{(a^2 E^{I(c + d\sqrt{x})})}{(b + \sqrt{-a^2 + b^2})} \right)] / (a^2 (-a^2 + b^2)^{3/2} d^6) - (10080 b x \text{PolyLog}[6, -\left( \frac{(a^2 E^{I(c + d\sqrt{x})})}{(b + \sqrt{-a^2 + b^2})} \right)] / (a^2 \sqrt{-a^2 + b^2} d^6) + (10080 b^2 \text{PolyLog}[7, -\left( \frac{(a^2 E^{I(c + d\sqrt{x})})}{(b - I\sqrt{a^2 - b^2})} \right)] / (a^2 (a^2 - b^2) d^8) + (10080 b^2 \text{PolyLog}[7, -\left( \frac{(a^2 E^{I(c + d\sqrt{x})})}{(b + I\sqrt{a^2 - b^2})} \right)] / (a^2 (a^2 - b^2) d^8) - ((10080 I) b^3 \sqrt{x} \text{PolyLog}[7, -\left( \frac{(a^2 E^{I(c + d\sqrt{x})})}{(b - \sqrt{-a^2 + b^2})} \right)] / (a^2 (-a^2 + b^2)^{3/2} d^7) + ((20160 I) b \sqrt{x} \text{PolyLog}[7, -\left( \frac{(a^2 E^{I(c + d\sqrt{x})})}{(b - \sqrt{-a^2 + b^2})} \right)] / (a^2 \sqrt{-a^2 + b^2} d^7) + ((10080 I) b^3 \sqrt{x} \text{PolyLog}[7, -\left( \frac{(a^2 E^{I(c + d\sqrt{x})})}{(b + \sqrt{-a^2 + b^2})} \right)] / (a^2 (-a^2 + b^2)^{3/2} d^7)
\end{aligned}$$

) - ((20160\*I)\*b\*Sqrt[x]\*PolyLog[7, -((a\*E^(I\*(c + d\*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))]/(a^2\*Sqrt[-a^2 + b^2]\*d^7) + (10080\*b^3\*PolyLog[8, -((a\*E^(I\*(c + d\*Sqrt[x])))/(b - Sqrt[-a^2 + b^2]))]/(a^2\*(-a^2 + b^2)^(3/2)\*d^8) - (20160\*b\*PolyLog[8, -((a\*E^(I\*(c + d\*Sqrt[x])))/(b - Sqrt[-a^2 + b^2]))]/(a^2\*Sqrt[-a^2 + b^2]\*d^8) - (10080\*b^3\*PolyLog[8, -((a\*E^(I\*(c + d\*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))]/(a^2\*(-a^2 + b^2)^(3/2)\*d^8) + (20160\*b\*PolyLog[8, -((a\*E^(I\*(c + d\*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))]/(a^2\*Sqrt[-a^2 + b^2]\*d^8) + (2\*b^2\*x^(7/2)\*Sin[c + d\*Sqrt[x]])/(a\*(a^2 - b^2)\*d\*(b + a\*Cos[c + d\*Sqrt[x]]))

### Rule 2190

Int[(((F\_)^((g\_)\*(e\_) + (f\_)\*(x\_)))^(n\_))\*((c\_) + (d\_)\*(x\_))^(m\_)]/((a\_) + (b\_)\*((F\_)^((g\_)\*(e\_) + (f\_)\*(x\_)))^(n\_)), x\_Symbol] := Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a]/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2264

Int[((F\_)^(u\_)\*((f\_) + (g\_)\*(x\_))^(m\_)]/((a\_) + (b\_)\*(F\_)^(u\_) + (c\_)\*(F\_)^(v\_)), x\_Symbol] := With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[(2\*c)/q, Int[(f + g\*x)^m\*F^u/(b - q + 2\*c\*F^u), x], x] - Dist[(2\*c)/q, Int[(f + g\*x)^m\*F^u/(b + q + 2\*c\*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2\*u] && LinearQ[u, x] && NeQ[b^2 - 4\*a\*c, 0] && IGtQ[m, 0]

### Rule 2282

Int[u\_, x\_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_)\*((a\_) + (b\_)\*x))\* (F\_)[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

### Rule 2531

Int[Log[1 + (e\_)\*((F\_)^((c\_)\*((a\_) + (b\_)\*(x\_)))^(n\_))]\*((f\_) + (g\_)\*(x\_))^(m\_), x\_Symbol] := -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x))))^n])/ (b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m - 1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

### Rule 3321

Int[((c\_) + (d\_)\*(x\_))^(m\_)]/((a\_) + (b\_)\*sin[(e\_) + Pi\*(k\_) + (f\_)\*(x\_)]), x\_Symbol] := Dist[2, Int[(c + d\*x)^m\*E^(I\*Pi\*(k - 1/2))\*E^(I\*(e + f

```
*x)))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(
e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

### Rule 3324

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2, x_
Symbol] := Simp[(b*(c + d*x)^m*Cos[e + f*x])/(f*(a^2 - b^2)*(a + b*Sin[e +
f*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
x] - Dist[(b*d*m)/(f*(a^2 - b^2)), Int[((c + d*x)^(m - 1)*Cos[e + f*x])/(a
+ b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

### Rule 4191

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x]^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

### Rule 4204

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

### Rule 4522

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)]/(Cos[(c_.) + (d_.)
*(x_)]*(b_.) + (a_.), x_Symbol] := Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1))
, x] + (Int[((e + f*x)^m*E^(I*(c + d*x)))/(I*a - Rt[-a^2 + b^2, 2] + I*b*E^
(I*(c + d*x))), x] + Int[((e + f*x)^m*E^(I*(c + d*x)))/(I*a + Rt[-a^2 + b^2
, 2] + I*b*E^(I*(c + d*x))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
0] && NegQ[a^2 - b^2]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rule 6609

```

Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{(a + b \sec(c + d\sqrt{x}))^2} dx &= 2 \text{Subst} \left( \int \frac{x^7}{(a + b \sec(c + dx))^2} dx, x, \sqrt{x} \right) \\
&= 2 \text{Subst} \left( \int \left( \frac{x^7}{a^2} + \frac{b^2 x^7}{a^2(b + a \cos(c + dx))^2} - \frac{2bx^7}{a^2(b + a \cos(c + dx))} \right) dx, x, \sqrt{x} \right) \\
&= \frac{x^4}{4a^2} - \frac{(4b) \text{Subst} \left( \int \frac{x^7}{b+a \cos(c+dx)} dx, x, \sqrt{x} \right)}{a^2} + \frac{(2b^2) \text{Subst} \left( \int \frac{x^7}{(b+a \cos(c+dx))^2} dx, x, \sqrt{x} \right)}{a^2} \\
&= \frac{x^4}{4a^2} + \frac{2b^2 x^{7/2} \sin(c + d\sqrt{x})}{a(a^2 - b^2)d(b + a \cos(c + d\sqrt{x}))} - \frac{(8b) \text{Subst} \left( \int \frac{e^{i(c+dx)} x^7}{a+2be^{i(c+dx)}+ae^{2i(c+dx)}} dx, x, \sqrt{x} \right)}{a^2} \\
&= -\frac{2ib^2 x^{7/2}}{a^2(a^2 - b^2)d} + \frac{x^4}{4a^2} + \frac{2b^2 x^{7/2} \sin(c + d\sqrt{x})}{a(a^2 - b^2)d(b + a \cos(c + d\sqrt{x}))} - \frac{(4b^3) \text{Subst} \left( \int \frac{e^{i(c+dx)} x^7}{a^2+2be^{i(c+dx)}+ae^{2i(c+dx)}} dx, x, \sqrt{x} \right)}{a^2} \\
&= -\frac{2ib^2 x^{7/2}}{a^2(a^2 - b^2)d} + \frac{x^4}{4a^2} + \frac{14b^2 x^3 \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b-i\sqrt{a^2-b^2}} \right)}{a^2(a^2 - b^2)d^2} + \frac{14b^2 x^3 \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b+i\sqrt{a^2-b^2}} \right)}{a^2(a^2 - b^2)d^2} \\
&= -\frac{2ib^2 x^{7/2}}{a^2(a^2 - b^2)d} + \frac{x^4}{4a^2} + \frac{14b^2 x^3 \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b-i\sqrt{a^2-b^2}} \right)}{a^2(a^2 - b^2)d^2} + \frac{14b^2 x^3 \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b+i\sqrt{a^2-b^2}} \right)}{a^2(a^2 - b^2)d^2} \\
&= -\frac{2ib^2 x^{7/2}}{a^2(a^2 - b^2)d} + \frac{x^4}{4a^2} + \frac{14b^2 x^3 \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b-i\sqrt{a^2-b^2}} \right)}{a^2(a^2 - b^2)d^2} + \frac{14b^2 x^3 \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b+i\sqrt{a^2-b^2}} \right)}{a^2(a^2 - b^2)d^2} \\
&= -\frac{2ib^2 x^{7/2}}{a^2(a^2 - b^2)d} + \frac{x^4}{4a^2} + \frac{14b^2 x^3 \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b-i\sqrt{a^2-b^2}} \right)}{a^2(a^2 - b^2)d^2} + \frac{14b^2 x^3 \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b+i\sqrt{a^2-b^2}} \right)}{a^2(a^2 - b^2)d^2} \\
&= -\frac{2ib^2 x^{7/2}}{a^2(a^2 - b^2)d} + \frac{x^4}{4a^2} + \frac{14b^2 x^3 \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b-i\sqrt{a^2-b^2}} \right)}{a^2(a^2 - b^2)d^2} + \frac{14b^2 x^3 \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b+i\sqrt{a^2-b^2}} \right)}{a^2(a^2 - b^2)d^2} \\
&= -\frac{2ib^2 x^{7/2}}{a^2(a^2 - b^2)d} + \frac{x^4}{4a^2} + \frac{14b^2 x^3 \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b-i\sqrt{a^2-b^2}} \right)}{a^2(a^2 - b^2)d^2} + \frac{14b^2 x^3 \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b+i\sqrt{a^2-b^2}} \right)}{a^2(a^2 - b^2)d^2} \\
&= -\frac{2ib^2 x^{7/2}}{a^2(a^2 - b^2)d} + \frac{x^4}{4a^2} + \frac{14b^2 x^3 \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b-i\sqrt{a^2-b^2}} \right)}{a^2(a^2 - b^2)d^2} + \frac{14b^2 x^3 \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b+i\sqrt{a^2-b^2}} \right)}{a^2(a^2 - b^2)d^2} \\
&= -\frac{2ib^2 x^{7/2}}{a^2(a^2 - b^2)d} + \frac{x^4}{4a^2} + \frac{14b^2 x^3 \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b-i\sqrt{a^2-b^2}} \right)}{a^2(a^2 - b^2)d^2} + \frac{14b^2 x^3 \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b+i\sqrt{a^2-b^2}} \right)}{a^2(a^2 - b^2)d^2} \\
&= -\frac{2ib^2 x^{7/2}}{a^2(a^2 - b^2)d} + \frac{x^4}{4a^2} + \frac{14b^2 x^3 \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b-i\sqrt{a^2-b^2}} \right)}{a^2(a^2 - b^2)d^2} + \frac{14b^2 x^3 \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b+i\sqrt{a^2-b^2}} \right)}{a^2(a^2 - b^2)d^2} \\
&= -\frac{2ib^2 x^{7/2}}{a^2(a^2 - b^2)d} + \frac{x^4}{4a^2} + \frac{14b^2 x^3 \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b-i\sqrt{a^2-b^2}} \right)}{a^2(a^2 - b^2)d^2} + \frac{14b^2 x^3 \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b+i\sqrt{a^2-b^2}} \right)}{a^2(a^2 - b^2)d^2}
\end{aligned}$$



**Mathematica** [A] time = 14.88, size = 3702, normalized size = 1.19

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[x^3/(a + b\*Sec[c + d\*Sqrt[x]])^2,x]

[Out]  $(x^4(b + a\cos[c + d\sqrt{x}])^2\sec[c + d\sqrt{x}]^2)/(4a^2(a + b\sec[c + d\sqrt{x}])^2) + (2bE^{Ic})(b + a\cos[c + d\sqrt{x}])^2((-2I)bE^{Ic}x^{7/2} + ((1 + E^{(2I)c}))(7bd^6\sqrt{-a^2 + b^2}E^{(2I)c})x^3\log[1 + (aE^{I(2c + d\sqrt{x})})/(bE^{Ic} - \sqrt{-a^2 + b^2}E^{(2I)c})]) + (2I)a^2d^7E^{Ic}x^{7/2}\log[1 + (aE^{I(2c + d\sqrt{x})})/(bE^{Ic} - \sqrt{-a^2 + b^2}E^{(2I)c})] - Ib^2d^7E^{Ic}x^{7/2}\log[1 + (aE^{I(2c + d\sqrt{x})})/(bE^{Ic} - \sqrt{-a^2 + b^2}E^{(2I)c})] + 7bd^6\sqrt{-a^2 + b^2}E^{(2I)c}x^3\log[1 + (aE^{I(2c + d\sqrt{x})})/(bE^{Ic} + \sqrt{-a^2 + b^2}E^{(2I)c})] - (2I)a^2d^7E^{Ic}x^{7/2}\log[1 + (aE^{I(2c + d\sqrt{x})})/(bE^{Ic} + \sqrt{-a^2 + b^2}E^{(2I)c})] + Ib^2d^7E^{Ic}x^{7/2}\log[1 + (aE^{I(2c + d\sqrt{x})})/(bE^{Ic} + \sqrt{-a^2 + b^2}E^{(2I)c})] - 7d^5((6I)b\sqrt{-a^2 + b^2}E^{(2I)c} - 2a^2dE^{Ic}\sqrt{x} + b^2dE^{Ic}\sqrt{x})x^{5/2}\text{PolyLog}[2, -(aE^{I(2c + d\sqrt{x})})/(bE^{Ic} - \sqrt{-a^2 + b^2}E^{(2I)c})] + 7d^5((-6I)b\sqrt{-a^2 + b^2}E^{(2I)c} - 2a^2dE^{Ic}\sqrt{x} + b^2dE^{Ic}\sqrt{x})x^{5/2}\text{PolyLog}[2, -(aE^{I(2c + d\sqrt{x})})/(bE^{Ic} + \sqrt{-a^2 + b^2}E^{(2I)c})] + 210bd^4\sqrt{-a^2 + b^2}E^{(2I)c}x^2\text{PolyLog}[3, -(aE^{I(2c + d\sqrt{x})})/(bE^{Ic} - \sqrt{-a^2 + b^2}E^{(2I)c})] + (84I)a^2d^5E^{Ic}x^{5/2}\text{PolyLog}[3, -(aE^{I(2c + d\sqrt{x})})/(bE^{Ic} - \sqrt{-a^2 + b^2}E^{(2I)c})] - (42I)b^2d^5E^{Ic}x^{5/2}\text{PolyLog}[3, -(aE^{I(2c + d\sqrt{x})})/(bE^{Ic} - \sqrt{-a^2 + b^2}E^{(2I)c})] + 210bd^4\sqrt{-a^2 + b^2}E^{(2I)c}x^2\text{PolyLog}[3, -(aE^{I(2c + d\sqrt{x})})/(bE^{Ic} + \sqrt{-a^2 + b^2}E^{(2I)c})] - (84I)a^2d^5E^{Ic}x^{5/2}\text{PolyLog}[3, -(aE^{I(2c + d\sqrt{x})})/(bE^{Ic} + \sqrt{-a^2 + b^2}E^{(2I)c})] + (42I)b^2d^5E^{Ic}x^{5/2}\text{PolyLog}[3, -(aE^{I(2c + d\sqrt{x})})/(bE^{Ic} + \sqrt{-a^2 + b^2}E^{(2I)c})] + (840I)bd^3\sqrt{-a^2 + b^2}E^{(2I)c}x^{3/2}\text{PolyLog}[4, -(aE^{I(2c + d\sqrt{x})})/(bE^{Ic} - \sqrt{-a^2 + b^2}E^{(2I)c})] - 420a^2d^4E^{Ic}x^2\text{PolyLog}[4, -(aE^{I(2c + d\sqrt{x})})/(bE^{Ic} - \sqrt{-a^2 + b^2}E^{(2I)c})] + 210b^2d^4E^{Ic}x^2\text{PolyLog}[4, -(aE^{I(2c + d\sqrt{x})})/(bE^{Ic} - \sqrt{-a^2 + b^2}E^{(2I)c})] + (840I)bd^3\sqrt{-a^2 + b^2}E^{(2I)c}x^{3/2}\text{PolyLog}[4, -(aE^{I(2c + d\sqrt{x})})/(bE^{Ic} + \sqrt{-a^2 + b^2}E^{(2I)c})] + 420a^2d^4E^{Ic}x^2\text{PolyLog}[4, -(aE^{I(2c + d\sqrt{x})})/(bE^{Ic} + \sqrt{-a^2 + b^2}E^{(2I)c})] - 210b^2d^4E^{Ic}x^2\text{PolyLog}[4, -(aE^{I(2c + d\sqrt{x})})/(bE^{Ic} + \sqrt{-a^2 + b^2}E^{(2I)c})] - 2$

$$\begin{aligned}
& 520*b*d^2*Sqrt[(-a^2 + b^2)*E^((2*I)*c)]*x*PolyLog[5, -((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] - (1680*I)*a^2*d^3* \\
& E^(I*c)*x^(3/2)*PolyLog[5, -((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] + (840*I)*b^2*d^3*E^(I*c)*x^(3/2)*PolyLog[5, - \\
& ((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] - 2520*b*d^2*Sqrt[(-a^2 + b^2)*E^((2*I)*c)]*x*PolyLog[5, -((a*E^(I*(2*c + \\
& d*Sqrt[x])))/(b*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] + (1680*I)*a^2* \\
& d^3*E^(I*c)*x^(3/2)*PolyLog[5, -((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + S \\
& qrt[(-a^2 + b^2)*E^((2*I)*c)]))] - (840*I)*b^2*d^3*E^(I*c)*x^(3/2)*PolyLog[ \\
& 5, -((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c) \\
& ]))] - (5040*I)*b*d*Sqrt[(-a^2 + b^2)*E^((2*I)*c)]*Sqrt[x]*PolyLog[6, -((a* \\
& E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] + 5 \\
& 040*a^2*d^2*E^(I*c)*x*PolyLog[6, -((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) - \\
& Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] - 2520*b^2*d^2*E^(I*c)*x*PolyLog[6, -((a \\
& *E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] - \\
& (5040*I)*b*d*Sqrt[(-a^2 + b^2)*E^((2*I)*c)]*Sqrt[x]*PolyLog[6, -((a*E^(I*(2 \\
& *c + d*Sqrt[x])))/(b*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] - 5040*a^2 \\
& *d^2*E^(I*c)*x*PolyLog[6, -((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + Sqrt[( \\
& -a^2 + b^2)*E^((2*I)*c)]))] + 2520*b^2*d^2*E^(I*c)*x*PolyLog[6, -((a*E^(I*( \\
& 2*c + d*Sqrt[x])))/(b*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] + 5040*b* \\
& Sqrt[(-a^2 + b^2)*E^((2*I)*c)]*PolyLog[7, -((a*E^(I*(2*c + d*Sqrt[x])))/(b* \\
& E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] + (10080*I)*a^2*d*E^(I*c)*Sqrt[ \\
& x]*PolyLog[7, -((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) - Sqrt[(-a^2 + b^2)* \\
& E^((2*I)*c)]))] - (5040*I)*b^2*d*E^(I*c)*Sqrt[x]*PolyLog[7, -((a*E^(I*(2*c \\
& + d*Sqrt[x])))/(b*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] + 5040*b*Sqrt \\
& [(-a^2 + b^2)*E^((2*I)*c)]*PolyLog[7, -((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I \\
& *c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] - (10080*I)*a^2*d*E^(I*c)*Sqrt[x]*P \\
& olyLog[7, -((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + Sqrt[(-a^2 + b^2)*E^(( \\
& 2*I)*c)]))] + (5040*I)*b^2*d*E^(I*c)*Sqrt[x]*PolyLog[7, -((a*E^(I*(2*c + d* \\
& Sqrt[x])))/(b*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] - 10080*a^2*E^(I* \\
& c)*PolyLog[8, -((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) - Sqrt[(-a^2 + b^2)* \\
& E^((2*I)*c)]))] + 5040*b^2*E^(I*c)*PolyLog[8, -((a*E^(I*(2*c + d*Sqrt[x])) \\
& )/(b*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] + 10080*a^2*E^(I*c)*PolyLog \\
& [8, -((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c \\
& ]))] - 5040*b^2*E^(I*c)*PolyLog[8, -((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c \\
& ) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] / (d^7*E^(I*c)*Sqrt[(-a^2 + b^2)*E^(( \\
& 2*I)*c)])*Sec[c + d*Sqrt[x]]^2 / (a^2*(a^2 - b^2)*d*(1 + E^((2*I)*c))*(a + \\
& b*Sec[c + d*Sqrt[x]]^2) + (2*(b + a*Cos[c + d*Sqrt[x]])*Sec[c + d*Sqrt[x]] \\
& ^2*(b^3*x^(7/2)*Sin[c] - a*b^2*x^(7/2)*Sin[d*Sqrt[x]])) / (a^2*(-a + b)*(a + \\
& b)*d*(a + b*Sec[c + d*Sqrt[x]]^2*(Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2 \\
& ]))
\end{aligned}$$

**fricas** [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^3}{b^2 \sec(d\sqrt{x} + c)^2 + 2ab \sec(d\sqrt{x} + c) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+b\*sec(c+d\*x^(1/2)))^2,x, algorithm="fricas")

[Out] integral(x^3/(b^2\*sec(d\*sqrt(x) + c)^2 + 2\*a\*b\*sec(d\*sqrt(x) + c) + a^2), x)

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+b\*sec(c+d\*x^(1/2)))^2,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x);;OUTPUT:Evaluation time: 1.64Not invertible Error: Bad Argument Value

**maple** [F] time = 1.37, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a + b \sec(c + d\sqrt{x}))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(a+b\*sec(c+d\*x^(1/2)))^2,x)

[Out] int(x^3/(a+b\*sec(c+d\*x^(1/2)))^2,x)

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+b\*sec(c+d\*x^(1/2)))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* h

elp (example of legal syntax is 'assume(4\*a^2-4\*b^2>0)', see 'assume?' for more details)Is 4\*a^2-4\*b^2 positive or negative?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\left(a + \frac{b}{\cos(c+d\sqrt{x})}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(a + b/cos(c + d\*x^(1/2)))^2,x)

[Out] int(x^3/(a + b/cos(c + d\*x^(1/2)))^2, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a + b \sec(c + d\sqrt{x}))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3/(a+b\*sec(c+d\*x\*\*(1/2)))\*\*2,x)

[Out] Integral(x\*\*3/(a + b\*sec(c + d\*sqrt(x)))\*\*2, x)

$$3.47 \quad \int \frac{x^2}{(a+b \sec(c+d\sqrt{x}))^2} dx$$

Optimal. Leaf size=2323

result too large to display

```
[Out] 10*b^2*x^2*ln(1+a*exp(I*(c+d*x^(1/2)))/(b-I*(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)
/d^2+10*b^2*x^2*ln(1+a*exp(I*(c+d*x^(1/2)))/(b+I*(a^2-b^2)^(1/2)))/a^2/(a^2
-b^2)/d^2-10*b^3*x^2*polylog(2,-a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2))
)/a^2/(-a^2+b^2)^(3/2)/d^2+10*b^3*x^2*polylog(2,-a*exp(I*(c+d*x^(1/2)))/(b+
(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^2+120*b^2*x*polylog(3,-a*exp(I*(c
+d*x^(1/2)))/(b-I*(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^4+120*b^2*x*polylog(3,-
a*exp(I*(c+d*x^(1/2)))/(b+I*(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^4+120*b^3*x*p
olylog(4,-a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)
/d^4-120*b^3*x*polylog(4,-a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/
(-a^2+b^2)^(3/2)/d^4+20*b*x^2*polylog(2,-a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^
2)^(1/2)))/a^2/d^2/(-a^2+b^2)^(1/2)-20*b*x^2*polylog(2,-a*exp(I*(c+d*x^(1/2)
)))/(b+(-a^2+b^2)^(1/2)))/a^2/d^2/(-a^2+b^2)^(1/2)-240*b*x*polylog(4,-a*exp
(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/d^4/(-a^2+b^2)^(1/2)+240*b*x*po
lylog(4,-a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/d^4/(-a^2+b^2)^(1
/2)-4*I*b*x^(5/2)*ln(1+a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/d/(
-a^2+b^2)^(1/2)-80*I*b*x^(3/2)*polylog(3,-a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b
^2)^(1/2)))/a^2/d^3/(-a^2+b^2)^(1/2)-240*I*b^3*polylog(5,-a*exp(I*(c+d*x^(1
/2)))/(b+(-a^2+b^2)^(1/2)))*x^(1/2)/a^2/(-a^2+b^2)^(3/2)/d^5-480*I*b*polylo
g(5,-a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))*x^(1/2)/a^2/d^5/(-a^2+b^2
)^(1/2)+2*b^2*x^(5/2)*sin(c+d*x^(1/2))/a/(a^2-b^2)/d/(b+a*cos(c+d*x^(1/2)))
-2*I*b^3*x^(5/2)*ln(1+a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^
2+b^2)^(3/2)/d-40*I*b^2*x^(3/2)*polylog(2,-a*exp(I*(c+d*x^(1/2)))/(b-I*(a^2
-b^2)^(1/2)))/a^2/(a^2-b^2)/d^3-40*I*b^2*x^(3/2)*polylog(2,-a*exp(I*(c+d*x^
(1/2)))/(b+I*(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^3-40*I*b^3*x^(3/2)*polylog(3
,-a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^3+240
*I*b^2*polylog(4,-a*exp(I*(c+d*x^(1/2)))/(b-I*(a^2-b^2)^(1/2)))*x^(1/2)/a^2
/(a^2-b^2)/d^5+240*I*b^2*polylog(4,-a*exp(I*(c+d*x^(1/2)))/(b+I*(a^2-b^2)^(
1/2)))*x^(1/2)/a^2/(a^2-b^2)/d^5+240*I*b^3*polylog(5,-a*exp(I*(c+d*x^(1/2))
)/(b-(-a^2+b^2)^(1/2)))*x^(1/2)/a^2/(-a^2+b^2)^(3/2)/d^5+480*I*b*polylog(5,
-a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))*x^(1/2)/a^2/d^5/(-a^2+b^2)^(1
/2)+2*I*b^3*x^(5/2)*ln(1+a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/(
-a^2+b^2)^(3/2)/d+40*I*b^3*x^(3/2)*polylog(3,-a*exp(I*(c+d*x^(1/2)))/(b+(-a
^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^3+4*I*b*x^(5/2)*ln(1+a*exp(I*(c+d*x^
(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/d/(-a^2+b^2)^(1/2)+80*I*b*x^(3/2)*polylog
(3,-a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/d^3/(-a^2+b^2)^(1/2)-2
*I*b^2*x^(5/2)/a^2/(a^2-b^2)/d+1/3*x^3/a^2-240*b^2*polylog(5,-a*exp(I*(c+d*
x^(1/2)))/(b-I*(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^6-240*b^2*polylog(5,-a*exp
(I*(c+d*x^(1/2)))/(b+I*(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^6-240*b^3*polylog(
```

$$6, -a \exp(I(c+d*x^{1/2})) / (b - (-a^2+b^2)^{1/2}) / a^2 / (-a^2+b^2)^{3/2} / d^6 + 240*b^3 \text{polylog}(6, -a \exp(I(c+d*x^{1/2})) / (b + (-a^2+b^2)^{1/2})) / a^2 / (-a^2+b^2)^{3/2} / d^6 + 480*b \text{polylog}(6, -a \exp(I(c+d*x^{1/2})) / (b - (-a^2+b^2)^{1/2})) / a^2 / d^6 / (-a^2+b^2)^{1/2} - 480*b \text{polylog}(6, -a \exp(I(c+d*x^{1/2})) / (b + (-a^2+b^2)^{1/2})) / a^2 / d^6 / (-a^2+b^2)^{1/2}$$

**Rubi [A]** time = 3.23, antiderivative size = 2323, normalized size of antiderivative = 1.00, number of steps used = 49, number of rules used = 11, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$ , Rules used = {4204, 4191, 3324, 3321, 2264, 2190, 2531, 6609, 2282, 6589, 4522}

result too large to display

Antiderivative was successfully verified.

[In] Int[x^2/(a + b\*Sec[c + d\*Sqrt[x]])^2,x]

[Out] 
$$\begin{aligned} &((-2*I)*b^2*x^{5/2})/(a^2*(a^2 - b^2)*d) + x^3/(3*a^2) + (10*b^2*x^2*\text{Log}[1 \\ &+ (a*E^{(I*(c + d*Sqrt[x]))})/(b - I*Sqrt[a^2 - b^2])])/(a^2*(a^2 - b^2)*d^2) \\ &+ (10*b^2*x^2*\text{Log}[1 + (a*E^{(I*(c + d*Sqrt[x]))})/(b + I*Sqrt[a^2 - b^2])]) / \\ &(a^2*(a^2 - b^2)*d^2) - ((2*I)*b^3*x^{5/2}*\text{Log}[1 + (a*E^{(I*(c + d*Sqrt[x]))}) \\ &)/(b - \text{Sqrt}[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^{3/2}*d) + ((4*I)*b*x^{5/2}* \text{Lo} \\ &\text{g}[1 + (a*E^{(I*(c + d*Sqrt[x]))})/(b - \text{Sqrt}[-a^2 + b^2])])/(a^2*\text{Sqrt}[-a^2 + b \\ &^2]*d) + ((2*I)*b^3*x^{5/2}*\text{Log}[1 + (a*E^{(I*(c + d*Sqrt[x]))})/(b + \text{Sqrt}[-a^ \\ &2 + b^2])])/(a^2*(-a^2 + b^2)^{3/2}*d) - ((4*I)*b*x^{5/2}*\text{Log}[1 + (a*E^{(I*(c \\ &+ d*Sqrt[x]))})/(b + \text{Sqrt}[-a^2 + b^2])])/(a^2*\text{Sqrt}[-a^2 + b^2]*d) - ((40*I \\ &)*b^2*x^{3/2}*\text{PolyLog}[2, -(a*E^{(I*(c + d*Sqrt[x]))})/(b - I*Sqrt[a^2 - b^2] \\ &)])/(a^2*(a^2 - b^2)*d^3) - ((40*I)*b^2*x^{3/2}*\text{PolyLog}[2, -(a*E^{(I*(c + \\ &d*Sqrt[x]))})/(b + I*Sqrt[a^2 - b^2])])/(a^2*(a^2 - b^2)*d^3) - (10*b^3*x^2 \\ &*\text{PolyLog}[2, -(a*E^{(I*(c + d*Sqrt[x]))})/(b - \text{Sqrt}[-a^2 + b^2])])/(a^2*(-a^ \\ &2 + b^2)^{3/2}*d^2) + (20*b*x^2*\text{PolyLog}[2, -(a*E^{(I*(c + d*Sqrt[x]))})/(b - \\ &\text{Sqrt}[-a^2 + b^2])])/(a^2*\text{Sqrt}[-a^2 + b^2]*d^2) + (10*b^3*x^2*\text{PolyLog}[2, - \\ &((a*E^{(I*(c + d*Sqrt[x]))})/(b + \text{Sqrt}[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^{3/2} \\ &)*d^2) - (20*b*x^2*\text{PolyLog}[2, -(a*E^{(I*(c + d*Sqrt[x]))})/(b + \text{Sqrt}[-a^2 + \\ &b^2])])/(a^2*\text{Sqrt}[-a^2 + b^2]*d^2) + (120*b^2*x*\text{PolyLog}[3, -(a*E^{(I*(c + \\ &d*Sqrt[x]))})/(b - I*Sqrt[a^2 - b^2])])/(a^2*(a^2 - b^2)*d^4) + (120*b^2*x* \\ &\text{PolyLog}[3, -(a*E^{(I*(c + d*Sqrt[x]))})/(b + I*Sqrt[a^2 - b^2])])/(a^2*(a^2 \\ &- b^2)*d^4) - ((40*I)*b^3*x^{3/2}*\text{PolyLog}[3, -(a*E^{(I*(c + d*Sqrt[x]))})/( \\ &b - \text{Sqrt}[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^{3/2}*d^3) + ((80*I)*b*x^{3/2}* \text{P} \\ &\text{olyLog}[3, -(a*E^{(I*(c + d*Sqrt[x]))})/(b - \text{Sqrt}[-a^2 + b^2])])/(a^2*\text{Sqr} \\ &t[-a^2 + b^2]*d^3) + ((40*I)*b^3*x^{3/2}*\text{PolyLog}[3, -(a*E^{(I*(c + d*Sqrt[x]))}) \\ &)/(b + \text{Sqrt}[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^{3/2}*d^3) - ((80*I)*b*x^{3/2} \\ &)*\text{PolyLog}[3, -(a*E^{(I*(c + d*Sqrt[x]))})/(b + \text{Sqrt}[-a^2 + b^2])])/(a^2*\text{Sqr} \\ &t[-a^2 + b^2]*d^3) + ((240*I)*b^2*\text{Sqrt}[x]*\text{PolyLog}[4, -(a*E^{(I*(c + d*Sqrt[ \\ &x]))})/(b - I*Sqrt[a^2 - b^2])])/(a^2*(a^2 - b^2)*d^5) + ((240*I)*b^2*\text{Sqrt}[ \\ &x]*\text{PolyLog}[4, -(a*E^{(I*(c + d*Sqrt[x]))})/(b + I*Sqrt[a^2 - b^2])])/(a^2*( \\ &a^2 - b^2)*d^5) + (120*b^3*x*\text{PolyLog}[4, -(a*E^{(I*(c + d*Sqrt[x]))})/(b - \text{Sq} \end{aligned}$$

```

rt[-a^2 + b^2]))]/(a^2*(-a^2 + b^2)^(3/2)*d^4) - (240*b*x*PolyLog[4, -((a*
E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2]))]/(a^2*Sqrt[-a^2 + b^2]*d^4)
- (120*b^3*x*PolyLog[4, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])
))]/(a^2*(-a^2 + b^2)^(3/2)*d^4) + (240*b*x*PolyLog[4, -((a*E^(I*(c + d*Sqr
t[x])))/(b + Sqrt[-a^2 + b^2]))]/(a^2*Sqrt[-a^2 + b^2]*d^4) - (240*b^2*Pol
yLog[5, -((a*E^(I*(c + d*Sqrt[x])))/(b - I*Sqrt[a^2 - b^2]))]/(a^2*(a^2 -
b^2)*d^6) - (240*b^2*PolyLog[5, -((a*E^(I*(c + d*Sqrt[x])))/(b + I*Sqrt[a^2
- b^2]))]/(a^2*(a^2 - b^2)*d^6) + ((240*I)*b^3*Sqrt[x]*PolyLog[5, -((a*E^
(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2]))]/(a^2*(-a^2 + b^2)^(3/2)*d^5)
- ((480*I)*b*Sqrt[x]*PolyLog[5, -((a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2
+ b^2]))]/(a^2*Sqrt[-a^2 + b^2]*d^5) - ((240*I)*b^3*Sqrt[x]*PolyLog[5, -(
(a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))]/(a^2*(-a^2 + b^2)^(3/2)
*d^5) + ((480*I)*b*Sqrt[x]*PolyLog[5, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt
[-a^2 + b^2]))]/(a^2*Sqrt[-a^2 + b^2]*d^5) - (240*b^3*PolyLog[6, -((a*E^(I
*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2]))]/(a^2*(-a^2 + b^2)^(3/2)*d^6) +
(480*b*PolyLog[6, -((a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2]))]/(a
^2*Sqrt[-a^2 + b^2]*d^6) + (240*b^3*PolyLog[6, -((a*E^(I*(c + d*Sqrt[x])))/
(b + Sqrt[-a^2 + b^2]))]/(a^2*(-a^2 + b^2)^(3/2)*d^6) - (480*b*PolyLog[6,
-((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))]/(a^2*Sqrt[-a^2 + b^2]
*d^6) + (2*b^2*x^(5/2)*Sin[c + d*Sqrt[x]])/(a*(a^2 - b^2)*d*(b + a*Cos[c +
d*Sqrt[x]]))

```

### Rule 2190

```

Int[(((F_)^((g_)*((e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_))/
((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_)))^(n_))), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

```

### Rule 2264

```

Int[((F_)^(u_)*((f_) + (g_)*(x_))^(m_))/((a_) + (b_)*(F_)^(u_) + (c_)
*(F_)^(v_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]

```

### Rule 2282

```

Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*
(F_)^v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*(a_.) + (b_.)*(x_)))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3321

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_.) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] := Dist[2, Int[((c + d*x)^m*E^(I*Pi*(k - 1/2))*E^(I*(e + f
*x)))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(
e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3324

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2, x_
Symbol] := Simp[(b*(c + d*x)^m*Cos[e + f*x])/(f*(a^2 - b^2)*(a + b*Sin[e +
f*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
x] - Dist[(b*d*m)/(f*(a^2 - b^2)), Int[((c + d*x)^(m - 1)*Cos[e + f*x])/(a
+ b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 4191

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x]^n)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 4204

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 4522

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)]/(Cos[(c_.) + (d_.)
*(x_)]*(b_.) + (a_.)), x_Symbol] := Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1))
, x] + (Int[((e + f*x)^m*E^(I*(c + d*x)))/(I*a - Rt[-a^2 + b^2, 2] + I*b*E^
(I*(c + d*x))), x] + Int[((e + f*x)^m*E^(I*(c + d*x)))/(I*a + Rt[-a^2 + b^2
```



, 2] + I\*b\*E^(I\*(c + d\*x)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NegQ[a^2 - b^2]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rule 6609

Int[((e\_.) + (f\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (d\_.)\*((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(p\_.)], x\_Symbol] :> Simp[((e + f\*x)^m\*PolyLog[n + 1, d\*(F^(c\*(a + b\*x)))^p])/(b\*c\*p\*Log[F]), x] - Dist[(f\*m)/(b\*c\*p\*Log[F]), Int[(e + f\*x)^(m - 1)\*PolyLog[n + 1, d\*(F^(c\*(a + b\*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]

### Rubi steps



**Mathematica** [A] time = 13.26, size = 2777, normalized size = 1.20

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[x^2/(a + b\*Sec[c + d\*Sqrt[x]])^2,x]

[Out] 
$$\begin{aligned} &((-4*I)*b^2*E^{((2*I)*c)}*x^{(5/2)}*(b + a*\text{Cos}[c + d*\text{Sqrt}[x]])^2*\text{Sec}[c + d*\text{Sqrt}[x]]^2)/(a^2*(a^2 - b^2)*d*(1 + E^{((2*I)*c)})*(a + b*\text{Sec}[c + d*\text{Sqrt}[x]])^2) \\ &+ (x^3*(b + a*\text{Cos}[c + d*\text{Sqrt}[x]])^2*\text{Sec}[c + d*\text{Sqrt}[x]]^2)/(3*a^2*(a + b*\text{Sec}[c + d*\text{Sqrt}[x]])^2) + (2*b*(b + a*\text{Cos}[c + d*\text{Sqrt}[x]])^2*(5*b*d^4*\text{Sqrt}[(-a^2 + b^2)*E^{((2*I)*c)}]*x^2*\text{Log}[1 + (a*E^{(I*(2*c + d*\text{Sqrt}[x]))})]/(b*E^{(I*c)} - \text{Sqrt}[(-a^2 + b^2)*E^{((2*I)*c)}]) + (2*I)*a^2*d^5*E^{(I*c)}*x^{(5/2)}*\text{Log}[1 + (a*E^{(I*(2*c + d*\text{Sqrt}[x]))})]/(b*E^{(I*c)} - \text{Sqrt}[(-a^2 + b^2)*E^{((2*I)*c)}]) - I*b^2*d^5*E^{(I*c)}*x^{(5/2)}*\text{Log}[1 + (a*E^{(I*(2*c + d*\text{Sqrt}[x]))})]/(b*E^{(I*c)} - \text{Sqrt}[(-a^2 + b^2)*E^{((2*I)*c)}]) + 5*b*d^4*\text{Sqrt}[(-a^2 + b^2)*E^{((2*I)*c)}]*x^2*\text{Log}[1 + (a*E^{(I*(2*c + d*\text{Sqrt}[x]))})]/(b*E^{(I*c)} + \text{Sqrt}[(-a^2 + b^2)*E^{((2*I)*c)}]) - (2*I)*a^2*d^5*E^{(I*c)}*x^{(5/2)}*\text{Log}[1 + (a*E^{(I*(2*c + d*\text{Sqrt}[x]))})]/(b*E^{(I*c)} + \text{Sqrt}[(-a^2 + b^2)*E^{((2*I)*c)}]) + I*b^2*d^5*E^{(I*c)}*x^{(5/2)}*\text{Log}[1 + (a*E^{(I*(2*c + d*\text{Sqrt}[x]))})]/(b*E^{(I*c)} + \text{Sqrt}[(-a^2 + b^2)*E^{((2*I)*c)}]) - 5*d^3*((4*I)*b*\text{Sqrt}[(-a^2 + b^2)*E^{((2*I)*c)}] - 2*a^2*d*E^{(I*c)}*\text{Sqrt}[x] + b^2*d*E^{(I*c)}*\text{Sqrt}[x])*x^{(3/2)}*\text{PolyLog}[2, -((a*E^{(I*(2*c + d*\text{Sqrt}[x]))})/(b*E^{(I*c)} - \text{Sqrt}[(-a^2 + b^2)*E^{((2*I)*c)}]) + 5*d^3*((-4*I)*b*\text{Sqrt}[(-a^2 + b^2)*E^{((2*I)*c)}] - 2*a^2*d*E^{(I*c)}*\text{Sqrt}[x] + b^2*d*E^{(I*c)}*\text{Sqrt}[x])*x^{(3/2)}*\text{PolyLog}[2, -((a*E^{(I*(2*c + d*\text{Sqrt}[x]))})/(b*E^{(I*c)} + \text{Sqrt}[(-a^2 + b^2)*E^{((2*I)*c)}]) + 60*b*d^2*\text{Sqrt}[(-a^2 + b^2)*E^{((2*I)*c)}]*x*\text{PolyLog}[3, -((a*E^{(I*(2*c + d*\text{Sqrt}[x]))})/(b*E^{(I*c)} - \text{Sqrt}[(-a^2 + b^2)*E^{((2*I)*c)}]) + (40*I)*a^2*d^3*E^{(I*c)}*x^{(3/2)}*\text{PolyLog}[3, -((a*E^{(I*(2*c + d*\text{Sqrt}[x]))})/(b*E^{(I*c)} - \text{Sqrt}[(-a^2 + b^2)*E^{((2*I)*c)}]) - (20*I)*b^2*d^3*E^{(I*c)}*x^{(3/2)}*\text{PolyLog}[3, -((a*E^{(I*(2*c + d*\text{Sqrt}[x]))})/(b*E^{(I*c)} - \text{Sqrt}[(-a^2 + b^2)*E^{((2*I)*c)}]) + 60*b*d^2*\text{Sqrt}[(-a^2 + b^2)*E^{((2*I)*c)}]*x*\text{PolyLog}[3, -((a*E^{(I*(2*c + d*\text{Sqrt}[x]))})/(b*E^{(I*c)} + \text{Sqrt}[(-a^2 + b^2)*E^{((2*I)*c)}]) - (40*I)*a^2*d^3*E^{(I*c)}*x^{(3/2)}*\text{PolyLog}[3, -((a*E^{(I*(2*c + d*\text{Sqrt}[x]))})/(b*E^{(I*c)} + \text{Sqrt}[(-a^2 + b^2)*E^{((2*I)*c)}]) + (20*I)*b^2*d^3*E^{(I*c)}*x^{(3/2)}*\text{PolyLog}[3, -((a*E^{(I*(2*c + d*\text{Sqrt}[x]))})/(b*E^{(I*c)} + \text{Sqrt}[(-a^2 + b^2)*E^{((2*I)*c)}]) + (120*I)*b*d*\text{Sqrt}[(-a^2 + b^2)*E^{((2*I)*c)}]*\text{Sqrt}[x]*\text{PolyLog}[4, -((a*E^{(I*(2*c + d*\text{Sqrt}[x]))})/(b*E^{(I*c)} - \text{Sqrt}[(-a^2 + b^2)*E^{((2*I)*c)}]) - 120*a^2*d^2*E^{(I*c)}*x*\text{PolyLog}[4, -((a*E^{(I*(2*c + d*\text{Sqrt}[x]))})/(b*E^{(I*c)} - \text{Sqrt}[(-a^2 + b^2)*E^{((2*I)*c)}]) + 60*b^2*d^2*E^{(I*c)}*x*\text{PolyLog}[4, -((a*E^{(I*(2*c + d*\text{Sqrt}[x]))})/(b*E^{(I*c)} - \text{Sqrt}[(-a^2 + b^2)*E^{((2*I)*c)}]) + (120*I)*b*d*\text{Sqrt}[(-a^2 + b^2)*E^{((2*I)*c)}]*\text{Sqrt}[x]*\text{PolyLog}[4, -((a*E^{(I*(2*c + d*\text{Sqrt}[x]))})/(b*E^{(I*c)} + \text{Sqrt}[(-a^2 + b^2)*E^{((2*I)*c)}]) + 120*a^2*d^2*E^{(I*c)}*x*\text{PolyLog}[4, -((a*E^{(I*(2*c + d*\text{Sqrt}[x]))})/(b*E^{(I*c)} + \text{Sqrt}[(-a^2 + b^2)*E^{((2*I)*c)}]) - 60*b^2*d^2*E^{(I*c)}*x*\text{PolyLog}[4, -(($$

$$\begin{aligned}
& a * E^{(I * (2 * c + d * \text{Sqrt}[x]))} / (b * E^{(I * c)} + \text{Sqrt}[(-a^2 + b^2) * E^{((2 * I) * c)}]) - \\
& 120 * b * \text{Sqrt}[(-a^2 + b^2) * E^{((2 * I) * c)}] * \text{PolyLog}[5, -((a * E^{(I * (2 * c + d * \text{Sqrt}[x]))}) / (b * E^{(I * c)} - \text{Sqrt}[(-a^2 + b^2) * E^{((2 * I) * c)}]) - \\
& ((a * E^{(I * (2 * c + d * \text{Sqrt}[x]))}) / (b * E^{(I * c)} - \text{Sqrt}[(-a^2 + b^2) * E^{((2 * I) * c)}]) - (240 * I) * a^2 * d * E^{(I * c)} * \\
& \text{Sqrt}[x] * \text{PolyLog}[5, -((a * E^{(I * (2 * c + d * \text{Sqrt}[x]))}) / (b * E^{(I * c)} - \text{Sqrt}[(-a^2 + b^2) * E^{((2 * I) * c)}]) - \\
& (240 * I) * a^2 * d * E^{(I * c)} * \text{Sqrt}[x] * \text{PolyLog}[5, -((a * E^{(I * (2 * c + d * \text{Sqrt}[x]))}) / (b * E^{(I * c)} - \text{Sqrt}[(-a^2 + b^2) * E^{((2 * I) * c)}]) - \\
& 120 * b * \text{Sqrt}[(-a^2 + b^2) * E^{((2 * I) * c)}] * \text{PolyLog}[5, -((a * E^{(I * (2 * c + d * \text{Sqrt}[x]))}) / (b * E^{(I * c)} + \text{Sqrt}[(-a^2 + b^2) * E^{((2 * I) * c)}]) - \\
& (240 * I) * a^2 * d * E^{(I * c)} * \text{Sqrt}[x] * \text{PolyLog}[5, -((a * E^{(I * (2 * c + d * \text{Sqrt}[x]))}) / (b * E^{(I * c)} + \text{Sqrt}[(-a^2 + b^2) * E^{((2 * I) * c)}]) - \\
& (120 * I) * b^2 * d * E^{(I * c)} * \text{Sqrt}[x] * \text{PolyLog}[5, -((a * E^{(I * (2 * c + d * \text{Sqrt}[x]))}) / (b * E^{(I * c)} + \text{Sqrt}[(-a^2 + b^2) * E^{((2 * I) * c)}]) - \\
& 240 * a^2 * E^{(I * c)} * \text{PolyLog}[6, -((a * E^{(I * (2 * c + d * \text{Sqrt}[x]))}) / (b * E^{(I * c)} - \text{Sqrt}[(-a^2 + b^2) * E^{((2 * I) * c)}]) - \\
& 120 * b^2 * E^{(I * c)} * \text{PolyLog}[6, -((a * E^{(I * (2 * c + d * \text{Sqrt}[x]))}) / (b * E^{(I * c)} - \text{Sqrt}[(-a^2 + b^2) * E^{((2 * I) * c)}]) - \\
& 240 * a^2 * E^{(I * c)} * \text{PolyLog}[6, -((a * E^{(I * (2 * c + d * \text{Sqrt}[x]))}) / (b * E^{(I * c)} + \text{Sqrt}[(-a^2 + b^2) * E^{((2 * I) * c)}]) - \\
& 120 * b^2 * E^{(I * c)} * \text{PolyLog}[6, -((a * E^{(I * (2 * c + d * \text{Sqrt}[x]))}) / (b * E^{(I * c)} + \text{Sqrt}[(-a^2 + b^2) * E^{((2 * I) * c)}]) - \\
& (a^2 * (a^2 - b^2) * d^6 * \text{Sqrt}[(-a^2 + b^2) * E^{((2 * I) * c)}] * (a + b * \text{Sec}[c + d * \text{Sqrt}[x]])^2) + (2 * (b + a * \text{Cos}[c + d * \text{Sqrt}[x]]) * \text{Sec}[c + d * \text{Sqrt}[x]]^2 * (b^3 * x^{(5/2)} * \text{Sin}[c] - a * b^2 * x^{(5/2)} * \text{Sin}[d * \text{Sqrt}[x]])) / (a^2 * (-a + b) * (a + b) * d * (a + b * \text{Sec}[c + d * \text{Sqrt}[x]])^2 * (\text{Cos}[c/2] - \text{Sin}[c/2]) * (\text{Cos}[c/2] + \text{Sin}[c/2]))
\end{aligned}$$

**fricas** [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{x^2}{b^2 \sec(d\sqrt{x} + c)^2 + 2ab \sec(d\sqrt{x} + c) + a^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b\*sec(c+d\*x^(1/2)))^2,x, algorithm="fricas")

[Out] integral(x^2/(b^2\*sec(d\*sqrt(x) + c)^2 + 2\*a\*b\*sec(d\*sqrt(x) + c) + a^2), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(b \sec(d\sqrt{x} + c) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b\*sec(c+d\*x^(1/2)))^2,x, algorithm="giac")

[Out] integrate(x^2/(b\*sec(d\*sqrt(x) + c) + a)^2, x)

**maple** [F] time = 1.30, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a + b \sec(c + d\sqrt{x}))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(a+b*sec(c+d*x^(1/2)))^2,x)`

[Out] `int(x^2/(a+b*sec(c+d*x^(1/2)))^2,x)`

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a^2-4\*b^2>0)', see `assume?` for more details)Is 4\*a^2-4\*b^2 positive or negative?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\left(a + \frac{b}{\cos(c+d\sqrt{x})}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(a + b/cos(c + d*x^(1/2)))^2,x)`

[Out] `int(x^2/(a + b/cos(c + d*x^(1/2)))^2, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a + b \sec(c + d\sqrt{x}))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(a+b*sec(c+d*x**(1/2)))**2,x)`

[Out] `Integral(x**2/(a + b*sec(c + d*sqrt(x)))**2, x)`

$$3.48 \quad \int \frac{x}{(a+b \sec(c+d\sqrt{x}))^2} dx$$

**Optimal.** Leaf size=1523

$$\frac{2ix^{3/2} \log\left(\frac{e^{i(c+d\sqrt{x})} a}{b-\sqrt{b^2-a^2}} + 1\right) b^3}{a^2 (b^2 - a^2)^{3/2} d} + \frac{2ix^{3/2} \log\left(\frac{e^{i(c+d\sqrt{x})} a}{b+\sqrt{b^2-a^2}} + 1\right) b^3}{a^2 (b^2 - a^2)^{3/2} d} - \frac{6x \operatorname{Li}_2\left(-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right) b^3}{a^2 (b^2 - a^2)^{3/2} d^2} + \frac{6x \operatorname{Li}_2\left(-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right) b^3}{a^2 (b^2 - a^2)^{3/2} d^2} - \frac{12i\sqrt{x}}{a}$$

[Out]  $-2*I*b^2*x^{(3/2)}/a^2/(a^2-b^2)/d+6*b^2*x*\ln(1+a*\exp(I*(c+d*x^{(1/2)})))/(b-I*(a^2-b^2)^{(1/2)})/a^2/(a^2-b^2)/d^2+6*b^2*x*\ln(1+a*\exp(I*(c+d*x^{(1/2)})))/(b+I*(a^2-b^2)^{(1/2)})/a^2/(a^2-b^2)/d^2+2*b^2*x^{(3/2)}*\sin(c+d*x^{(1/2)})/a/(a^2-b^2)/d/(b+a*\cos(c+d*x^{(1/2)}))-2*I*b^3*x^{(3/2)}*\ln(1+a*\exp(I*(c+d*x^{(1/2)})))/(b-(-a^2+b^2)^{(1/2)})/a^2/(-a^2+b^2)^{(3/2)}/d-4*I*b*x^{(3/2)}*\ln(1+a*\exp(I*(c+d*x^{(1/2)})))/(b+(-a^2+b^2)^{(1/2)})/a^2/d/(-a^2+b^2)^{(1/2)}-12*I*b^2*polylog(2,-a*\exp(I*(c+d*x^{(1/2)})))/(b-I*(a^2-b^2)^{(1/2)})*x^{(1/2)}/a^2/(a^2-b^2)/d^3-12*I*b^2*polylog(2,-a*\exp(I*(c+d*x^{(1/2)})))/(b+I*(a^2-b^2)^{(1/2)})*x^{(1/2)}/a^2/(a^2-b^2)/d^3-12*I*b^3*polylog(3,-a*\exp(I*(c+d*x^{(1/2)})))/(b-(-a^2+b^2)^{(1/2)})*x^{(1/2)}/a^2/(-a^2+b^2)^{(3/2)}/d^3-24*I*b*polylog(3,-a*\exp(I*(c+d*x^{(1/2)})))/(b+(-a^2+b^2)^{(1/2)})*x^{(1/2)}/a^2/d^3/(-a^2+b^2)^{(1/2)}+2*I*b^3*x^{(3/2)}*\ln(1+a*\exp(I*(c+d*x^{(1/2)})))/(b+(-a^2+b^2)^{(1/2)})/a^2/(-a^2+b^2)^{(3/2)}/d+4*I*b*x^{(3/2)}*\ln(1+a*\exp(I*(c+d*x^{(1/2)})))/(b-(-a^2+b^2)^{(1/2)})/a^2/d/(-a^2+b^2)^{(1/2)}+12*I*b^3*polylog(3,-a*\exp(I*(c+d*x^{(1/2)})))/(b+(-a^2+b^2)^{(1/2)})*x^{(1/2)}/a^2/(-a^2+b^2)^{(3/2)}/d^3+24*I*b*polylog(3,-a*\exp(I*(c+d*x^{(1/2)})))/(b-(-a^2+b^2)^{(1/2)})*x^{(1/2)}/a^2/d^3/(-a^2+b^2)^{(1/2)}+1/2*x^2/a^2-6*b^3*x*polylog(2,-a*\exp(I*(c+d*x^{(1/2)})))/(b-(-a^2+b^2)^{(1/2)})/a^2/(-a^2+b^2)^{(3/2)}/d^2+6*b^3*x*polylog(2,-a*\exp(I*(c+d*x^{(1/2)})))/(b+(-a^2+b^2)^{(1/2)})/a^2/(-a^2+b^2)^{(3/2)}/d^2+12*b*x*polylog(2,-a*\exp(I*(c+d*x^{(1/2)})))/(b-(-a^2+b^2)^{(1/2)})/a^2/d^2/(-a^2+b^2)^{(1/2)}-12*b*x*polylog(2,-a*\exp(I*(c+d*x^{(1/2)})))/(b+(-a^2+b^2)^{(1/2)})/a^2/d^2/(-a^2+b^2)^{(1/2)}+12*b^2*polylog(3,-a*\exp(I*(c+d*x^{(1/2)})))/(b-I*(a^2-b^2)^{(1/2)})/a^2/(a^2-b^2)/d^4+12*b^2*polylog(3,-a*\exp(I*(c+d*x^{(1/2)})))/(b+I*(a^2-b^2)^{(1/2)})/a^2/(a^2-b^2)/d^4+12*b^3*polylog(4,-a*\exp(I*(c+d*x^{(1/2)})))/(b-(-a^2+b^2)^{(1/2)})/a^2/(-a^2+b^2)^{(3/2)}/d^4-12*b^3*polylog(4,-a*\exp(I*(c+d*x^{(1/2)})))/(b+(-a^2+b^2)^{(1/2)})/a^2/(-a^2+b^2)^{(3/2)}/d^4-24*b*polylog(4,-a*\exp(I*(c+d*x^{(1/2)})))/(b-(-a^2+b^2)^{(1/2)})/a^2/d^4/(-a^2+b^2)^{(1/2)}+24*b*polylog(4,-a*\exp(I*(c+d*x^{(1/2)})))/(b+(-a^2+b^2)^{(1/2)})/a^2/d^4/(-a^2+b^2)^{(1/2)}$

**Rubi [A]** time = 2.50, antiderivative size = 1523, normalized size of antiderivative = 1.00, number of steps used = 37, number of rules used = 11, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$ , Rules used = {4204, 4191, 3324, 3321, 2264, 2190, 2531, 6609, 2282, 6589, 4522}

result too large to display

Antiderivative was successfully verified.

[In] Int[x/(a + b\*Sec[c + d\*Sqrt[x]])^2,x]

[Out] 
$$\begin{aligned} &((-2*I)*b^2*x^{(3/2)})/(a^2*(a^2 - b^2)*d) + x^2/(2*a^2) + (6*b^2*x*Log[1 + ( \\ &a*E^{(I*(c + d*Sqrt[x]))})/(b - I*Sqrt[a^2 - b^2])])/(a^2*(a^2 - b^2)*d^2) + \\ &(6*b^2*x*Log[1 + (a*E^{(I*(c + d*Sqrt[x]))})/(b + I*Sqrt[a^2 - b^2])])/(a^2*( \\ &a^2 - b^2)*d^2) - ((2*I)*b^3*x^{(3/2)*Log[1 + (a*E^{(I*(c + d*Sqrt[x]))})/(b - \\ &Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^{(3/2)*d) + ((4*I)*b*x^{(3/2)*Log[1 + \\ &(a*E^{(I*(c + d*Sqrt[x]))})/(b - Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d) \\ &+ ((2*I)*b^3*x^{(3/2)*Log[1 + (a*E^{(I*(c + d*Sqrt[x]))})/(b + Sqrt[-a^2 + b^ \\ &2])])/(a^2*(-a^2 + b^2)^{(3/2)*d) - ((4*I)*b*x^{(3/2)*Log[1 + (a*E^{(I*(c + d* \\ &Sqrt[x]))})/(b + Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d) - ((12*I)*b^2* \\ &Sqrt[x]*PolyLog[2, -(a*E^{(I*(c + d*Sqrt[x]))})/(b - I*Sqrt[a^2 - b^2])])/( \\ &a^2*(a^2 - b^2)*d^3) - ((12*I)*b^2*Sqrt[x]*PolyLog[2, -(a*E^{(I*(c + d*Sqrt \\ &[x]))})/(b + I*Sqrt[a^2 - b^2])])/(a^2*(a^2 - b^2)*d^3) - (6*b^3*x*PolyLog[ \\ &2, -(a*E^{(I*(c + d*Sqrt[x]))})/(b - Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^ \\ &(3/2)*d^2) + (12*b*x*PolyLog[2, -(a*E^{(I*(c + d*Sqrt[x]))})/(b - Sqrt[-a^2 \\ &+ b^2])])/(a^2*Sqrt[-a^2 + b^2]*d^2) + (6*b^3*x*PolyLog[2, -(a*E^{(I*(c + \\ &d*Sqrt[x]))})/(b + Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^{(3/2)*d^2) - (12*b \\ &*x*PolyLog[2, -(a*E^{(I*(c + d*Sqrt[x]))})/(b + Sqrt[-a^2 + b^2])])/(a^2*Sq \\ &rt[-a^2 + b^2]*d^2) + (12*b^2*PolyLog[3, -(a*E^{(I*(c + d*Sqrt[x]))})/(b - I \\ &*Sqrt[a^2 - b^2])])/(a^2*(a^2 - b^2)*d^4) + (12*b^2*PolyLog[3, -(a*E^{(I*( \\ &c + d*Sqrt[x]))})/(b + I*Sqrt[a^2 - b^2])])/(a^2*(a^2 - b^2)*d^4) - ((12*I) \\ &*b^3*Sqrt[x]*PolyLog[3, -(a*E^{(I*(c + d*Sqrt[x]))})/(b - Sqrt[-a^2 + b^2])]) \\ &]/(a^2*(-a^2 + b^2)^{(3/2)*d^3) + ((24*I)*b*Sqrt[x]*PolyLog[3, -(a*E^{(I*(c \\ &+ d*Sqrt[x]))})/(b - Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d^3) + ((12 \\ &*I)*b^3*Sqrt[x]*PolyLog[3, -(a*E^{(I*(c + d*Sqrt[x]))})/(b + Sqrt[-a^2 + b^2 \\ &])])/(a^2*(-a^2 + b^2)^{(3/2)*d^3) - ((24*I)*b*Sqrt[x]*PolyLog[3, -(a*E^{(I \\ &*(c + d*Sqrt[x]))})/(b + Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d^3) + ( \\ &12*b^3*PolyLog[4, -(a*E^{(I*(c + d*Sqrt[x]))})/(b - Sqrt[-a^2 + b^2])])/(a^ \\ &2*(-a^2 + b^2)^{(3/2)*d^4) - (24*b*PolyLog[4, -(a*E^{(I*(c + d*Sqrt[x]))})/(b \\ &- Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d^4) - (12*b^3*PolyLog[4, -( \\ &a*E^{(I*(c + d*Sqrt[x]))})/(b + Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^{(3/2)* \\ &d^4) + (24*b*PolyLog[4, -(a*E^{(I*(c + d*Sqrt[x]))})/(b + Sqrt[-a^2 + b^2])]) \\ &]/(a^2*Sqrt[-a^2 + b^2]*d^4) + (2*b^2*x^{(3/2)*Sin[c + d*Sqrt[x])})/(a*(a^2 \\ &- b^2)*d*(b + a*Cos[c + d*Sqrt[x])]) \end{aligned}$$

### Rule 2190

$$\text{Int}[(((F_)^{((g_)*(e_)+(f_)*(x_)))})^{(n_)*((c_)+(d_)*(x_))^{(m_)}))/((a_)+(b_)*((F_)^{((g_)*(e_)+(f_)*(x_)))})^{(n_)}), x\_Symbol] \rightarrow \text{Simp} [((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - \text{Dist} [(d*m)/(b*f*g*n*Log[F]), \text{Int}[(c + d*x)^{(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a}], x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x] \&\& \text{IGtQ}[m, 0]$$

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^(c_.)*((a_.) + (b_.)*(x_)))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3321

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] := Dist[2, Int[((c + d*x)^m*E^(I*Pi*(k - 1/2))*E^(I*(e + f
*x)))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(
e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3324

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2, x_
Symbol] := Simp[(b*(c + d*x)^m*Cos[e + f*x])/(f*(a^2 - b^2)*(a + b*Sin[e +
f*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
x] - Dist[(b*d*m)/(f*(a^2 - b^2)), Int[((c + d*x)^(m - 1)*Cos[e + f*x])/(a
+ b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 4191

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
```



$n[e + f*x]^n), x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x\} \ \&\& \ \text{ILtQ}[n, 0] \ \&\& \ \text{IGtQ}[m, 0]$

#### Rule 4204

$\text{Int}[(x_)^{(m_.)}*((a_.) + (b_.)*\text{Sec}[(c_.) + (d_.)*(x_)^{(n_.)}])^{(p_.)}, x\_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)*(a + b*\text{Sec}[c + d*x])^p}, x], x, x^n], x] /; \text{FreeQ}\{a, b, c, d, m, n, p\}, x\} \ \&\& \ \text{IGtQ}[\text{Simplify}[(m + 1)/n], 0] \ \&\& \ \text{IntegerQ}[p]$

#### Rule 4522

$\text{Int}[(((e_.) + (f_.)*(x_))^{(m_.)}*\text{Sin}[(c_.) + (d_.)*(x_)]) / (\text{Cos}[(c_.) + (d_.)*(x_)]*(b_.) + (a_.)), x\_Symbol] \rightarrow \text{Simp}[(I*(e + f*x)^{(m + 1)}) / (b*f*(m + 1)), x] + (\text{Int}[(e + f*x)^m * E^{(I*(c + d*x))} / (I*a - \text{Rt}[-a^2 + b^2, 2] + I*b * E^{(I*(c + d*x))}), x] + \text{Int}[(e + f*x)^m * E^{(I*(c + d*x))} / (I*a + \text{Rt}[-a^2 + b^2, 2] + I*b * E^{(I*(c + d*x))}), x]) /; \text{FreeQ}\{a, b, c, d, e, f\}, x\} \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ \text{NegQ}[a^2 - b^2]$

#### Rule 6589

$\text{Int}[\text{PolyLog}[n_, (c_.)*((a_.) + (b_.)*(x_))^{(p_.)}] / ((d_.) + (e_.)*(x_)), x\_Symbol] \rightarrow \text{Simp}[\text{PolyLog}[n + 1, c*(a + b*x)^p] / (e*p), x] /; \text{FreeQ}\{a, b, c, d, e, n, p\}, x\} \ \&\& \ \text{EqQ}[b*d, a*e]$

#### Rule 6609

$\text{Int}[(((e_.) + (f_.)*(x_))^{(m_.)}*\text{PolyLog}[n_, (d_.)*((F_)^{(c_.)*((a_.) + (b_.)*(x_))})^{(p_.)}]), x\_Symbol] \rightarrow \text{Simp}[(e + f*x)^m * \text{PolyLog}[n + 1, d*(F^{(c*(a + b*x))})^p] / (b*c*p*\text{Log}[F]), x] - \text{Dist}[(f*m) / (b*c*p*\text{Log}[F]), \text{Int}[(e + f*x)^{(m - 1)} * \text{PolyLog}[n + 1, d*(F^{(c*(a + b*x))})^p], x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, n, p\}, x\} \ \&\& \ \text{GtQ}[m, 0]$

#### Rubi steps

$$\begin{aligned}
\int \frac{x}{(a + b \sec(c + d\sqrt{x}))^2} dx &= 2 \operatorname{Subst} \left( \int \frac{x^3}{(a + b \sec(c + dx))^2} dx, x, \sqrt{x} \right) \\
&= 2 \operatorname{Subst} \left( \int \left( \frac{x^3}{a^2} + \frac{b^2 x^3}{a^2 (b + a \cos(c + dx))^2} - \frac{2bx^3}{a^2 (b + a \cos(c + dx))} \right) dx, x, \sqrt{x} \right) \\
&= \frac{x^2}{2a^2} - \frac{(4b) \operatorname{Subst} \left( \int \frac{x^3}{b+a \cos(c+dx)} dx, x, \sqrt{x} \right)}{a^2} + \frac{(2b^2) \operatorname{Subst} \left( \int \frac{x^3}{(b+a \cos(c+dx))^2} dx, x, \sqrt{x} \right)}{a^2} \\
&= \frac{x^2}{2a^2} + \frac{2b^2 x^{3/2} \sin(c + d\sqrt{x})}{a(a^2 - b^2)d(b + a \cos(c + d\sqrt{x}))} - \frac{(8b) \operatorname{Subst} \left( \int \frac{e^{i(c+dx)} x^3}{a+2be^{i(c+dx)}+ae^{2i(c+dx)}} dx, x, \sqrt{x} \right)}{a^2} \\
&= -\frac{2ib^2 x^{3/2}}{a^2(a^2 - b^2)d} + \frac{x^2}{2a^2} + \frac{2b^2 x^{3/2} \sin(c + d\sqrt{x})}{a(a^2 - b^2)d(b + a \cos(c + d\sqrt{x}))} - \frac{(4b^3) \operatorname{Subst} \left( \int \frac{x^3}{a+2be^{i(c+dx)}+ae^{2i(c+dx)}} dx, x, \sqrt{x} \right)}{a^2} \\
&= -\frac{2ib^2 x^{3/2}}{a^2(a^2 - b^2)d} + \frac{x^2}{2a^2} + \frac{6b^2 x \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b-i\sqrt{a^2-b^2}} \right)}{a^2(a^2 - b^2)d^2} + \frac{6b^2 x \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b+i\sqrt{a^2-b^2}} \right)}{a^2(a^2 - b^2)d^2} + \dots \\
&= -\frac{2ib^2 x^{3/2}}{a^2(a^2 - b^2)d} + \frac{x^2}{2a^2} + \frac{6b^2 x \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b-i\sqrt{a^2-b^2}} \right)}{a^2(a^2 - b^2)d^2} + \frac{6b^2 x \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b+i\sqrt{a^2-b^2}} \right)}{a^2(a^2 - b^2)d^2} - \dots \\
&= -\frac{2ib^2 x^{3/2}}{a^2(a^2 - b^2)d} + \frac{x^2}{2a^2} + \frac{6b^2 x \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b-i\sqrt{a^2-b^2}} \right)}{a^2(a^2 - b^2)d^2} + \frac{6b^2 x \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b+i\sqrt{a^2-b^2}} \right)}{a^2(a^2 - b^2)d^2} - \dots \\
&= -\frac{2ib^2 x^{3/2}}{a^2(a^2 - b^2)d} + \frac{x^2}{2a^2} + \frac{6b^2 x \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b-i\sqrt{a^2-b^2}} \right)}{a^2(a^2 - b^2)d^2} + \frac{6b^2 x \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b+i\sqrt{a^2-b^2}} \right)}{a^2(a^2 - b^2)d^2} - \dots \\
&= -\frac{2ib^2 x^{3/2}}{a^2(a^2 - b^2)d} + \frac{x^2}{2a^2} + \frac{6b^2 x \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b-i\sqrt{a^2-b^2}} \right)}{a^2(a^2 - b^2)d^2} + \frac{6b^2 x \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b+i\sqrt{a^2-b^2}} \right)}{a^2(a^2 - b^2)d^2} - \dots \\
&= -\frac{2ib^2 x^{3/2}}{a^2(a^2 - b^2)d} + \frac{x^2}{2a^2} + \frac{6b^2 x \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b-i\sqrt{a^2-b^2}} \right)}{a^2(a^2 - b^2)d^2} + \frac{6b^2 x \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b+i\sqrt{a^2-b^2}} \right)}{a^2(a^2 - b^2)d^2} - \dots
\end{aligned}$$

**Mathematica [A]** time = 15.94, size = 1767, normalized size = 1.16

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[x/(a + b\*Sec[c + d\*Sqrt[x]])^2,x]

[Out]  $(x^2(b + a\cos[c + d\sqrt{x}])^2\sec[c + d\sqrt{x}]^2)/(2a^2(a + b\sec[c + d\sqrt{x}])^2) + (2b(b + a\cos[c + d\sqrt{x}])^2((-2I)b^3d^3E^{(2I)c}x^{3/2})/(1 + E^{(2I)c}) + (3b^2d^2\sqrt{-a^2 + b^2}E^{(2I)c})x\log[1 + (aE^{I(2c + d\sqrt{x})})/(bE^{Ic} - \sqrt{-a^2 + b^2}E^{(2I)c})] + (2I)a^2d^3E^{Ic}x^{3/2}\log[1 + (aE^{I(2c + d\sqrt{x})})/(bE^{Ic} - \sqrt{-a^2 + b^2}E^{(2I)c})] - Ib^2d^3E^{Ic}x^{3/2}\log[1 + (aE^{I(2c + d\sqrt{x})})/(bE^{Ic} - \sqrt{-a^2 + b^2}E^{(2I)c})] + 3b^2d^2\sqrt{-a^2 + b^2}E^{(2I)c}x\log[1 + (aE^{I(2c + d\sqrt{x})})/(bE^{Ic} + \sqrt{-a^2 + b^2}E^{(2I)c})] - (2I)a^2d^3E^{Ic}x^{3/2}\log[1 + (aE^{I(2c + d\sqrt{x})})/(bE^{Ic} + \sqrt{-a^2 + b^2}E^{(2I)c})] + Ib^2d^3E^{Ic}x^{3/2}\log[1 + (aE^{I(2c + d\sqrt{x})})/(bE^{Ic} + \sqrt{-a^2 + b^2}E^{(2I)c})] - 3d^2((2I)b\sqrt{-a^2 + b^2}E^{(2I)c} - 2a^2dE^{Ic}\sqrt{x} + b^2dE^{Ic}\sqrt{x})\sqrt{x}\text{PolyLog}[2, -(aE^{I(2c + d\sqrt{x})})/(bE^{Ic} - \sqrt{-a^2 + b^2}E^{(2I)c})] + 3d^2((-2I)b\sqrt{-a^2 + b^2}E^{(2I)c} - 2a^2dE^{Ic}\sqrt{x} + b^2dE^{Ic}\sqrt{x})\sqrt{x}\text{PolyLog}[2, -(aE^{I(2c + d\sqrt{x})})/(bE^{Ic} + \sqrt{-a^2 + b^2}E^{(2I)c})] + 6b\sqrt{-a^2 + b^2}E^{(2I)c}\text{PolyLog}[3, -(aE^{I(2c + d\sqrt{x})})/(bE^{Ic} - \sqrt{-a^2 + b^2}E^{(2I)c})] + (12I)a^2dE^{Ic}\sqrt{x}\text{PolyLog}[3, -(aE^{I(2c + d\sqrt{x})})/(bE^{Ic} - \sqrt{-a^2 + b^2}E^{(2I)c})] - (6I)b^2dE^{Ic}\sqrt{x}\text{PolyLog}[3, -(aE^{I(2c + d\sqrt{x})})/(bE^{Ic} - \sqrt{-a^2 + b^2}E^{(2I)c})] + 6b\sqrt{-a^2 + b^2}E^{(2I)c}\text{PolyLog}[3, -(aE^{I(2c + d\sqrt{x})})/(bE^{Ic} + \sqrt{-a^2 + b^2}E^{(2I)c})] - (12I)a^2dE^{Ic}\sqrt{x}\text{PolyLog}[3, -(aE^{I(2c + d\sqrt{x})})/(bE^{Ic} + \sqrt{-a^2 + b^2}E^{(2I)c})] + (6I)b^2dE^{Ic}\sqrt{x}\text{PolyLog}[3, -(aE^{I(2c + d\sqrt{x})})/(bE^{Ic} + \sqrt{-a^2 + b^2}E^{(2I)c})] - 12a^2E^{Ic}\text{PolyLog}[4, -(aE^{I(2c + d\sqrt{x})})/(bE^{Ic} - \sqrt{-a^2 + b^2}E^{(2I)c})] + 6b^2E^{Ic}\text{PolyLog}[4, -(aE^{I(2c + d\sqrt{x})})/(bE^{Ic} - \sqrt{-a^2 + b^2}E^{(2I)c})] + 12a^2E^{Ic}\text{PolyLog}[4, -(aE^{I(2c + d\sqrt{x})})/(bE^{Ic} + \sqrt{-a^2 + b^2}E^{(2I)c})] - 6b^2E^{Ic}\text{PolyLog}[4, -(aE^{I(2c + d\sqrt{x})})/(bE^{Ic} + \sqrt{-a^2 + b^2}E^{(2I)c})])/\sqrt{-a^2 + b^2}E^{(2I)c}\sec[c + d\sqrt{x}]^2/(a^2(a^2 - b^2)d^4(a + b\sec[c + d\sqrt{x}])^2) + (2(b + a\cos[c + d\sqrt{x}])\sec[c + d\sqrt{x}]^2(b^3x^{3/2}\sin[c] - ab^2x^{3/2}\sin[d\sqrt{x}]))/(a^2(-a + b)(a + b)d^2(a + b\sec[c + d\sqrt{x}])^2(\cos[c/2] - \sin[c/2])*(\cos[c/2] + \sin[c/2]))$

**fricas** [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x}{b^2 \sec(d\sqrt{x} + c)^2 + 2ab \sec(d\sqrt{x} + c) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*sec(c+d\*x^(1/2)))^2,x, algorithm="fricas")

[Out] integral(x/(b^2\*sec(d\*sqrt(x) + c)^2 + 2\*a\*b\*sec(d\*sqrt(x) + c) + a^2), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(b \sec(d\sqrt{x} + c) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*sec(c+d\*x^(1/2)))^2,x, algorithm="giac")

[Out] integrate(x/(b\*sec(d\*sqrt(x) + c) + a)^2, x)

**maple** [F] time = 1.31, size = 0, normalized size = 0.00

$$\int \frac{x}{(a + b \sec(c + d\sqrt{x}))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a+b\*sec(c+d\*x^(1/2)))^2,x)

[Out] int(x/(a+b\*sec(c+d\*x^(1/2)))^2,x)

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*sec(c+d\*x^(1/2)))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a^2-4\*b^2>0)', see `assume?` for more details)Is 4\*a^2-4\*b^2 positive or negative?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\left(a + \frac{b}{\cos(c+d\sqrt{x})}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a + b/cos(c + d\*x^(1/2)))^2,x)

[Out] int(x/(a + b/cos(c + d\*x^(1/2)))^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a + b \sec(c + d\sqrt{x})\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*sec(c+d\*x\*\*(1/2)))\*\*2,x)

[Out] Integral(x/(a + b\*sec(c + d\*sqrt(x)))\*\*2, x)

$$3.49 \quad \int \frac{1}{x(a+b \sec(c+d\sqrt{x}))^2} dx$$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{1}{x(a+b \sec(c+d\sqrt{x}))^2}, x\right)$$

[Out] Unintegrable(1/x/(a+b\*sec(c+d\*x^(1/2)))^2,x)

**Rubi [A]** time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{x(a+b \sec(c+d\sqrt{x}))^2} dx$$

Verification is Not applicable to the result.

[In] Int[1/(x\*(a + b\*Sec[c + d\*Sqrt[x]]))^2],x]

[Out] Defer[Int][1/(x\*(a + b\*Sec[c + d\*Sqrt[x]]))^2], x]

Rubi steps

$$\int \frac{1}{x(a+b \sec(c+d\sqrt{x}))^2} dx = \int \frac{1}{x(a+b \sec(c+d\sqrt{x}))^2} dx$$

**Mathematica [A]** time = 44.42, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a+b \sec(c+d\sqrt{x}))^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x\*(a + b\*Sec[c + d\*Sqrt[x]]))^2],x]

[Out] Integrate[1/(x\*(a + b\*Sec[c + d\*Sqrt[x]]))^2], x]

**fricas [A]** time = 1.39, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{b^2x \sec(d\sqrt{x} + c)^2 + 2abx \sec(d\sqrt{x} + c) + a^2x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*sec(c+d\*x^(1/2)))^2,x, algorithm="fricas")

[Out] integral(1/(b^2\*x\*sec(d\*sqrt(x) + c)^2 + 2\*a\*b\*x\*sec(d\*sqrt(x) + c) + a^2\*x), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \sec(d\sqrt{x} + c) + a)^2 x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*sec(c+d\*x^(1/2)))^2,x, algorithm="giac")

[Out] integrate(1/((b\*sec(d\*sqrt(x) + c) + a)^2\*x), x)

**maple** [A] time = 1.30, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a + b \sec(c + d\sqrt{x}))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(a+b\*sec(c+d\*x^(1/2)))^2,x)

[Out] int(1/x/(a+b\*sec(c+d\*x^(1/2)))^2,x)

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*sec(c+d\*x^(1/2)))^2,x, algorithm="maxima")

[Out] Timed out

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x \left( a + \frac{b}{\cos(c+d\sqrt{x})} \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x*(a + b/cos(c + d*x^(1/2))))^2),x)`

[Out] `int(1/(x*(a + b/cos(c + d*x^(1/2))))^2), x)`

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a + b \sec(c + d\sqrt{x}))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(a+b*sec(c+d*x**(1/2))))**2,x)`

[Out] `Integral(1/(x*(a + b*sec(c + d*sqrt(x))))**2), x)`



$$3.50 \quad \int \frac{1}{x^2(a+b \sec(c+d\sqrt{x}))^2} dx$$

Optimal. Leaf size=23

$$\text{Int} \left( \frac{1}{x^2 (a + b \sec(c + d\sqrt{x}))^2}, x \right)$$

[Out] Unintegrable(1/x^2/(a+b\*sec(c+d\*x^(1/2)))^2,x)

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{x^2(a+b \sec(c+d\sqrt{x}))^2} dx$$

Verification is Not applicable to the result.

[In] Int[1/(x^2\*(a + b\*Sec[c + d\*Sqrt[x]]))^2],x]

[Out] Defer[Int][1/(x^2\*(a + b\*Sec[c + d\*Sqrt[x]]))^2], x]

Rubi steps

$$\int \frac{1}{x^2(a+b \sec(c+d\sqrt{x}))^2} dx = \int \frac{1}{x^2(a+b \sec(c+d\sqrt{x}))^2} dx$$

Mathematica [A] time = 31.57, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2(a+b \sec(c+d\sqrt{x}))^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x^2\*(a + b\*Sec[c + d\*Sqrt[x]]))^2],x]

[Out] Integrate[1/(x^2\*(a + b\*Sec[c + d\*Sqrt[x]]))^2], x]

fricas [A] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{1}{b^2 x^2 \sec(d\sqrt{x} + c)^2 + 2 ab x^2 \sec(d\sqrt{x} + c) + a^2 x^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b\*sec(c+d\*x^(1/2)))^2,x, algorithm="fricas")

[Out] integral(1/(b^2\*x^2\*sec(d\*sqrt(x) + c)^2 + 2\*a\*b\*x^2\*sec(d\*sqrt(x) + c) + a^2\*x^2), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \sec(d\sqrt{x} + c) + a)^2 x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b\*sec(c+d\*x^(1/2)))^2,x, algorithm="giac")

[Out] integrate(1/((b\*sec(d\*sqrt(x) + c) + a)^2\*x^2), x)

**maple** [A] time = 1.28, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (a + b \sec(c + d\sqrt{x}))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(a+b\*sec(c+d\*x^(1/2)))^2,x)

[Out] int(1/x^2/(a+b\*sec(c+d\*x^(1/2)))^2,x)

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b\*sec(c+d\*x^(1/2)))^2,x, algorithm="maxima")

[Out] Timed out

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x^2 \left( a + \frac{b}{\cos(c+d\sqrt{x})} \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^2*(a + b/cos(c + d*x^(1/2))))^2), x)`

[Out] `int(1/(x^2*(a + b/cos(c + d*x^(1/2))))^2), x)`

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (a + b \sec(c + d\sqrt{x}))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**2/(a+b*sec(c+d*x**(1/2))))**2, x)`

[Out] `Integral(1/(x**2*(a + b*sec(c + d*sqrt(x))))**2), x)`

### 3.51 $\int x^{3/2} (a + b \sec(c + d\sqrt{x})) dx$

**Optimal.** Leaf size=284

$$\frac{2}{5}ax^{5/2} + \frac{48b\text{Li}_5\left(-ie^{i(c+d\sqrt{x})}\right)}{d^5} - \frac{48b\text{Li}_5\left(ie^{i(c+d\sqrt{x})}\right)}{d^5} - \frac{48ib\sqrt{x}\text{Li}_4\left(-ie^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{48ib\sqrt{x}\text{Li}_4\left(ie^{i(c+d\sqrt{x})}\right)}{d^4} - \frac{24bx\text{Li}_3\left(-ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{24bx\text{Li}_3\left(ie^{i(c+d\sqrt{x})}\right)}{d^3}$$

[Out]  $\frac{2}{5}ax^{5/2} - 4I*b*x^2*\arctan(\exp(I*(c+d*x^{(1/2)})))/d + 8I*b*x^{(3/2)}*\text{polylog}(2, -I*\exp(I*(c+d*x^{(1/2)})))/d^2 - 8I*b*x^{(3/2)}*\text{polylog}(2, I*\exp(I*(c+d*x^{(1/2)})))/d^2 - 24*b*x*\text{polylog}(3, -I*\exp(I*(c+d*x^{(1/2)})))/d^3 + 24*b*x*\text{polylog}(3, I*\exp(I*(c+d*x^{(1/2)})))/d^3 + 48*b*\text{polylog}(5, -I*\exp(I*(c+d*x^{(1/2)})))/d^5 - 48*b*\text{polylog}(5, I*\exp(I*(c+d*x^{(1/2)})))/d^5 - 48I*b*\text{polylog}(4, -I*\exp(I*(c+d*x^{(1/2)})))*x^{(1/2)}/d^4 + 48I*b*\text{polylog}(4, I*\exp(I*(c+d*x^{(1/2)})))*x^{(1/2)}/d^4$

**Rubi [A]** time = 0.25, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$ , Rules used = {14, 4204, 4181, 2531, 6609, 2282, 6589}

$$\frac{8ibx^{3/2}\text{PolyLog}\left(2, -ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{8ibx^{3/2}\text{PolyLog}\left(2, ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{24bx\text{PolyLog}\left(3, -ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{24bx\text{PolyLog}\left(3, ie^{i(c+d\sqrt{x})}\right)}{d^3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^{(3/2)}*(a + b*\text{Sec}[c + d*\text{Sqrt}[x]]), x]$

[Out]  $(2*a*x^{(5/2)})/5 - ((4*I)*b*x^2*\text{ArcTan}[E^{(I*(c + d*\text{Sqrt}[x])})])/d + ((8*I)*b*x^{(3/2)}*\text{PolyLog}[2, (-I)*E^{(I*(c + d*\text{Sqrt}[x])})])/d^2 - ((8*I)*b*x^{(3/2)}*\text{PolyLog}[2, I*E^{(I*(c + d*\text{Sqrt}[x])})])/d^2 - (24*b*x*\text{PolyLog}[3, (-I)*E^{(I*(c + d*\text{Sqrt}[x])})])/d^3 + (24*b*x*\text{PolyLog}[3, I*E^{(I*(c + d*\text{Sqrt}[x])})])/d^3 - ((48*I)*b*\text{Sqrt}[x]*\text{PolyLog}[4, (-I)*E^{(I*(c + d*\text{Sqrt}[x])})])/d^4 + ((48*I)*b*\text{Sqrt}[x]*\text{PolyLog}[4, I*E^{(I*(c + d*\text{Sqrt}[x])})])/d^4 + (48*b*\text{PolyLog}[5, (-I)*E^{(I*(c + d*\text{Sqrt}[x])})])/d^5 - (48*b*\text{PolyLog}[5, I*E^{(I*(c + d*\text{Sqrt}[x])})])/d^5$

#### Rule 14

$\text{Int}[(u_)*((c_)*(x_))^{(m_)}, x\_Symbol] := \text{Int}[\text{ExpandIntegrand}[(c*x)^m*u, x], x] /; \text{FreeQ}\{c, m\}, x \ \&\& \ \text{SumQ}[u] \ \&\& \ !\text{LinearQ}[u, x] \ \&\& \ !\text{MatchQ}[u, (a_)+ (b_)*(v_)] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{InverseFunctionQ}[v]$

#### Rule 2282

$\text{Int}[u_, x\_Symbol] := \text{With}\{v = \text{FunctionOfExponential}[u, x]\}, \text{Dist}[v/D[v, x], \text{Subst}[\text{Int}[\text{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x] /; \text{FunctionOfExponentialQ}[u, x] \ \&\& \ !\text{MatchQ}[u, (w_)*((a_)*(v_))^{(n_)}] /; \text{FreeQ}[$

{a, m, n}, x] && IntegerQ[m\*n]] && !MatchQ[u, E^((c\_.)\*((a\_.) + (b\_.)\*x))\*  
(F\_) [v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

### Rule 2531

Int[Log[1 + (e\_.)\*((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(n\_.)]\*(f\_.) + (g\_.)  
\*(x\_)^(m\_.), x\_Symbol] := -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))  
)^n])]/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m -  
1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f,  
g, n}, x] && GtQ[m, 0]

### Rule 4181

Int[csc[(e\_.) + Pi\*(k\_.) + (f\_.)\*(x\_)]\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol  
] := Simp[(-2\*(c + d\*x)^m\*ArcTanh[E^(I\*k\*Pi)\*E^(I\*(e + f\*x))]/f, x] + (-Di  
st[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Log[1 - E^(I\*k\*Pi)\*E^(I\*(e + f\*x))], x],  
x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Log[1 + E^(I\*k\*Pi)\*E^(I\*(e + f\*x))  
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2\*k] && IGtQ[m, 0]

### Rule 4204

Int[(x\_)^(m\_.)\*((a\_.) + (b\_.)\*Sec[(c\_.) + (d\_.)\*(x\_)^(n\_)])^(p\_.), x\_Symbol  
] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*Sec[c + d\*x])^p  
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +  
1)/n], 0] && IntegerQ[p]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_S  
ymbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d,  
e, n, p}, x] && EqQ[b\*d, a\*e]

### Rule 6609

Int[((e\_.) + (f\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (d\_.)\*((F\_)^((c\_.)\*((a\_.) + (b\_.  
)\*(x\_))))^(p\_.)], x\_Symbol] := Simp[((e + f\*x)^m\*PolyLog[n + 1, d\*(F^(c\*(a  
+ b\*x)))^p])/ (b\*c\*p\*Log[F]), x] - Dist[(f\*m)/(b\*c\*p\*Log[F]), Int[(e + f\*x)^(  
m - 1)\*PolyLog[n + 1, d\*(F^(c\*(a + b\*x)))^p], x], x] /; FreeQ[{F, a, b, c,  
d, e, f, n, p}, x] && GtQ[m, 0]

### Rubi steps

$$\begin{aligned}
\int x^{3/2} (a + b \sec(c + d\sqrt{x})) dx &= \int (ax^{3/2} + bx^{3/2} \sec(c + d\sqrt{x})) dx \\
&= \frac{2}{5} ax^{5/2} + b \int x^{3/2} \sec(c + d\sqrt{x}) dx \\
&= \frac{2}{5} ax^{5/2} + (2b) \text{Subst} \left( \int x^4 \sec(c + dx) dx, x, \sqrt{x} \right) \\
&= \frac{2}{5} ax^{5/2} - \frac{4ibx^2 \tan^{-1} \left( e^{i(c+d\sqrt{x})} \right)}{d} - \frac{(8b) \text{Subst} \left( \int x^3 \log(1 - ie^{i(c+dx)}) dx, x, \sqrt{x} \right)}{d} \\
&= \frac{2}{5} ax^{5/2} - \frac{4ibx^2 \tan^{-1} \left( e^{i(c+d\sqrt{x})} \right)}{d} + \frac{8ibx^{3/2} \text{Li}_2 \left( -ie^{i(c+d\sqrt{x})} \right)}{d^2} - \frac{8ibx^{3/2} \text{Li}_2 \left( ie^{i(c+d\sqrt{x})} \right)}{d^2} \\
&= \frac{2}{5} ax^{5/2} - \frac{4ibx^2 \tan^{-1} \left( e^{i(c+d\sqrt{x})} \right)}{d} + \frac{8ibx^{3/2} \text{Li}_2 \left( -ie^{i(c+d\sqrt{x})} \right)}{d^2} - \frac{8ibx^{3/2} \text{Li}_2 \left( ie^{i(c+d\sqrt{x})} \right)}{d^2} \\
&= \frac{2}{5} ax^{5/2} - \frac{4ibx^2 \tan^{-1} \left( e^{i(c+d\sqrt{x})} \right)}{d} + \frac{8ibx^{3/2} \text{Li}_2 \left( -ie^{i(c+d\sqrt{x})} \right)}{d^2} - \frac{8ibx^{3/2} \text{Li}_2 \left( ie^{i(c+d\sqrt{x})} \right)}{d^2} \\
&= \frac{2}{5} ax^{5/2} - \frac{4ibx^2 \tan^{-1} \left( e^{i(c+d\sqrt{x})} \right)}{d} + \frac{8ibx^{3/2} \text{Li}_2 \left( -ie^{i(c+d\sqrt{x})} \right)}{d^2} - \frac{8ibx^{3/2} \text{Li}_2 \left( ie^{i(c+d\sqrt{x})} \right)}{d^2} \\
&= \frac{2}{5} ax^{5/2} - \frac{4ibx^2 \tan^{-1} \left( e^{i(c+d\sqrt{x})} \right)}{d} + \frac{8ibx^{3/2} \text{Li}_2 \left( -ie^{i(c+d\sqrt{x})} \right)}{d^2} - \frac{8ibx^{3/2} \text{Li}_2 \left( ie^{i(c+d\sqrt{x})} \right)}{d^2}
\end{aligned}$$

**Mathematica [A]** time = 0.25, size = 281, normalized size = 0.99

$$2 \left( ad^5 x^{5/2} - 10ibd^4 x^2 \tan^{-1} \left( e^{i(c+d\sqrt{x})} \right) \right) + 20ibd^3 x^{3/2} \text{Li}_2 \left( -ie^{i(c+d\sqrt{x})} \right) - 20ibd^3 x^{3/2} \text{Li}_2 \left( ie^{i(c+d\sqrt{x})} \right) - 60bd^2 x \text{Li}_3 \left( -ie^{i(c+d\sqrt{x})} \right) + 60bd^2 x \text{Li}_3 \left( ie^{i(c+d\sqrt{x})} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^(3/2)\*(a + b\*Sec[c + d\*Sqrt[x]]),x]

[Out] (2\*(a\*d^5\*x^(5/2) - (10\*I)\*b\*d^4\*x^2\*ArcTan[E^(I\*(c + d\*Sqrt[x]))]) + (20\*I)\*b\*d^3\*x^(3/2)\*PolyLog[2, (-I)\*E^(I\*(c + d\*Sqrt[x]))] - (20\*I)\*b\*d^3\*x^(3/2)\*PolyLog[2, I\*E^(I\*(c + d\*Sqrt[x]))] - 60\*b\*d^2\*x\*PolyLog[3, (-I)\*E^(I\*(c + d\*Sqrt[x]))] + 60\*b\*d^2\*x\*PolyLog[3, I\*E^(I\*(c + d\*Sqrt[x]))] - (120\*I)\*b\*d\*Sqrt[x]\*PolyLog[4, (-I)\*E^(I\*(c + d\*Sqrt[x]))] + (120\*I)\*b\*d\*Sqrt[x]\*PolyLog[4, I\*E^(I\*(c + d\*Sqrt[x]))] + 120\*b\*PolyLog[5, (-I)\*E^(I\*(c + d\*Sqrt[x]))]) - 120\*b\*PolyLog[5, I\*E^(I\*(c + d\*Sqrt[x]))]))/(5\*d^5)

**fricas** [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(bx^{\frac{3}{2}} \sec(d\sqrt{x} + c) + ax^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)\*(a+b\*sec(c+d\*x^(1/2))),x, algorithm="fricas")

[Out] integral(b\*x^(3/2)\*sec(d\*sqrt(x) + c) + a\*x^(3/2), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sec(d\sqrt{x} + c) + a)x^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)\*(a+b\*sec(c+d\*x^(1/2))),x, algorithm="giac")

[Out] integrate((b\*sec(d\*sqrt(x) + c) + a)\*x^(3/2), x)

**maple** [F] time = 1.20, size = 0, normalized size = 0.00

$$\int x^{\frac{3}{2}} (a + b \sec(c + d\sqrt{x})) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)\*(a+b\*sec(c+d\*x^(1/2))),x)

[Out] int(x^(3/2)\*(a+b\*sec(c+d\*x^(1/2))),x)

**maxima** [B] time = 0.97, size = 738, normalized size = 2.60

$$2(d\sqrt{x} + c)^5 a - 10(d\sqrt{x} + c)^4 ac + 20(d\sqrt{x} + c)^3 ac^2 - 20(d\sqrt{x} + c)^2 ac^3 + 10(d\sqrt{x} + c)ac^4 + 10bc^4 \log(\sec$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)\*(a+b\*sec(c+d\*x^(1/2))),x, algorithm="maxima")

[Out] 1/5\*(2\*(d\*sqrt(x) + c)^5\*a - 10\*(d\*sqrt(x) + c)^4\*a\*c + 20\*(d\*sqrt(x) + c)^3\*a\*c^2 - 20\*(d\*sqrt(x) + c)^2\*a\*c^3 + 10\*(d\*sqrt(x) + c)\*a\*c^4 + 10\*b\*c^4\*log(sec(d\*sqrt(x) + c) + tan(d\*sqrt(x) + c)) + 5\*(-2\*I\*(d\*sqrt(x) + c)^4\*b + 8\*I\*(d\*sqrt(x) + c)^3\*b\*c - 12\*I\*(d\*sqrt(x) + c)^2\*b\*c^2 + 8\*I\*(d\*sqrt(x) + c)\*b\*c^3)\*arctan2(cos(d\*sqrt(x) + c), sin(d\*sqrt(x) + c) + 1) + 5\*(-2\*I\*(d\*sqrt(x) + c)^4\*b + 8\*I\*(d\*sqrt(x) + c)^3\*b\*c - 12\*I\*(d\*sqrt(x) + c)^2\*b\*

```

c^2 + 8*I*(d*sqrt(x) + c)*b*c^3)*arctan2(cos(d*sqrt(x) + c), -sin(d*sqrt(x)
+ c) + 1) + 5*(-8*I*(d*sqrt(x) + c)^3*b + 24*I*(d*sqrt(x) + c)^2*b*c - 24*
I*(d*sqrt(x) + c)*b*c^2 + 8*I*b*c^3)*dilog(I*e^(I*d*sqrt(x) + I*c)) + 5*(8*
I*(d*sqrt(x) + c)^3*b - 24*I*(d*sqrt(x) + c)^2*b*c + 24*I*(d*sqrt(x) + c)*
b*c^2 - 8*I*b*c^3)*dilog(-I*e^(I*d*sqrt(x) + I*c)) + 5*((d*sqrt(x) + c)^4*b
- 4*(d*sqrt(x) + c)^3*b*c + 6*(d*sqrt(x) + c)^2*b*c^2 - 4*(d*sqrt(x) + c)*
b*c^3)*log(cos(d*sqrt(x) + c)^2 + sin(d*sqrt(x) + c)^2 + 2*sin(d*sqrt(x) + c
) + 1) - 5*((d*sqrt(x) + c)^4*b - 4*(d*sqrt(x) + c)^3*b*c + 6*(d*sqrt(x) +
c)^2*b*c^2 - 4*(d*sqrt(x) + c)*b*c^3)*log(cos(d*sqrt(x) + c)^2 + sin(d*sqrt
(x) + c)^2 - 2*sin(d*sqrt(x) + c) + 1) - 240*b*polylog(5, I*e^(I*d*sqrt(x)
+ I*c)) + 240*b*polylog(5, -I*e^(I*d*sqrt(x) + I*c)) + 5*(48*I*(d*sqrt(x) +
c)*b - 48*I*b*c)*polylog(4, I*e^(I*d*sqrt(x) + I*c)) + 5*(-48*I*(d*sqrt(x)
+ c)*b + 48*I*b*c)*polylog(4, -I*e^(I*d*sqrt(x) + I*c)) + 120*((d*sqrt(x)
+ c)^2*b - 2*(d*sqrt(x) + c)*b*c + b*c^2)*polylog(3, I*e^(I*d*sqrt(x) + I*c
)) - 120*((d*sqrt(x) + c)^2*b - 2*(d*sqrt(x) + c)*b*c + b*c^2)*polylog(3, -
I*e^(I*d*sqrt(x) + I*c)))/d^5

```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^{3/2} \left( a + \frac{b}{\cos(c + d\sqrt{x})} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)\*(a + b/cos(c + d\*x^(1/2))),x)

[Out] int(x^(3/2)\*(a + b/cos(c + d\*x^(1/2))), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3/2} (a + b \sec(c + d\sqrt{x})) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(3/2)\*(a+b\*sec(c+d\*x\*\*(1/2))),x)

[Out] Integral(x\*\*(3/2)\*(a + b\*sec(c + d\*sqrt(x))), x)



### 3.52 $\int \sqrt{x} \left( a + b \sec \left( c + d\sqrt{x} \right) \right) dx$

**Optimal.** Leaf size=158

$$\frac{2}{3}ax^{3/2} - \frac{4b\text{Li}_3\left(-ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{4b\text{Li}_3\left(ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{4ib\sqrt{x}\text{Li}_2\left(-ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{4ib\sqrt{x}\text{Li}_2\left(ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{4ibx \tan^{-1}\left(e^{i(c+d\sqrt{x})}\right)}{d}$$

[Out]  $2/3*a*x^{(3/2)}-4*I*b*x*\arctan(\exp(I*(c+d*x^{(1/2))}))/d-4*b*\text{polylog}(3,-I*\exp(I*(c+d*x^{(1/2))}))/d^3+4*b*\text{polylog}(3,I*\exp(I*(c+d*x^{(1/2))}))/d^3+4*I*b*\text{polylog}(2,-I*\exp(I*(c+d*x^{(1/2))}))*x^{(1/2)}/d^2-4*I*b*\text{polylog}(2,I*\exp(I*(c+d*x^{(1/2))}))*x^{(1/2)}/d^2$

**Rubi [A]** time = 0.13, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$ , Rules used = {14, 4204, 4181, 2531, 2282, 6589}

$$\frac{4ib\sqrt{x}\text{PolyLog}\left(2,-ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{4ib\sqrt{x}\text{PolyLog}\left(2,ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{4b\text{PolyLog}\left(3,-ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{4b\text{PolyLog}\left(3,ie^{i(c+d\sqrt{x})}\right)}{d^3}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x]\*(a + b\*Sec[c + d\*Sqrt[x]]), x]

[Out]  $(2*a*x^{(3/2)})/3 - ((4*I)*b*x*\text{ArcTan}[E^{(I*(c + d*\text{Sqrt}[x])})])/d + ((4*I)*b*\text{Sqrt}[x]*\text{PolyLog}[2, (-I)*E^{(I*(c + d*\text{Sqrt}[x])})])/d^2 - ((4*I)*b*\text{Sqrt}[x]*\text{PolyLog}[2, I*E^{(I*(c + d*\text{Sqrt}[x])})])/d^2 - (4*b*\text{PolyLog}[3, (-I)*E^{(I*(c + d*\text{Sqrt}[x])})])/d^3 + (4*b*\text{PolyLog}[3, I*E^{(I*(c + d*\text{Sqrt}[x])})])/d^3$

#### Rule 14

Int[(u\_)\*((c\_)\*(x\_))^(m\_), x\_Symbol] := Int[ExpandIntegrand[(c\*x)^m\*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a\_ + (b\_)\*(v\_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

#### Rule 2282

Int[u\_, x\_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_)\*((a\_)+(b\_)\*x))\*(F\_)[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

#### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*(a_.) + (b_.)*(x_)))^(n_.)]*((f_.) + (g_.)
*(x_)^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

#### Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_)^(m_.), x_Symbol
] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

#### Rule 4204

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

#### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_)^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

#### Rubi steps

$$\begin{aligned}
\int \sqrt{x} (a + b \sec(c + d\sqrt{x})) dx &= \int (a\sqrt{x} + b\sqrt{x} \sec(c + d\sqrt{x})) dx \\
&= \frac{2}{3}ax^{3/2} + b \int \sqrt{x} \sec(c + d\sqrt{x}) dx \\
&= \frac{2}{3}ax^{3/2} + (2b) \text{Subst} \left( \int x^2 \sec(c + dx) dx, x, \sqrt{x} \right) \\
&= \frac{2}{3}ax^{3/2} - \frac{4ibx \tan^{-1} \left( e^{i(c+d\sqrt{x})} \right)}{d} - \frac{(4b) \text{Subst} \left( \int x \log(1 - ie^{i(c+dx)}) dx, x, \sqrt{x} \right)}{d} \\
&= \frac{2}{3}ax^{3/2} - \frac{4ibx \tan^{-1} \left( e^{i(c+d\sqrt{x})} \right)}{d} + \frac{4ib\sqrt{x} \text{Li}_2 \left( -ie^{i(c+d\sqrt{x})} \right)}{d^2} - \frac{4ib\sqrt{x} \text{Li}_2 \left( ie^{i(c+d\sqrt{x})} \right)}{d^2} \\
&= \frac{2}{3}ax^{3/2} - \frac{4ibx \tan^{-1} \left( e^{i(c+d\sqrt{x})} \right)}{d} + \frac{4ib\sqrt{x} \text{Li}_2 \left( -ie^{i(c+d\sqrt{x})} \right)}{d^2} - \frac{4ib\sqrt{x} \text{Li}_2 \left( ie^{i(c+d\sqrt{x})} \right)}{d^2} \\
&= \frac{2}{3}ax^{3/2} - \frac{4ibx \tan^{-1} \left( e^{i(c+d\sqrt{x})} \right)}{d} + \frac{4ib\sqrt{x} \text{Li}_2 \left( -ie^{i(c+d\sqrt{x})} \right)}{d^2} - \frac{4ib\sqrt{x} \text{Li}_2 \left( ie^{i(c+d\sqrt{x})} \right)}{d^2}
\end{aligned}$$

**Mathematica [A]** time = 0.14, size = 155, normalized size = 0.98

$$\frac{2 \left( ad^3 x^{3/2} - 6ibd^2 x \tan^{-1} \left( e^{i(c+d\sqrt{x})} \right) \right) + 6ibd\sqrt{x} \text{Li}_2 \left( -ie^{i(c+d\sqrt{x})} \right) - 6ibd\sqrt{x} \text{Li}_2 \left( ie^{i(c+d\sqrt{x})} \right) - 6b\text{Li}_3 \left( -ie^{i(c+d\sqrt{x})} \right)}{3d^3}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x]\*(a + b\*Sec[c + d\*Sqrt[x]]),x]

[Out] (2\*(a\*d^3\*x^(3/2) - (6\*I)\*b\*d^2\*x\*ArcTan[E^(I\*(c + d\*Sqrt[x]))]) + (6\*I)\*b\*d\*Sqrt[x]\*PolyLog[2, (-I)\*E^(I\*(c + d\*Sqrt[x]))] - (6\*I)\*b\*d\*Sqrt[x]\*PolyLog[2, I\*E^(I\*(c + d\*Sqrt[x]))] - 6\*b\*PolyLog[3, (-I)\*E^(I\*(c + d\*Sqrt[x]))] + 6\*b\*PolyLog[3, I\*E^(I\*(c + d\*Sqrt[x]))]))/(3\*d^3)

**fricas [F]** time = 0.71, size = 0, normalized size = 0.00

$$\text{integral} \left( b\sqrt{x} \sec(d\sqrt{x} + c) + a\sqrt{x}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sec(c+d\*x^(1/2)))\*x^(1/2),x, algorithm="fricas")

[Out] integral(b\*sqrt(x)\*sec(d\*sqrt(x) + c) + a\*sqrt(x), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sec(d\sqrt{x} + c) + a)\sqrt{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sec(c+d\*x^(1/2)))\*x^(1/2),x, algorithm="giac")

[Out] integrate((b\*sec(d\*sqrt(x) + c) + a)\*sqrt(x), x)

**maple** [F] time = 1.18, size = 0, normalized size = 0.00

$$\int (a + b \sec(c + d\sqrt{x})) \sqrt{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*sec(c+d\*x^(1/2)))\*x^(1/2),x)

[Out] int((a+b\*sec(c+d\*x^(1/2)))\*x^(1/2),x)

**maxima** [B] time = 0.98, size = 374, normalized size = 2.37

$$2(d\sqrt{x} + c)^3 a - 6(d\sqrt{x} + c)^2 ac + 6(d\sqrt{x} + c)ac^2 + 6bc^2 \log(\sec(d\sqrt{x} + c) + \tan(d\sqrt{x} + c)) + 3(-2i(d\sqrt{x} + c) \dots)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sec(c+d\*x^(1/2)))\*x^(1/2),x, algorithm="maxima")

[Out] 1/3\*(2\*(d\*sqrt(x) + c)^3\*a - 6\*(d\*sqrt(x) + c)^2\*a\*c + 6\*(d\*sqrt(x) + c)\*a\*c^2 + 6\*b\*c^2\*log(sec(d\*sqrt(x) + c) + tan(d\*sqrt(x) + c)) + 3\*(-2\*I\*(d\*sqrt(x) + c)^2\*b + 4\*I\*(d\*sqrt(x) + c)\*b\*c)\*arctan2(cos(d\*sqrt(x) + c), sin(d\*sqrt(x) + c) + 1) + 3\*(-2\*I\*(d\*sqrt(x) + c)^2\*b + 4\*I\*(d\*sqrt(x) + c)\*b\*c)\*arctan2(cos(d\*sqrt(x) + c), -sin(d\*sqrt(x) + c) + 1) + 3\*(-4\*I\*(d\*sqrt(x) + c)\*b + 4\*I\*b\*c)\*dilog(I\*e^(I\*d\*sqrt(x) + I\*c)) + 3\*(4\*I\*(d\*sqrt(x) + c)\*b - 4\*I\*b\*c)\*dilog(-I\*e^(I\*d\*sqrt(x) + I\*c)) + 3\*((d\*sqrt(x) + c)^2\*b - 2\*(d\*sqrt(x) + c)\*b\*c)\*log(cos(d\*sqrt(x) + c)^2 + sin(d\*sqrt(x) + c)^2 + 2\*sin(d\*sqrt(x) + c) + 1) - 3\*((d\*sqrt(x) + c)^2\*b - 2\*(d\*sqrt(x) + c)\*b\*c)\*log(cos(d\*sqrt(x) + c)^2 + sin(d\*sqrt(x) + c)^2 - 2\*sin(d\*sqrt(x) + c) + 1) + 12\*b\*polylog(3, I\*e^(I\*d\*sqrt(x) + I\*c)) - 12\*b\*polylog(3, -I\*e^(I\*d\*sqrt(x) + I\*c)))/d^3

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x} \left( a + \frac{b}{\cos(c + d\sqrt{x})} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a + b/cos(c + d*x^(1/2))), x)
```

```
[Out] int(x^(1/2)*(a + b/cos(c + d*x^(1/2))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x} (a + b \sec(c + d\sqrt{x})) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(c+d*x**(1/2)))*x**(1/2), x)
```

```
[Out] Integral(sqrt(x)*(a + b*sec(c + d*sqrt(x))), x)
```

$$3.53 \quad \int \frac{a+b \sec(c+d\sqrt{x})}{\sqrt{x}} dx$$

Optimal. Leaf size=26

$$2a\sqrt{x} + \frac{2b \tanh^{-1}(\sin(c+d\sqrt{x}))}{d}$$

[Out] 2\*b\*arctanh(sin(c+d\*x^(1/2)))/d+2\*a\*x^(1/2)

**Rubi [A]** time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$ , Rules used = {14, 4204, 3770}

$$2a\sqrt{x} + \frac{2b \tanh^{-1}(\sin(c+d\sqrt{x}))}{d}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Sec[c + d\*Sqrt[x]])/Sqrt[x], x]

[Out] 2\*a\*Sqrt[x] + (2\*b\*ArcTanh[Sin[c + d\*Sqrt[x]]])/d

#### Rule 14

```
Int[(u_)*((c_.)*(x_)^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

#### Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]
```

#### Rule 4204

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

#### Rubi steps

$$\begin{aligned}
\int \frac{a + b \sec(c + d\sqrt{x})}{\sqrt{x}} dx &= \int \left( \frac{a}{\sqrt{x}} + \frac{b \sec(c + d\sqrt{x})}{\sqrt{x}} \right) dx \\
&= 2a\sqrt{x} + b \int \frac{\sec(c + d\sqrt{x})}{\sqrt{x}} dx \\
&= 2a\sqrt{x} + (2b) \text{Subst} \left( \int \sec(c + dx) dx, x, \sqrt{x} \right) \\
&= 2a\sqrt{x} + \frac{2b \tanh^{-1}(\sin(c + d\sqrt{x}))}{d}
\end{aligned}$$

**Mathematica [A]** time = 0.08, size = 26, normalized size = 1.00

$$2a\sqrt{x} + \frac{2b \tanh^{-1}(\sin(c + d\sqrt{x}))}{d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Sec[c + d\*Sqrt[x]])/Sqrt[x], x]

[Out] 2\*a\*Sqrt[x] + (2\*b\*ArcTanh[Sin[c + d\*Sqrt[x]])/d

**fricas [A]** time = 0.66, size = 41, normalized size = 1.58

$$\frac{2ad\sqrt{x} + b \log(\sin(d\sqrt{x} + c) + 1) - b \log(-\sin(d\sqrt{x} + c) + 1)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sec(c+d\*x^(1/2)))/x^(1/2), x, algorithm="fricas")

[Out] (2\*a\*d\*sqrt(x) + b\*log(sin(d\*sqrt(x) + c) + 1) - b\*log(-sin(d\*sqrt(x) + c) + 1))/d

**giac [B]** time = 0.85, size = 50, normalized size = 1.92

$$\frac{2 \left( (d\sqrt{x} + c)a + b \log \left( \left| \tan \left( \frac{1}{2} d\sqrt{x} + \frac{1}{2} c \right) + 1 \right| \right) - b \log \left( \left| \tan \left( \frac{1}{2} d\sqrt{x} + \frac{1}{2} c \right) - 1 \right| \right) \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sec(c+d\*x^(1/2)))/x^(1/2), x, algorithm="giac")

[Out]  $2*((d*\sqrt{x} + c)*a + b*\log(\text{abs}(\tan(1/2*d*\sqrt{x} + 1/2*c) + 1)) - b*\log(\text{abs}(\tan(1/2*d*\sqrt{x} + 1/2*c) - 1)))/d$

**maple** [A] time = 0.26, size = 32, normalized size = 1.23

$$2a\sqrt{x} + \frac{2b \ln(\sec(c + d\sqrt{x}) + \tan(c + d\sqrt{x}))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*sec(c+d*x^(1/2)))/x^(1/2),x)`

[Out]  $2*a*x^(1/2)+2*b/d*\ln(\sec(c+d*x^(1/2))+\tan(c+d*x^(1/2)))$

**maxima** [A] time = 0.37, size = 31, normalized size = 1.19

$$2a\sqrt{x} + \frac{2b \log(\sec(d\sqrt{x} + c) + \tan(d\sqrt{x} + c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sec(c+d*x^(1/2)))/x^(1/2),x, algorithm="maxima")`

[Out]  $2*a*\sqrt{x} + 2*b*\log(\sec(d*\sqrt{x} + c) + \tan(d*\sqrt{x} + c))/d$

**mupad** [B] time = 2.94, size = 71, normalized size = 2.73

$$2a\sqrt{x} - \frac{2b \ln\left(\frac{b^{2i} - 2b e^{d\sqrt{x}} e^{ci}}{\sqrt{x}}\right)}{d} + \frac{2b \ln\left(\frac{b^{2i} + 2b e^{d\sqrt{x}} e^{ci}}{\sqrt{x}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b/cos(c + d*x^(1/2)))/x^(1/2),x)`

[Out]  $2*a*x^(1/2) - (2*b*\log((b^{2i} - 2*b*\exp(d*x^(1/2)*1i)*\exp(c*1i))/x^(1/2)))/d + (2*b*\log((b^{2i} + 2*b*\exp(d*x^(1/2)*1i)*\exp(c*1i))/x^(1/2)))/d$

**sympy** [A] time = 4.74, size = 56, normalized size = 2.15

$$\begin{cases} \frac{2a(c+d\sqrt{x})+2b \log(\tan(c+d\sqrt{x})+\sec(c+d\sqrt{x}))}{d} & \text{for } d \neq 0 \\ -\sqrt{x}(-2a - 2b \sec(c)) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sec(c+d*x**(1/2)))/x**(1/2),x)`

[Out] `Piecewise(((2*a*(c + d*sqrt(x)) + 2*b*log(tan(c + d*sqrt(x)) + sec(c + d*sqrt(x))))/d, Ne(d, 0)), (-sqrt(x)*(-2*a - 2*b*sec(c)), True))`



$$3.54 \quad \int \frac{a+b \sec(c+d\sqrt{x})}{x^{3/2}} dx$$

Optimal. Leaf size=30

$$b \operatorname{Int} \left( \frac{\sec(c+d\sqrt{x})}{x^{3/2}}, x \right) - \frac{2a}{\sqrt{x}}$$

[Out]  $-2*a/x^{(1/2)}+b*\operatorname{Unintegrable}(\sec(c+d*x^{(1/2)})/x^{(3/2)}, x)$

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{a+b \sec(c+d\sqrt{x})}{x^{3/2}} dx$$

Verification is Not applicable to the result.

[In]  $\operatorname{Int}[(a + b*\operatorname{Sec}[c + d*\operatorname{Sqrt}[x]])/x^{(3/2)}, x]$

[Out]  $(-2*a)/\operatorname{Sqrt}[x] + b*\operatorname{Defer}[\operatorname{Int}[\operatorname{Sec}[c + d*\operatorname{Sqrt}[x]]/x^{(3/2)}, x]$

Rubi steps

$$\begin{aligned} \int \frac{a+b \sec(c+d\sqrt{x})}{x^{3/2}} dx &= \int \left( \frac{a}{x^{3/2}} + \frac{b \sec(c+d\sqrt{x})}{x^{3/2}} \right) dx \\ &= -\frac{2a}{\sqrt{x}} + b \int \frac{\sec(c+d\sqrt{x})}{x^{3/2}} dx \end{aligned}$$

Mathematica [A] time = 2.90, size = 0, normalized size = 0.00

$$\int \frac{a+b \sec(c+d\sqrt{x})}{x^{3/2}} dx$$

Verification is Not applicable to the result.

[In]  $\operatorname{Integrate}[(a + b*\operatorname{Sec}[c + d*\operatorname{Sqrt}[x]])/x^{(3/2)}, x]$

[Out]  $\operatorname{Integrate}[(a + b*\operatorname{Sec}[c + d*\operatorname{Sqrt}[x]])/x^{(3/2)}, x]$

fricas [A] time = 0.59, size = 0, normalized size = 0.00

$$\operatorname{integral} \left( \frac{b\sqrt{x} \sec(d\sqrt{x} + c) + a\sqrt{x}}{x^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sec(c+d\*x^(1/2)))/x^(3/2),x, algorithm="fricas")

[Out] integral((b\*sqrt(x)\*sec(d\*sqrt(x) + c) + a\*sqrt(x))/x^2, x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \sec(d\sqrt{x} + c) + a}{x^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sec(c+d\*x^(1/2)))/x^(3/2),x, algorithm="giac")

[Out] integrate((b\*sec(d\*sqrt(x) + c) + a)/x^(3/2), x)

**maple** [A] time = 1.38, size = 0, normalized size = 0.00

$$\int \frac{a + b \sec(c + d\sqrt{x})}{x^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*sec(c+d\*x^(1/2)))/x^(3/2),x)

[Out] int((a+b\*sec(c+d\*x^(1/2)))/x^(3/2),x)

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sec(c+d\*x^(1/2)))/x^(3/2),x, algorithm="maxima")

[Out] Timed out

**mupad** [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{a + \frac{b}{\cos(c + d\sqrt{x})}}{x^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cos(c + d\*x^(1/2)))/x^(3/2),x)

[Out] `int((a + b/cos(c + d*x^(1/2)))/x^(3/2), x)`

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \sec(c + d\sqrt{x})}{x^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sec(c+d*x**(1/2)))/x**(3/2), x)`

[Out] `Integral((a + b*sec(c + d*sqrt(x)))/x**(3/2), x)`

$$3.55 \quad \int \frac{a+b \sec(c+d\sqrt{x})}{x^{5/2}} dx$$

Optimal. Leaf size=32

$$b \operatorname{Int}\left(\frac{\sec(c+d\sqrt{x})}{x^{5/2}}, x\right) - \frac{2a}{3x^{3/2}}$$

[Out]  $-2/3*a/x^{(3/2)}+b*\operatorname{Unintegrable}(\sec(c+d*x^{(1/2)})/x^{(5/2)}, x)$

**Rubi [A]** time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{a+b \sec(c+d\sqrt{x})}{x^{5/2}} dx$$

Verification is Not applicable to the result.

[In]  $\operatorname{Int}[(a + b*\operatorname{Sec}[c + d*\operatorname{Sqrt}[x]])/x^{(5/2)}, x]$

[Out]  $(-2*a)/(3*x^{(3/2)}) + b*\operatorname{Defer}[\operatorname{Int}[\operatorname{Sec}[c + d*\operatorname{Sqrt}[x]]/x^{(5/2)}, x]$

Rubi steps

$$\begin{aligned} \int \frac{a+b \sec(c+d\sqrt{x})}{x^{5/2}} dx &= \int \left( \frac{a}{x^{5/2}} + \frac{b \sec(c+d\sqrt{x})}{x^{5/2}} \right) dx \\ &= -\frac{2a}{3x^{3/2}} + b \int \frac{\sec(c+d\sqrt{x})}{x^{5/2}} dx \end{aligned}$$

**Mathematica [A]** time = 2.83, size = 0, normalized size = 0.00

$$\int \frac{a+b \sec(c+d\sqrt{x})}{x^{5/2}} dx$$

Verification is Not applicable to the result.

[In]  $\operatorname{Integrate}[(a + b*\operatorname{Sec}[c + d*\operatorname{Sqrt}[x]])/x^{(5/2)}, x]$

[Out]  $\operatorname{Integrate}[(a + b*\operatorname{Sec}[c + d*\operatorname{Sqrt}[x]])/x^{(5/2)}, x]$

**fricas [A]** time = 0.55, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{b\sqrt{x} \sec(d\sqrt{x} + c) + a\sqrt{x}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sec(c+d\*x^(1/2)))/x^(5/2),x, algorithm="fricas")

[Out] integral((b\*sqrt(x)\*sec(d\*sqrt(x) + c) + a\*sqrt(x))/x^3, x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \sec(d\sqrt{x} + c) + a}{x^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sec(c+d\*x^(1/2)))/x^(5/2),x, algorithm="giac")

[Out] integrate((b\*sec(d\*sqrt(x) + c) + a)/x^(5/2), x)

**maple** [A] time = 1.46, size = 0, normalized size = 0.00

$$\int \frac{a + b \sec(c + d\sqrt{x})}{x^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*sec(c+d\*x^(1/2)))/x^(5/2),x)

[Out] int((a+b\*sec(c+d\*x^(1/2)))/x^(5/2),x)

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sec(c+d\*x^(1/2)))/x^(5/2),x, algorithm="maxima")

[Out] Timed out

**mupad** [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{a + \frac{b}{\cos(c+d\sqrt{x})}}{x^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cos(c + d\*x^(1/2)))/x^(5/2),x)

[Out] `int((a + b/cos(c + d*x^(1/2)))/x^(5/2), x)`

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \sec(c + d\sqrt{x})}{x^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sec(c+d*x**(1/2)))/x**(5/2), x)`

[Out] `Integral((a + b*sec(c + d*sqrt(x)))/x**(5/2), x)`

### 3.56 $\int x^{3/2} \left( a + b \sec \left( c + d\sqrt{x} \right) \right)^2 dx$

**Optimal.** Leaf size=451

$$\frac{2}{5}a^2x^{5/2} + \frac{96ab\text{Li}_5\left(-ie^{i(c+d\sqrt{x})}\right)}{d^5} - \frac{96ab\text{Li}_5\left(ie^{i(c+d\sqrt{x})}\right)}{d^5} - \frac{96iab\sqrt{x}\text{Li}_4\left(-ie^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{96iab\sqrt{x}\text{Li}_4\left(ie^{i(c+d\sqrt{x})}\right)}{d^4} - 48$$

```
[Out] 16*I*a*b*x^(3/2)*polylog(2,-I*exp(I*(c+d*x^(1/2))))/d^2+2/5*a^2*x^(5/2)-12*I*b^2*x*polylog(2,-exp(2*I*(c+d*x^(1/2))))/d^3+8*b^2*x^(3/2)*ln(1+exp(2*I*(c+d*x^(1/2))))/d^2-8*I*a*b*x^2*arctan(exp(I*(c+d*x^(1/2))))/d+96*I*a*b*polylog(4,I*exp(I*(c+d*x^(1/2))))*x^(1/2)/d^4-16*I*a*b*x^(3/2)*polylog(2,I*exp(I*(c+d*x^(1/2))))/d^2-48*a*b*x*polylog(3,-I*exp(I*(c+d*x^(1/2))))/d^3+48*a*b*x*polylog(3,I*exp(I*(c+d*x^(1/2))))/d^3-2*I*b^2*x^2/d+96*a*b*polylog(5,-I*exp(I*(c+d*x^(1/2))))/d^5-96*a*b*polylog(5,I*exp(I*(c+d*x^(1/2))))/d^5+12*b^2*polylog(3,-exp(2*I*(c+d*x^(1/2))))*x^(1/2)/d^4-96*I*a*b*polylog(4,-I*exp(I*(c+d*x^(1/2))))*x^(1/2)/d^4+6*I*b^2*polylog(4,-exp(2*I*(c+d*x^(1/2))))/d^5+2*b^2*x^2*tan(c+d*x^(1/2))/d
```

**Rubi [A]** time = 0.54, antiderivative size = 451, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 10, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$ , Rules used = {4204, 4190, 4181, 2531, 6609, 2282, 6589, 4184, 3719, 2190}

$$\frac{16iabx^{3/2}\text{PolyLog}\left(2,-ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{16iabx^{3/2}\text{PolyLog}\left(2,ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{48abx\text{PolyLog}\left(3,-ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{48abx\text{PolyLog}\left(3,ie^{i(c+d\sqrt{x})}\right)}{d^3} - 48$$

Antiderivative was successfully verified.

[In] Int[x^(3/2)\*(a + b\*Sec[c + d\*Sqrt[x]])^2,x]

```
[Out] ((-2*I)*b^2*x^2)/d + (2*a^2*x^(5/2))/5 - ((8*I)*a*b*x^2*ArcTan[E^(I*(c + d*Sqrt[x]))])/d + (8*b^2*x^(3/2)*Log[1 + E^((2*I)*(c + d*Sqrt[x]))])/d^2 + ((16*I)*a*b*x^(3/2)*PolyLog[2, (-I)*E^(I*(c + d*Sqrt[x]))])/d^2 - ((16*I)*a*b*x^(3/2)*PolyLog[2, I*E^(I*(c + d*Sqrt[x]))])/d^2 - ((12*I)*b^2*x*polylog(2, -E^((2*I)*(c + d*Sqrt[x]))))/d^3 - (48*a*b*x*polylog(3, (-I)*E^(I*(c + d*Sqrt[x]))))/d^3 + (48*a*b*x*polylog(3, I*E^(I*(c + d*Sqrt[x]))))/d^3 + (12*b^2*Sqrt[x]*polylog(3, -E^((2*I)*(c + d*Sqrt[x]))))/d^4 - ((96*I)*a*b*Sqrt[x]*polylog(4, (-I)*E^(I*(c + d*Sqrt[x]))))/d^4 + ((96*I)*a*b*Sqrt[x]*polylog(4, I*E^(I*(c + d*Sqrt[x]))))/d^4 + ((6*I)*b^2*polylog(4, -E^((2*I)*(c + d*Sqrt[x]))))/d^5 + (96*a*b*polylog(5, (-I)*E^(I*(c + d*Sqrt[x]))))/d^5 - (96*a*b*polylog(5, I*E^(I*(c + d*Sqrt[x]))))/d^5 + (2*b^2*x^2*Tan[c + d*Sqrt[x]])/d
```

**Rule 2190**

```
Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 2531

```
Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_))))^(n_)]*((f_) + (g_)
*(x_))^(m_), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 3719

```
Int[((c_) + (d_)*(x_))^(m_)*tan[(e_) + (f_)*(x_)], x_Symbol] := Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

### Rule 4181

```
Int[csc[(e_) + Pi*(k_) + (f_)*(x_)]*((c_) + (d_)*(x_))^(m_), x_Symbol
] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x)
)], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

### Rule 4184

```
Int[csc[(e_) + (f_)*(x_)]^2*((c_) + (d_)*(x_))^(m_), x_Symbol] := -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

### Rule 4190



```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

#### Rule 4204

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol]
:= Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

#### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

#### Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_)))^(p_.)]], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

#### Rubi steps

$$\begin{aligned}
\int x^{3/2} (a + b \sec(c + d\sqrt{x}))^2 dx &= 2 \operatorname{Subst} \left( \int x^4 (a + b \sec(c + dx))^2 dx, x, \sqrt{x} \right) \\
&= 2 \operatorname{Subst} \left( \int (a^2 x^4 + 2abx^4 \sec(c + dx) + b^2 x^4 \sec^2(c + dx)) dx, x, \sqrt{x} \right) \\
&= \frac{2}{5} a^2 x^{5/2} + (4ab) \operatorname{Subst} \left( \int x^4 \sec(c + dx) dx, x, \sqrt{x} \right) + (2b^2) \operatorname{Subst} \left( \int x^4 \sec^2(c + dx) dx, x, \sqrt{x} \right) \\
&= \frac{2}{5} a^2 x^{5/2} - \frac{8iabx^2 \tan^{-1} \left( e^{i(c+d\sqrt{x})} \right)}{d} + \frac{2b^2 x^2 \tan(c + d\sqrt{x})}{d} - \frac{(16ab) \operatorname{Subst} \left( \int x^4 \sec(c + dx) dx, x, \sqrt{x} \right)}{d} \\
&= -\frac{2ib^2 x^2}{d} + \frac{2}{5} a^2 x^{5/2} - \frac{8iabx^2 \tan^{-1} \left( e^{i(c+d\sqrt{x})} \right)}{d} + \frac{16iabx^{3/2} \operatorname{Li}_2 \left( -ie^{i(c+d\sqrt{x})} \right)}{d^2} \\
&= -\frac{2ib^2 x^2}{d} + \frac{2}{5} a^2 x^{5/2} - \frac{8iabx^2 \tan^{-1} \left( e^{i(c+d\sqrt{x})} \right)}{d} + \frac{8b^2 x^{3/2} \log \left( 1 + e^{2i(c+d\sqrt{x})} \right)}{d^2} \\
&= -\frac{2ib^2 x^2}{d} + \frac{2}{5} a^2 x^{5/2} - \frac{8iabx^2 \tan^{-1} \left( e^{i(c+d\sqrt{x})} \right)}{d} + \frac{8b^2 x^{3/2} \log \left( 1 + e^{2i(c+d\sqrt{x})} \right)}{d^2} \\
&= -\frac{2ib^2 x^2}{d} + \frac{2}{5} a^2 x^{5/2} - \frac{8iabx^2 \tan^{-1} \left( e^{i(c+d\sqrt{x})} \right)}{d} + \frac{8b^2 x^{3/2} \log \left( 1 + e^{2i(c+d\sqrt{x})} \right)}{d^2} \\
&= -\frac{2ib^2 x^2}{d} + \frac{2}{5} a^2 x^{5/2} - \frac{8iabx^2 \tan^{-1} \left( e^{i(c+d\sqrt{x})} \right)}{d} + \frac{8b^2 x^{3/2} \log \left( 1 + e^{2i(c+d\sqrt{x})} \right)}{d^2} \\
&= -\frac{2ib^2 x^2}{d} + \frac{2}{5} a^2 x^{5/2} - \frac{8iabx^2 \tan^{-1} \left( e^{i(c+d\sqrt{x})} \right)}{d} + \frac{8b^2 x^{3/2} \log \left( 1 + e^{2i(c+d\sqrt{x})} \right)}{d^2}
\end{aligned}$$

**Mathematica [A]** time = 1.58, size = 443, normalized size = 0.98

$$\frac{2 \left( a^2 d^5 x^{5/2} - 20iab d^4 x^2 \tan^{-1} \left( e^{i(c+d\sqrt{x})} \right) \right) + 40iab d^3 x^{3/2} \operatorname{Li}_2 \left( -ie^{i(c+d\sqrt{x})} \right) - 40iab d^3 x^{3/2} \operatorname{Li}_2 \left( ie^{i(c+d\sqrt{x})} \right) - 120abd^2 x^{3/2} \log \left( 1 + e^{2i(c+d\sqrt{x})} \right)}{d^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^(3/2)\*(a + b\*Sec[c + d\*Sqrt[x]])^2,x]

[Out] (2\*((-5\*I)\*b^2\*d^4\*x^2 + a^2\*d^5\*x^(5/2) - (20\*I)\*a\*b\*d^4\*x^2\*ArcTan[E^(I\*(c + d\*Sqrt[x]))]) + 20\*b^2\*d^3\*x^(3/2)\*Log[1 + E^((2\*I)\*(c + d\*Sqrt[x]))] + (40\*I)\*a\*b\*d^3\*x^(3/2)\*PolyLog[2, (-I)\*E^(I\*(c + d\*Sqrt[x]))] - (40\*I)\*a\*b\*d^3\*x^(3/2)\*PolyLog[2, I\*E^(I\*(c + d\*Sqrt[x]))] - (30\*I)\*b^2\*d^2\*x\*PolyLog[2, -E^((2\*I)\*(c + d\*Sqrt[x]))] - 120\*a\*b\*d^2\*x\*PolyLog[3, (-I)\*E^(I\*(c + d\*Sqrt[x]))])

Sqrt[x]))] + 120\*a\*b\*d^2\*x\*PolyLog[3, I\*E^(I\*(c + d\*Sqrt[x]))] + 30\*b^2\*d\*Sqrt[x]\*PolyLog[3, -E^((2\*I)\*(c + d\*Sqrt[x]))] - (240\*I)\*a\*b\*d\*Sqrt[x]\*PolyLog[4, (-I)\*E^(I\*(c + d\*Sqrt[x]))] + (240\*I)\*a\*b\*d\*Sqrt[x]\*PolyLog[4, I\*E^(I\*(c + d\*Sqrt[x]))] + (15\*I)\*b^2\*PolyLog[4, -E^((2\*I)\*(c + d\*Sqrt[x]))] + 240\*a\*b\*PolyLog[5, (-I)\*E^(I\*(c + d\*Sqrt[x]))] - 240\*a\*b\*PolyLog[5, I\*E^(I\*(c + d\*Sqrt[x]))] + 5\*b^2\*d^4\*x^2\*Tan[c + d\*Sqrt[x]])/(5\*d^5)

**fricas** [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(b^2x^{\frac{3}{2}}\sec(d\sqrt{x}+c)^2+2abx^{\frac{3}{2}}\sec(d\sqrt{x}+c)+a^2x^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)\*(a+b\*sec(c+d\*x^(1/2)))^2,x, algorithm="fricas")

[Out] integral(b^2\*x^(3/2)\*sec(d\*sqrt(x) + c)^2 + 2\*a\*b\*x^(3/2)\*sec(d\*sqrt(x) + c) + a^2\*x^(3/2), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sec(d\sqrt{x} + c) + a)^2 x^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)\*(a+b\*sec(c+d\*x^(1/2)))^2,x, algorithm="giac")

[Out] integrate((b\*sec(d\*sqrt(x) + c) + a)^2\*x^(3/2), x)

**maple** [F] time = 1.29, size = 0, normalized size = 0.00

$$\int x^{\frac{3}{2}} (a + b \sec(c + d\sqrt{x}))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)\*(a+b\*sec(c+d\*x^(1/2)))^2,x)

[Out] int(x^(3/2)\*(a+b\*sec(c+d\*x^(1/2)))^2,x)

**maxima** [B] time = 0.87, size = 2876, normalized size = 6.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)\*(a+b\*sec(c+d\*x^(1/2)))^2,x, algorithm="maxima")

```
[Out] 2/5*((d*sqrt(x) + c)^5*a^2 - 5*(d*sqrt(x) + c)^4*a^2*c + 10*(d*sqrt(x) + c)^3*a^2*c^2 - 10*(d*sqrt(x) + c)^2*a^2*c^3 + 5*(d*sqrt(x) + c)*a^2*c^4 + 10*a*b*c^4*log(sec(d*sqrt(x) + c) + tan(d*sqrt(x) + c)) + 5*(6*b^2*c^4 - (6*(d*sqrt(x) + c)^4*a*b - 24*(d*sqrt(x) + c)^3*a*b*c + 36*(d*sqrt(x) + c)^2*a*b*c^2 - 24*(d*sqrt(x) + c)*a*b*c^3 + 6*((d*sqrt(x) + c)^4*a*b - 4*(d*sqrt(x) + c)^3*a*b*c + 6*(d*sqrt(x) + c)^2*a*b*c^2 - 4*(d*sqrt(x) + c)*a*b*c^3)*cos(2*d*sqrt(x) + 2*c) - (-6*I*(d*sqrt(x) + c)^4*a*b + 24*I*(d*sqrt(x) + c)^3*a*b*c - 36*I*(d*sqrt(x) + c)^2*a*b*c^2 + 24*I*(d*sqrt(x) + c)*a*b*c^3)*sin(2*d*sqrt(x) + 2*c))*arctan2(cos(d*sqrt(x) + c), sin(d*sqrt(x) + c) + 1) - (6*(d*sqrt(x) + c)^4*a*b - 24*(d*sqrt(x) + c)^3*a*b*c + 36*(d*sqrt(x) + c)^2*a*b*c^2 - 24*(d*sqrt(x) + c)*a*b*c^3 + 6*((d*sqrt(x) + c)^4*a*b - 4*(d*sqrt(x) + c)^3*a*b*c + 6*(d*sqrt(x) + c)^2*a*b*c^2 - 4*(d*sqrt(x) + c)*a*b*c^3)*cos(2*d*sqrt(x) + 2*c) - (-6*I*(d*sqrt(x) + c)^4*a*b + 24*I*(d*sqrt(x) + c)^3*a*b*c - 36*I*(d*sqrt(x) + c)^2*a*b*c^2 + 24*I*(d*sqrt(x) + c)*a*b*c^3)*sin(2*d*sqrt(x) + 2*c))*arctan2(cos(d*sqrt(x) + c), -sin(d*sqrt(x) + c) + 1) + (16*(d*sqrt(x) + c)^3*b^2 - 36*(d*sqrt(x) + c)^2*b^2*c + 36*(d*sqrt(x) + c)*b^2*c^2 - 12*b^2*c^3 + 4*(4*(d*sqrt(x) + c)^3*b^2 - 9*(d*sqrt(x) + c)^2*b^2*c + 9*(d*sqrt(x) + c)*b^2*c^2 - 3*b^2*c^3)*cos(2*d*sqrt(x) + 2*c) + (16*I*(d*sqrt(x) + c)^3*b^2 - 36*I*(d*sqrt(x) + c)^2*b^2*c + 36*I*(d*sqrt(x) + c)*b^2*c^2 - 12*I*b^2*c^3)*sin(2*d*sqrt(x) + 2*c))*arctan2(sin(2*d*sqrt(x) + 2*c), cos(2*d*sqrt(x) + 2*c) + 1) - 6*((d*sqrt(x) + c)^4*b^2 - 4*(d*sqrt(x) + c)^3*b^2*c + 6*(d*sqrt(x) + c)^2*b^2*c^2 - 4*(d*sqrt(x) + c)*b^2*c^3)*cos(2*d*sqrt(x) + 2*c) - (24*(d*sqrt(x) + c)^2*b^2 - 36*(d*sqrt(x) + c)*b^2*c + 18*b^2*c^2 + 6*(4*(d*sqrt(x) + c)^2*b^2 - 6*(d*sqrt(x) + c)*b^2*c + 3*b^2*c^2)*cos(2*d*sqrt(x) + 2*c) - (-24*I*(d*sqrt(x) + c)^2*b^2 + 36*I*(d*sqrt(x) + c)*b^2*c - 18*I*b^2*c^2)*sin(2*d*sqrt(x) + 2*c))*dilog(-e^(2*I*d*sqrt(x) + 2*I*c)) - (24*(d*sqrt(x) + c)^3*a*b - 72*(d*sqrt(x) + c)^2*a*b*c + 72*(d*sqrt(x) + c)*a*b*c^2 - 24*a*b*c^3 + 24*((d*sqrt(x) + c)^3*a*b - 3*(d*sqrt(x) + c)^2*a*b*c + 3*(d*sqrt(x) + c)*a*b*c^2 - a*b*c^3)*cos(2*d*sqrt(x) + 2*c) - (-24*I*(d*sqrt(x) + c)^3*a*b + 72*I*(d*sqrt(x) + c)^2*a*b*c - 72*I*(d*sqrt(x) + c)*a*b*c^2 + 24*I*a*b*c^3)*sin(2*d*sqrt(x) + 2*c))*dilog(I*e^(I*d*sqrt(x) + I*c)) + (24*(d*sqrt(x) + c)^3*a*b - 72*(d*sqrt(x) + c)^2*a*b*c + 72*(d*sqrt(x) + c)*a*b*c^2 - 24*a*b*c^3 + 24*((d*sqrt(x) + c)^3*a*b - 3*(d*sqrt(x) + c)^2*a*b*c + 3*(d*sqrt(x) + c)*a*b*c^2 - a*b*c^3)*cos(2*d*sqrt(x) + 2*c) + (24*I*(d*sqrt(x) + c)^3*a*b - 72*I*(d*sqrt(x) + c)^2*a*b*c + 72*I*(d*sqrt(x) + c)*a*b*c^2 - 24*I*a*b*c^3)*sin(2*d*sqrt(x) + 2*c))*dilog(-I*e^(I*d*sqrt(x) + I*c)) + (-8*I*(d*sqrt(x) + c)^3*b^2 + 18*I*(d*sqrt(x) + c)^2*b^2*c - 18*I*(d*sqrt(x) + c)*b^2*c^2 + 6*I*b^2*c^3 + (-8*I*(d*sqrt(x) + c)^3*b^2 + 18*I*(d*sqrt(x) + c)^2*b^2*c - 18*I*(d*sqrt(x) + c)*b^2*c^2 + 6*I*b^2*c^3)*cos(2*d*sqrt(x) + 2*c) + 2*(4*(d*sqrt(x) + c)^3*b^2 - 9*(d*sqrt(x) + c)^2*b^2*c + 9*(d*sqrt(x) + c)*b^2*c^2 - 3*b^2*c^3)*sin(2*d*sqrt(x) + 2*c))*log(cos(2*d*sqrt(x) + 2*c)^2 + sin(2*d*sqrt(x) + 2*c)^2 + 2*cos(2*d*sqrt(x) + 2*c) + 1) + (-3*I*(d*sqrt(x) + c)^4*a*b + 12*I*(d*sqrt(x) + c)^3*a*b*c - 18*I*(d*sqrt(x) + c)^2*a*b*c^2 + 12*I*(d*sqrt(x) + c)*a*b*c^3 + (-3*I*(d*sqrt(x) + c)^4*a*b + 12*I*(d*sqrt(x) + c)^3*a*b*c - 18*I*(d
```

```

sqrt(x) + c)^2*a*b*c^2 + 12*I*(d*sqrt(x) + c)*a*b*c^3)*cos(2*d*sqrt(x) + 2*
c) + 3*((d*sqrt(x) + c)^4*a*b - 4*(d*sqrt(x) + c)^3*a*b*c + 6*(d*sqrt(x) +
c)^2*a*b*c^2 - 4*(d*sqrt(x) + c)*a*b*c^3)*sin(2*d*sqrt(x) + 2*c))*log(cos(d
*sqrt(x) + c)^2 + sin(d*sqrt(x) + c)^2 + 2*sin(d*sqrt(x) + c) + 1) + (3*I*(
d*sqrt(x) + c)^4*a*b - 12*I*(d*sqrt(x) + c)^3*a*b*c + 18*I*(d*sqrt(x) + c)^
2*a*b*c^2 - 12*I*(d*sqrt(x) + c)*a*b*c^3 + (3*I*(d*sqrt(x) + c)^4*a*b - 12*
I*(d*sqrt(x) + c)^3*a*b*c + 18*I*(d*sqrt(x) + c)^2*a*b*c^2 - 12*I*(d*sqrt(x)
) + c)*a*b*c^3)*cos(2*d*sqrt(x) + 2*c) - 3*((d*sqrt(x) + c)^4*a*b - 4*(d*sq
rt(x) + c)^3*a*b*c + 6*(d*sqrt(x) + c)^2*a*b*c^2 - 4*(d*sqrt(x) + c)*a*b*c^
3)*sin(2*d*sqrt(x) + 2*c))*log(cos(d*sqrt(x) + c)^2 + sin(d*sqrt(x) + c)^2
- 2*sin(d*sqrt(x) + c) + 1) + (144*I*a*b*cos(2*d*sqrt(x) + 2*c) - 144*a*b*s
in(2*d*sqrt(x) + 2*c) + 144*I*a*b)*polylog(5, I*e^(I*d*sqrt(x) + I*c)) + (-
144*I*a*b*cos(2*d*sqrt(x) + 2*c) + 144*a*b*sin(2*d*sqrt(x) + 2*c) - 144*I*a
*b)*polylog(5, -I*e^(I*d*sqrt(x) + I*c)) + 12*(b^2*cos(2*d*sqrt(x) + 2*c) +
I*b^2*sin(2*d*sqrt(x) + 2*c) + b^2)*polylog(4, -e^(2*I*d*sqrt(x) + 2*I*c))
+ (144*(d*sqrt(x) + c)*a*b - 144*a*b*c + 144*((d*sqrt(x) + c)*a*b - a*b*c)
*cos(2*d*sqrt(x) + 2*c) + (144*I*(d*sqrt(x) + c)*a*b - 144*I*a*b*c)*sin(2*d
*sqrt(x) + 2*c))*polylog(4, I*e^(I*d*sqrt(x) + I*c)) - (144*(d*sqrt(x) + c)
*a*b - 144*a*b*c + 144*((d*sqrt(x) + c)*a*b - a*b*c)*cos(2*d*sqrt(x) + 2*c)
- (-144*I*(d*sqrt(x) + c)*a*b + 144*I*a*b*c)*sin(2*d*sqrt(x) + 2*c))*polyl
og(4, -I*e^(I*d*sqrt(x) + I*c)) + (-24*I*(d*sqrt(x) + c)*b^2 + 18*I*b^2*c +
(-24*I*(d*sqrt(x) + c)*b^2 + 18*I*b^2*c)*cos(2*d*sqrt(x) + 2*c) + 6*(4*(d*
sqrt(x) + c)*b^2 - 3*b^2*c)*sin(2*d*sqrt(x) + 2*c))*polylog(3, -e^(2*I*d*sq
rt(x) + 2*I*c)) + (-72*I*(d*sqrt(x) + c)^2*a*b + 144*I*(d*sqrt(x) + c)*a*b*
c - 72*I*a*b*c^2 + (-72*I*(d*sqrt(x) + c)^2*a*b + 144*I*(d*sqrt(x) + c)*a*b
*c - 72*I*a*b*c^2)*cos(2*d*sqrt(x) + 2*c) + 72*((d*sqrt(x) + c)^2*a*b - 2*(
d*sqrt(x) + c)*a*b*c + a*b*c^2)*sin(2*d*sqrt(x) + 2*c))*polylog(3, I*e^(I*d
*sqrt(x) + I*c)) + (72*I*(d*sqrt(x) + c)^2*a*b - 144*I*(d*sqrt(x) + c)*a*b*
c + 72*I*a*b*c^2 + (72*I*(d*sqrt(x) + c)^2*a*b - 144*I*(d*sqrt(x) + c)*a*b*
c + 72*I*a*b*c^2)*cos(2*d*sqrt(x) + 2*c) - 72*((d*sqrt(x) + c)^2*a*b - 2*(d
*sqrt(x) + c)*a*b*c + a*b*c^2)*sin(2*d*sqrt(x) + 2*c))*polylog(3, -I*e^(I*d
*sqrt(x) + I*c)) + (-6*I*(d*sqrt(x) + c)^4*b^2 + 24*I*(d*sqrt(x) + c)^3*b^2
*c - 36*I*(d*sqrt(x) + c)^2*b^2*c^2 + 24*I*(d*sqrt(x) + c)*b^2*c^3)*sin(2*d
*sqrt(x) + 2*c))/(-3*I*cos(2*d*sqrt(x) + 2*c) + 3*sin(2*d*sqrt(x) + 2*c) -
3*I))/d^5

```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int x^{3/2} \left( a + \frac{b}{\cos(c + d\sqrt{x})} \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)\*(a + b/cos(c + d\*x^(1/2)))^2,x)

```
[Out] int(x^(3/2)*(a + b/cos(c + d*x^(1/2)))^2, x)
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int x^{\frac{3}{2}} (a + b \sec(c + d\sqrt{x}))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(a+b*sec(c+d*x**(1/2)))**2,x)
```

```
[Out] Integral(x**(3/2)*(a + b*sec(c + d*sqrt(x)))**2, x)
```

### 3.57 $\int \sqrt{x} \left( a + b \sec \left( c + d\sqrt{x} \right) \right)^2 dx$

**Optimal.** Leaf size=255

$$\frac{2}{3}a^2x^{3/2} - \frac{8ab\text{Li}_3\left(-ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{8ab\text{Li}_3\left(ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{8iab\sqrt{x}\text{Li}_2\left(-ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{8iab\sqrt{x}\text{Li}_2\left(ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{8iabx \tan\left(c + d\sqrt{x}\right)}{d}$$

[Out]  $-2*I*b^2*x/d + 2/3*a^2*x^{(3/2)} - 8*I*a*b*x*\arctan(\exp(I*(c+d*x^{(1/2)})))/d - 2*I*b^2*\text{polylog}(2, -\exp(2*I*(c+d*x^{(1/2)})))/d^3 - 8*a*b*\text{polylog}(3, -I*\exp(I*(c+d*x^{(1/2)})))/d^3 + 8*a*b*\text{polylog}(3, I*\exp(I*(c+d*x^{(1/2)})))/d^3 + 4*b^2*\ln(1+\exp(2*I*(c+d*x^{(1/2)})))*x^{(1/2)}/d^2 + 8*I*a*b*\text{polylog}(2, -I*\exp(I*(c+d*x^{(1/2)})))*x^{(1/2)}/d^2 - 8*I*a*b*\text{polylog}(2, I*\exp(I*(c+d*x^{(1/2)})))*x^{(1/2)}/d^2 + 2*b^2*x*\tan(c + d*x^{(1/2)})/d$

**Rubi [A]** time = 0.32, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {4204, 4190, 4181, 2531, 2282, 6589, 4184, 3719, 2190, 2279, 2391}

$$\frac{8iab\sqrt{x}\text{PolyLog}\left(2, -ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{8iab\sqrt{x}\text{PolyLog}\left(2, ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{8ab\text{PolyLog}\left(3, -ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{8ab\text{PolyLog}\left(3, ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{8iabx \tan\left(c + d\sqrt{x}\right)}{d}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x]\*(a + b\*Sec[c + d\*Sqrt[x]])^2, x]

[Out]  $((-2*I)*b^2*x)/d + (2*a^2*x^{(3/2)})/3 - ((8*I)*a*b*x*\text{ArcTan}[E^{(I*(c + d*\text{Sqrt}[x]))}])/d + (4*b^2*\text{Sqrt}[x]*\text{Log}[1 + E^{((2*I)*(c + d*\text{Sqrt}[x]))}])/d^2 + ((8*I)*a*b*\text{Sqrt}[x]*\text{PolyLog}[2, (-I)*E^{(I*(c + d*\text{Sqrt}[x]))}])/d^2 - ((8*I)*a*b*\text{Sqrt}[x]*\text{PolyLog}[2, I*E^{(I*(c + d*\text{Sqrt}[x]))}])/d^2 - ((2*I)*b^2*\text{PolyLog}[2, -E^{((2*I)*(c + d*\text{Sqrt}[x]))}])/d^3 - (8*a*b*\text{PolyLog}[3, (-I)*E^{(I*(c + d*\text{Sqrt}[x]))}])/d^3 + (8*a*b*\text{PolyLog}[3, I*E^{(I*(c + d*\text{Sqrt}[x]))}])/d^3 + (2*b^2*x*\text{Tan}[c + d*\text{Sqrt}[x]])/d$

#### Rule 2190

Int[(((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)), x\_Symbol] := Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a]/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_)))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] :=> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_.))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :=> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_)))^(n_.)]*((f_.) + (g_.)
*(x_)^(m_.), x_Symbol] :=> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 3719

```
Int[((c_.) + (d_.)*(x_)^(m_.))*tan[(e_.) + (f_.)*(x_)], x_Symbol] :=> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
+ f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

### Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_)^(m_.), x_Symbol
] :=> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x)
)], x], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

### Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_)^(m_.), x_Symbol] :=> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
```



`t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]`

#### Rule 4190

`Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]`

#### Rule 4204

`Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]`

#### Rule 6589

`Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]`

#### Rubi steps

$$\begin{aligned}
\int \sqrt{x} (a + b \sec(c + d\sqrt{x}))^2 dx &= 2 \operatorname{Subst} \left( \int x^2 (a + b \sec(c + dx))^2 dx, x, \sqrt{x} \right) \\
&= 2 \operatorname{Subst} \left( \int (a^2 x^2 + 2abx^2 \sec(c + dx) + b^2 x^2 \sec^2(c + dx)) dx, x, \sqrt{x} \right) \\
&= \frac{2}{3} a^2 x^{3/2} + (4ab) \operatorname{Subst} \left( \int x^2 \sec(c + dx) dx, x, \sqrt{x} \right) + (2b^2) \operatorname{Subst} \left( \int x^2 \sec^2(c + dx) dx, x, \sqrt{x} \right) \\
&= \frac{2}{3} a^2 x^{3/2} - \frac{8iabx \tan^{-1} \left( e^{i(c+d\sqrt{x})} \right)}{d} + \frac{2b^2 x \tan(c + d\sqrt{x})}{d} - \frac{(8ab) \operatorname{Subst} \left( \int x \sec(c + dx) dx, x, \sqrt{x} \right)}{d} \\
&= -\frac{2ib^2 x}{d} + \frac{2}{3} a^2 x^{3/2} - \frac{8iabx \tan^{-1} \left( e^{i(c+d\sqrt{x})} \right)}{d} + \frac{8iab\sqrt{x} \operatorname{Li}_2 \left( -ie^{i(c+d\sqrt{x})} \right)}{d^2} - \frac{8iab\sqrt{x} \operatorname{Li}_2 \left( ie^{i(c+d\sqrt{x})} \right)}{d^2} \\
&= -\frac{2ib^2 x}{d} + \frac{2}{3} a^2 x^{3/2} - \frac{8iabx \tan^{-1} \left( e^{i(c+d\sqrt{x})} \right)}{d} + \frac{4b^2 \sqrt{x} \log \left( 1 + e^{2i(c+d\sqrt{x})} \right)}{d^2} + \frac{4b^2 \sqrt{x} \log \left( 1 + e^{2i(c+d\sqrt{x})} \right)}{d^2} \\
&= -\frac{2ib^2 x}{d} + \frac{2}{3} a^2 x^{3/2} - \frac{8iabx \tan^{-1} \left( e^{i(c+d\sqrt{x})} \right)}{d} + \frac{4b^2 \sqrt{x} \log \left( 1 + e^{2i(c+d\sqrt{x})} \right)}{d^2} + \frac{4b^2 \sqrt{x} \log \left( 1 + e^{2i(c+d\sqrt{x})} \right)}{d^2} \\
&= -\frac{2ib^2 x}{d} + \frac{2}{3} a^2 x^{3/2} - \frac{8iabx \tan^{-1} \left( e^{i(c+d\sqrt{x})} \right)}{d} + \frac{4b^2 \sqrt{x} \log \left( 1 + e^{2i(c+d\sqrt{x})} \right)}{d^2} + \frac{4b^2 \sqrt{x} \log \left( 1 + e^{2i(c+d\sqrt{x})} \right)}{d^2}
\end{aligned}$$

**Mathematica [A]** time = 0.82, size = 247, normalized size = 0.97

$$2 \left( a^2 d^3 x^{3/2} - 12iab d^2 x \tan^{-1} \left( e^{i(c+d\sqrt{x})} \right) + 12iab d \sqrt{x} \operatorname{Li}_2 \left( -ie^{i(c+d\sqrt{x})} \right) - 12iab d \sqrt{x} \operatorname{Li}_2 \left( ie^{i(c+d\sqrt{x})} \right) - 12ab \operatorname{Li}_3 \left( -ie^{i(c+d\sqrt{x})} \right) + 12ab \operatorname{Li}_3 \left( ie^{i(c+d\sqrt{x})} \right) \right) / (3d^3)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x]\*(a + b\*Sec[c + d\*Sqrt[x]])^2,x]

[Out] (2\*((-3\*I)\*b^2\*d^2\*x + a^2\*d^3\*x^(3/2) - (12\*I)\*a\*b\*d^2\*x\*ArcTan[E^(I\*(c + d\*Sqrt[x]))] + 6\*b^2\*d\*Sqrt[x]\*Log[1 + E^((2\*I)\*(c + d\*Sqrt[x]))] + (12\*I)\*a\*b\*d\*Sqrt[x]\*PolyLog[2, (-I)\*E^(I\*(c + d\*Sqrt[x]))] - (12\*I)\*a\*b\*d\*Sqrt[x]\*PolyLog[2, I\*E^(I\*(c + d\*Sqrt[x]))] - (3\*I)\*b^2\*PolyLog[2, -E^((2\*I)\*(c + d\*Sqrt[x]))] - 12\*a\*b\*PolyLog[3, (-I)\*E^(I\*(c + d\*Sqrt[x]))] + 12\*a\*b\*PolyLog[3, I\*E^(I\*(c + d\*Sqrt[x]))] + 3\*b^2\*d^2\*x\*Tan[c + d\*Sqrt[x]]))/(3\*d^3)

**fricas [F]** time = 0.74, size = 0, normalized size = 0.00

$$\operatorname{integral} \left( b^2 \sqrt{x} \sec(d\sqrt{x} + c)^2 + 2ab\sqrt{x} \sec(d\sqrt{x} + c) + a^2 \sqrt{x}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sec(c+d\*x^(1/2)))^2\*x^(1/2),x, algorithm="fricas")

[Out] integral(b^2\*sqrt(x)\*sec(d\*sqrt(x) + c)^2 + 2\*a\*b\*sqrt(x)\*sec(d\*sqrt(x) + c) + a^2\*sqrt(x), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sec(d\sqrt{x} + c) + a)^2 \sqrt{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sec(c+d\*x^(1/2)))^2\*x^(1/2),x, algorithm="giac")

[Out] integrate((b\*sec(d\*sqrt(x) + c) + a)^2\*sqrt(x), x)

**maple** [F] time = 1.29, size = 0, normalized size = 0.00

$$\int (a + b \sec(c + d\sqrt{x}))^2 \sqrt{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*sec(c+d\*x^(1/2)))^2\*x^(1/2),x)

[Out] int((a+b\*sec(c+d\*x^(1/2)))^2\*x^(1/2),x)

**maxima** [B] time = 1.00, size = 1279, normalized size = 5.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sec(c+d\*x^(1/2)))^2\*x^(1/2),x, algorithm="maxima")

[Out] 
$$\begin{aligned} & 2/3*((d*\sqrt{x} + c)^3*a^2 - 3*(d*\sqrt{x} + c)^2*a^2*c + 3*(d*\sqrt{x} + c)* \\ & a^2*c^2 + 6*a*b*c^2*\log(\sec(d*\sqrt{x} + c) + \tan(d*\sqrt{x} + c)) + 3*(2*b^2 \\ & *c^2 - (2*(d*\sqrt{x} + c)^2*a*b - 4*(d*\sqrt{x} + c)*a*b*c + 2*((d*\sqrt{x} + \\ & c)^2*a*b - 2*(d*\sqrt{x} + c)*a*b*c)*\cos(2*d*\sqrt{x} + 2*c) - (-2*I*(d*\sqrt{x} \\ & (x) + c)^2*a*b + 4*I*(d*\sqrt{x} + c)*a*b*c)*\sin(2*d*\sqrt{x} + 2*c))*\arctan2 \\ & (\cos(d*\sqrt{x} + c), \sin(d*\sqrt{x} + c) + 1) - (2*(d*\sqrt{x} + c)^2*a*b - 4 \\ & *(d*\sqrt{x} + c)*a*b*c + 2*((d*\sqrt{x} + c)^2*a*b - 2*(d*\sqrt{x} + c)*a*b*c \\ & )*\cos(2*d*\sqrt{x} + 2*c) - (-2*I*(d*\sqrt{x} + c)^2*a*b + 4*I*(d*\sqrt{x} + c \\ & )*a*b*c)*\sin(2*d*\sqrt{x} + 2*c))*\arctan2(\cos(d*\sqrt{x} + c), -\sin(d*\sqrt{x} \\ & + c) + 1) + (2*(d*\sqrt{x} + c)*b^2 - 2*b^2*c + 2*((d*\sqrt{x} + c)*b^2 - b^ \\ & 2*c)*\cos(2*d*\sqrt{x} + 2*c) + (2*I*(d*\sqrt{x} + c)*b^2 - 2*I*b^2*c)*\sin(2*d \\ & *sqrt{x} + 2*c))*\arctan2(\sin(2*d*\sqrt{x} + 2*c), \cos(2*d*\sqrt{x} + 2*c) + 1 \\ & ) - 2*((d*\sqrt{x} + c)^2*b^2 - 2*(d*\sqrt{x} + c)*b^2*c)*\cos(2*d*\sqrt{x} + 2 \end{aligned}$$

```

*c) - (b^2*cos(2*d*sqrt(x) + 2*c) + I*b^2*sin(2*d*sqrt(x) + 2*c) + b^2)*dilog(-e^(2*I*d*sqrt(x) + 2*I*c)) - (4*(d*sqrt(x) + c)*a*b - 4*a*b*c + 4*((d*sqrt(x) + c)*a*b - a*b*c)*cos(2*d*sqrt(x) + 2*c) - (-4*I*(d*sqrt(x) + c)*a*b + 4*I*a*b*c)*sin(2*d*sqrt(x) + 2*c))*dilog(I*e^(I*d*sqrt(x) + I*c)) + (4*(d*sqrt(x) + c)*a*b - 4*a*b*c + 4*((d*sqrt(x) + c)*a*b - a*b*c)*cos(2*d*sqrt(x) + 2*c) + (4*I*(d*sqrt(x) + c)*a*b - 4*I*a*b*c)*sin(2*d*sqrt(x) + 2*c))*dilog(-I*e^(I*d*sqrt(x) + I*c)) + (-I*(d*sqrt(x) + c)*b^2 + I*b^2*c + (-I*(d*sqrt(x) + c)*b^2 + I*b^2*c)*cos(2*d*sqrt(x) + 2*c) + ((d*sqrt(x) + c)*b^2 - b^2*c)*sin(2*d*sqrt(x) + 2*c))*log(cos(2*d*sqrt(x) + 2*c)^2 + sin(2*d*sqrt(x) + 2*c)^2 + 2*cos(2*d*sqrt(x) + 2*c) + 1) + (-I*(d*sqrt(x) + c)^2*a*b + 2*I*(d*sqrt(x) + c)*a*b*c + (-I*(d*sqrt(x) + c)^2*a*b + 2*I*(d*sqrt(x) + c)*a*b*c)*cos(2*d*sqrt(x) + 2*c) + ((d*sqrt(x) + c)^2*a*b - 2*(d*sqrt(x) + c)*a*b*c)*sin(2*d*sqrt(x) + 2*c))*log(cos(d*sqrt(x) + c)^2 + sin(d*sqrt(x) + c)^2 + 2*sin(d*sqrt(x) + c) + 1) + (I*(d*sqrt(x) + c)^2*a*b - 2*I*(d*sqrt(x) + c)*a*b*c + (I*(d*sqrt(x) + c)^2*a*b - 2*I*(d*sqrt(x) + c)*a*b*c)*cos(2*d*sqrt(x) + 2*c) - ((d*sqrt(x) + c)^2*a*b - 2*(d*sqrt(x) + c)*a*b*c)*sin(2*d*sqrt(x) + 2*c))*log(cos(d*sqrt(x) + c)^2 + sin(d*sqrt(x) + c)^2 - 2*sin(d*sqrt(x) + c) + 1) + (-4*I*a*b*cos(2*d*sqrt(x) + 2*c) + 4*a*b*sin(2*d*sqrt(x) + 2*c) - 4*I*a*b)*polylog(3, I*e^(I*d*sqrt(x) + I*c)) + (4*I*a*b*cos(2*d*sqrt(x) + 2*c) - 4*a*b*sin(2*d*sqrt(x) + 2*c) + 4*I*a*b)*polylog(3, -I*e^(I*d*sqrt(x) + I*c)) + (-2*I*(d*sqrt(x) + c)^2*b^2 + 4*I*(d*sqrt(x) + c)*b^2*c)*sin(2*d*sqrt(x) + 2*c))/(-I*cos(2*d*sqrt(x) + 2*c) + sin(2*d*sqrt(x) + 2*c) - I))/d^3

```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{x} \left( a + \frac{b}{\cos(c + d\sqrt{x})} \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)\*(a + b/cos(c + d\*x^(1/2)))^2,x)

[Out] int(x^(1/2)\*(a + b/cos(c + d\*x^(1/2)))^2, x)

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x} \left( a + b \sec(c + d\sqrt{x}) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sec(c+d\*x\*\*(1/2)))\*2\*x\*\*(1/2),x)

[Out] Integral(sqrt(x)\*(a + b\*sec(c + d\*sqrt(x)))\*2, x)

$$3.58 \quad \int \frac{(a+b \sec(c+d\sqrt{x}))^2}{\sqrt{x}} dx$$

Optimal. Leaf size=47

$$2a^2\sqrt{x} + \frac{4ab \tanh^{-1}(\sin(c+d\sqrt{x}))}{d} + \frac{2b^2 \tan(c+d\sqrt{x})}{d}$$

[Out]  $4*a*b*\operatorname{arctanh}(\sin(c+d*x^{(1/2)}))/d+2*a^2*x^{(1/2)}+2*b^2*\tan(c+d*x^{(1/2)})/d$

**Rubi [A]** time = 0.05, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$ , Rules used = {4204, 3773, 3770, 3767, 8}

$$2a^2\sqrt{x} + \frac{4ab \tanh^{-1}(\sin(c+d\sqrt{x}))}{d} + \frac{2b^2 \tan(c+d\sqrt{x})}{d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + b*\operatorname{Sec}[c + d*\operatorname{Sqrt}[x]])^2/\operatorname{Sqrt}[x], x]$

[Out]  $2*a^2*\operatorname{Sqrt}[x] + (4*a*b*\operatorname{ArcTanh}[\operatorname{Sin}[c + d*\operatorname{Sqrt}[x]])/d + (2*b^2*\operatorname{Tan}[c + d*\operatorname{Sqrt}[x]])/d$

Rule 8

$\operatorname{Int}[a_, x\_Symbol] \rightarrow \operatorname{Simp}[a*x, x] /; \operatorname{FreeQ}[a, x]$

Rule 3767

$\operatorname{Int}[\operatorname{csc}[(c_.) + (d_.)*(x_.)]^{(n_.)}, x\_Symbol] \rightarrow -\operatorname{Dist}[d^{(-1)}, \operatorname{Subst}[\operatorname{Int}[\operatorname{ExpandIntegrand}[(1 + x^2)^{(n/2 - 1)}, x], x], x, \operatorname{Cot}[c + d*x]], x] /; \operatorname{FreeQ}[\{c, d\}, x] \ \&\& \ \operatorname{IGtQ}[n/2, 0]$

Rule 3770

$\operatorname{Int}[\operatorname{csc}[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{ArcTanh}[\operatorname{Cos}[c + d*x]]/d, x] /; \operatorname{FreeQ}[\{c, d\}, x]$

Rule 3773

$\operatorname{Int}[(\operatorname{csc}[(c_.) + (d_.)*(x_.)]*(b_.) + (a_.))^2, x\_Symbol] \rightarrow \operatorname{Simp}[a^2*x, x] + (\operatorname{Dist}[2*a*b, \operatorname{Int}[\operatorname{Csc}[c + d*x], x], x] + \operatorname{Dist}[b^2, \operatorname{Int}[\operatorname{Csc}[c + d*x]^2, x], x]) /; \operatorname{FreeQ}[\{a, b, c, d\}, x]$

Rule 4204

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_.)])^(p_.), x_Symbol
] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned} \int \frac{(a + b \sec(c + d\sqrt{x}))^2}{\sqrt{x}} dx &= 2 \operatorname{Subst} \left( \int (a + b \sec(c + dx))^2 dx, x, \sqrt{x} \right) \\ &= 2a^2\sqrt{x} + (4ab) \operatorname{Subst} \left( \int \sec(c + dx) dx, x, \sqrt{x} \right) + (2b^2) \operatorname{Subst} \left( \int \sec^2(c + dx) dx, x, \sqrt{x} \right) \\ &= 2a^2\sqrt{x} + \frac{4ab \tanh^{-1}(\sin(c + d\sqrt{x}))}{d} - \frac{(2b^2) \operatorname{Subst}(\int 1 dx, x, -\tan(c + d\sqrt{x}))}{d} \\ &= 2a^2\sqrt{x} + \frac{4ab \tanh^{-1}(\sin(c + d\sqrt{x}))}{d} + \frac{2b^2 \tan(c + d\sqrt{x})}{d} \end{aligned}$$

**Mathematica [A]** time = 0.28, size = 45, normalized size = 0.96

$$\frac{2(a^2d\sqrt{x} + 2ab \tanh^{-1}(\sin(c + d\sqrt{x})) + b^2 \tan(c + d\sqrt{x}))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*Sqrt[x]])^2/Sqrt[x], x]
```

```
[Out] (2*(a^2*d*Sqrt[x] + 2*a*b*ArcTanh[Sin[c + d*Sqrt[x]]) + b^2*Tan[c + d*Sqrt[x]])/d
```

**fricas [B]** time = 0.79, size = 91, normalized size = 1.94

$$\frac{2(a^2d\sqrt{x} \cos(d\sqrt{x} + c) + ab \cos(d\sqrt{x} + c) \log(\sin(d\sqrt{x} + c) + 1) - ab \cos(d\sqrt{x} + c) \log(-\sin(d\sqrt{x} + c) + 1) + b^2 \sin(d\sqrt{x} + c))}{d \cos(d\sqrt{x} + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(c+d*x^(1/2)))^2/x^(1/2), x, algorithm="fricas")
```

```
[Out] 2*(a^2*d*sqrt(x)*cos(d*sqrt(x) + c) + a*b*cos(d*sqrt(x) + c)*log(sin(d*sqrt(x) + c) + 1) - a*b*cos(d*sqrt(x) + c)*log(-sin(d*sqrt(x) + c) + 1) + b^2*sin(d*sqrt(x) + c))/(d*cos(d*sqrt(x) + c))
```

**giac** [B] time = 0.75, size = 88, normalized size = 1.87

$$\frac{2 \left( (d\sqrt{x} + c)a^2 + 2ab \log \left( \left| \tan \left( \frac{1}{2} d\sqrt{x} + \frac{1}{2} c \right) + 1 \right| \right) - 2ab \log \left( \left| \tan \left( \frac{1}{2} d\sqrt{x} + \frac{1}{2} c \right) - 1 \right| \right) - \frac{2b^2 \tan \left( \frac{1}{2} d\sqrt{x} + \frac{1}{2} c \right)}{\tan \left( \frac{1}{2} d\sqrt{x} + \frac{1}{2} c \right)^2 - 1} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sec(c+d\*x^(1/2)))^2/x^(1/2),x, algorithm="giac")

[Out] 2\*((d\*sqrt(x) + c)\*a^2 + 2\*a\*b\*log(abs(tan(1/2\*d\*sqrt(x) + 1/2\*c) + 1)) - 2\*a\*b\*log(abs(tan(1/2\*d\*sqrt(x) + 1/2\*c) - 1)) - 2\*b^2\*tan(1/2\*d\*sqrt(x) + 1/2\*c)/(tan(1/2\*d\*sqrt(x) + 1/2\*c)^2 - 1))/d

**maple** [A] time = 0.76, size = 60, normalized size = 1.28

$$2a^2\sqrt{x} + \frac{2b^2 \tan(c + d\sqrt{x})}{d} + \frac{4ab \ln(\sec(c + d\sqrt{x}) + \tan(c + d\sqrt{x}))}{d} + \frac{2a^2c}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*sec(c+d\*x^(1/2)))^2/x^(1/2),x)

[Out] 2\*a^2\*x^(1/2)+2\*b^2\*tan(c+d\*x^(1/2))/d+4/d\*a\*b\*ln(sec(c+d\*x^(1/2))+tan(c+d\*x^(1/2)))+2/d\*a^2\*c

**maxima** [A] time = 0.53, size = 50, normalized size = 1.06

$$2a^2\sqrt{x} + \frac{4ab \log(\sec(d\sqrt{x} + c) + \tan(d\sqrt{x} + c))}{d} + \frac{2b^2 \tan(d\sqrt{x} + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sec(c+d\*x^(1/2)))^2/x^(1/2),x, algorithm="maxima")

[Out] 2\*a^2\*sqrt(x) + 4\*a\*b\*log(sec(d\*sqrt(x) + c) + tan(d\*sqrt(x) + c))/d + 2\*b^2\*tan(d\*sqrt(x) + c)/d

**mupad** [B] time = 2.68, size = 109, normalized size = 2.32

$$2a^2\sqrt{x} + \frac{b^2 4i}{d(e^{c2i+d\sqrt{x}2i} + 1)} + \frac{4ab \ln\left(-\frac{ab4i}{\sqrt{x}} - \frac{4ab e^{d\sqrt{x}1i} e^{c1i}}{\sqrt{x}}\right)}{d} - \frac{4ab \ln\left(\frac{ab4i}{\sqrt{x}} - \frac{4ab e^{d\sqrt{x}1i} e^{c1i}}{\sqrt{x}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x^(1/2)))^2/x^(1/2),x)
```

```
[Out] 2*a^2*x^(1/2) + (b^2*4i)/(d*(exp(c*2i + d*x^(1/2)*2i) + 1)) + (4*a*b*log(-
(a*b*4i)/x^(1/2) - (4*a*b*exp(d*x^(1/2)*1i)*exp(c*1i))/x^(1/2)))/d - (4*a*b
*log((a*b*4i)/x^(1/2) - (4*a*b*exp(d*x^(1/2)*1i)*exp(c*1i))/x^(1/2)))/d
```

**sympy [A]** time = 9.75, size = 88, normalized size = 1.87

$$\begin{cases} \frac{2a^2(c+d\sqrt{x})+4ab\log(\tan(c+d\sqrt{x})+\sec(c+d\sqrt{x}))+2b^2\tan(c+d\sqrt{x})}{d} & \text{for } d \neq 0 \\ -\sqrt{x}(-2a^2 - 4ab\sec(c) - 2b^2\sec^2(c)) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(c+d*x**(1/2)))**2/x**(1/2),x)
```

```
[Out] Piecewise(((2*a**2*(c + d*sqrt(x)) + 4*a*b*log(tan(c + d*sqrt(x)) + sec(c +
d*sqrt(x))) + 2*b**2*tan(c + d*sqrt(x)))/d, Ne(d, 0)), (-sqrt(x)*(-2*a**2
- 4*a*b*sec(c) - 2*b**2*sec(c)**2), True))
```



$$3.59 \quad \int \frac{(a+b \sec(c+d\sqrt{x}))^2}{x^{3/2}} dx$$

Optimal. Leaf size=25

$$\text{Int} \left( \frac{(a+b \sec(c+d\sqrt{x}))^2}{x^{3/2}}, x \right)$$

[Out] Unintegrable((a+b\*sec(c+d\*x^(1/2)))^2/x^(3/2), x)

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{(a+b \sec(c+d\sqrt{x}))^2}{x^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*Sec[c + d\*Sqrt[x]])^2/x^(3/2), x]

[Out] Defer[Int] [(a + b\*Sec[c + d\*Sqrt[x]])^2/x^(3/2), x]

Rubi steps

$$\int \frac{(a+b \sec(c+d\sqrt{x}))^2}{x^{3/2}} dx = \int \frac{(a+b \sec(c+d\sqrt{x}))^2}{x^{3/2}} dx$$

Mathematica [A] time = 23.13, size = 0, normalized size = 0.00

$$\int \frac{(a+b \sec(c+d\sqrt{x}))^2}{x^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*Sec[c + d\*Sqrt[x]])^2/x^(3/2), x]

[Out] Integrate[(a + b\*Sec[c + d\*Sqrt[x]])^2/x^(3/2), x]

fricas [A] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{b^2\sqrt{x} \sec(d\sqrt{x} + c)^2 + 2ab\sqrt{x} \sec(d\sqrt{x} + c) + a^2\sqrt{x}}{x^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sec(c+d\*x^(1/2)))^2/x^(3/2),x, algorithm="fricas")

[Out] integral((b^2\*sqrt(x)\*sec(d\*sqrt(x) + c)^2 + 2\*a\*b\*sqrt(x)\*sec(d\*sqrt(x) + c) + a^2\*sqrt(x))/x^2, x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \sec(d\sqrt{x} + c) + a)^2}{x^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sec(c+d\*x^(1/2)))^2/x^(3/2),x, algorithm="giac")

[Out] integrate((b\*sec(d\*sqrt(x) + c) + a)^2/x^(3/2), x)

**maple** [A] time = 1.29, size = 0, normalized size = 0.00

$$\int \frac{(a + b \sec(c + d\sqrt{x}))^2}{x^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*sec(c+d\*x^(1/2)))^2/x^(3/2),x)

[Out] int((a+b\*sec(c+d\*x^(1/2)))^2/x^(3/2),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$4b^2 \sin(2d\sqrt{x} + 2c) + 4 \left( d \cos(2d\sqrt{x} + 2c) \int \frac{b^2 \sin(2d\sqrt{x} + 2c) + (abd \cos(2d\sqrt{x} + 2c) \cos(d\sqrt{x} + c) + abd \sin(2d\sqrt{x} + 2c) \sin(d\sqrt{x} + c))}{(d \cos(2d\sqrt{x} + 2c)^2 + d \sin(2d\sqrt{x} + 2c)^2 + 2d \cos(2d\sqrt{x} + 2c))} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sec(c+d\*x^(1/2)))^2/x^(3/2),x, algorithm="maxima")

[Out] (4\*b^2\*sin(2\*d\*sqrt(x) + 2\*c) + (d\*cos(2\*d\*sqrt(x) + 2\*c)^2\*integrate(4\*(b^2\*sin(2\*d\*sqrt(x) + 2\*c) + (a\*b\*d\*cos(2\*d\*sqrt(x) + 2\*c)\*cos(d\*sqrt(x) + c) + a\*b\*d\*sin(2\*d\*sqrt(x) + 2\*c)\*sin(d\*sqrt(x) + c) + a\*b\*d\*cos(d\*sqrt(x) + c))\*sqrt(x))/((d\*cos(2\*d\*sqrt(x) + 2\*c)^2 + d\*sin(2\*d\*sqrt(x) + 2\*c)^2 + 2\*d\*cos(2\*d\*sqrt(x) + 2\*c) + d)\*x^2), x) + d\*integrate(4\*(b^2\*sin(2\*d\*sqrt(x) + 2\*c) + (a\*b\*d\*cos(2\*d\*sqrt(x) + 2\*c)\*cos(d\*sqrt(x) + c) + a\*b\*d\*sin(2\*d\*sqrt(x) + 2\*c)\*sin(d\*sqrt(x) + c) + a\*b\*d\*cos(d\*sqrt(x) + c))\*sqrt(x))/((d\*cos(2\*d\*sqrt(x) + 2\*c)^2 + d\*sin(2\*d\*sqrt(x) + 2\*c)^2 + 2\*d\*cos(2\*d\*sqrt(x) + 2\*c) + d)\*x^2), x)

```

+ 2*c) + (a*b*d*cos(2*d*sqrt(x) + 2*c)*cos(d*sqrt(x) + c) + a*b*d*sin(2*d*
sqrt(x) + 2*c)*sin(d*sqrt(x) + c) + a*b*d*cos(d*sqrt(x) + c))*sqrt(x))/((d*
cos(2*d*sqrt(x) + 2*c)^2 + d*sin(2*d*sqrt(x) + 2*c)^2 + 2*d*cos(2*d*sqrt(x)
+ 2*c) + d)*x^2), x)*sin(2*d*sqrt(x) + 2*c)^2 + 2*d*cos(2*d*sqrt(x) + 2*c)
*integrate(4*(b^2*sin(2*d*sqrt(x) + 2*c) + (a*b*d*cos(2*d*sqrt(x) + 2*c)*co
s(d*sqrt(x) + c) + a*b*d*sin(2*d*sqrt(x) + 2*c)*sin(d*sqrt(x) + c) + a*b*d*
cos(d*sqrt(x) + c))*sqrt(x))/((d*cos(2*d*sqrt(x) + 2*c)^2 + d*sin(2*d*sqrt(
x) + 2*c)^2 + 2*d*cos(2*d*sqrt(x) + 2*c) + d)*x^2), x) + d*integrate(4*(b^2
*sin(2*d*sqrt(x) + 2*c) + (a*b*d*cos(2*d*sqrt(x) + 2*c)*cos(d*sqrt(x) + c)
+ a*b*d*sin(2*d*sqrt(x) + 2*c)*sin(d*sqrt(x) + c) + a*b*d*cos(d*sqrt(x) + c
))*sqrt(x))/((d*cos(2*d*sqrt(x) + 2*c)^2 + d*sin(2*d*sqrt(x) + 2*c)^2 + 2*d
*cos(2*d*sqrt(x) + 2*c) + d)*x^2), x))*x - 2*(a^2*d*cos(2*d*sqrt(x) + 2*c)^
2 + a^2*d*sin(2*d*sqrt(x) + 2*c)^2 + 2*a^2*d*cos(2*d*sqrt(x) + 2*c) + a^2*d
)*sqrt(x))/((d*cos(2*d*sqrt(x) + 2*c)^2 + d*sin(2*d*sqrt(x) + 2*c)^2 + 2*d*
cos(2*d*sqrt(x) + 2*c) + d)*x)

```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a + \frac{b}{\cos(c+d\sqrt{x})}\right)^2}{x^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cos(c + d\*x^(1/2)))^2/x^(3/2), x)

[Out] int((a + b/cos(c + d\*x^(1/2)))^2/x^(3/2), x)

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \sec\left(c + d\sqrt{x}\right)\right)^2}{x^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sec(c+d\*x\*\*(1/2)))\*\*2/x\*\*(3/2), x)

[Out] Integral((a + b\*sec(c + d\*sqrt(x)))\*\*2/x\*\*(3/2), x)

$$3.60 \quad \int \frac{(a+b \sec(c+d\sqrt{x}))^2}{x^{5/2}} dx$$

Optimal. Leaf size=25

$$\text{Int} \left( \frac{(a+b \sec(c+d\sqrt{x}))^2}{x^{5/2}}, x \right)$$

[Out] Unintegrable((a+b\*sec(c+d\*x^(1/2)))^2/x^(5/2), x)

**Rubi [A]** time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{(a+b \sec(c+d\sqrt{x}))^2}{x^{5/2}} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*Sec[c + d\*Sqrt[x]])^2/x^(5/2), x]

[Out] Defer[Int][(a + b\*Sec[c + d\*Sqrt[x]])^2/x^(5/2), x]

Rubi steps

$$\int \frac{(a+b \sec(c+d\sqrt{x}))^2}{x^{5/2}} dx = \int \frac{(a+b \sec(c+d\sqrt{x}))^2}{x^{5/2}} dx$$

**Mathematica [A]** time = 23.81, size = 0, normalized size = 0.00

$$\int \frac{(a+b \sec(c+d\sqrt{x}))^2}{x^{5/2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*Sec[c + d\*Sqrt[x]])^2/x^(5/2), x]

[Out] Integrate[(a + b\*Sec[c + d\*Sqrt[x]])^2/x^(5/2), x]

**fricas [A]** time = 0.70, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{b^2 \sqrt{x} \sec(d\sqrt{x} + c)^2 + 2ab\sqrt{x} \sec(d\sqrt{x} + c) + a^2 \sqrt{x}}{x^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sec(c+d\*x^(1/2)))^2/x^(5/2),x, algorithm="fricas")

[Out] integral((b^2\*sqrt(x)\*sec(d\*sqrt(x) + c)^2 + 2\*a\*b\*sqrt(x)\*sec(d\*sqrt(x) + c) + a^2\*sqrt(x))/x^3, x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \sec(d\sqrt{x} + c) + a)^2}{x^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sec(c+d\*x^(1/2)))^2/x^(5/2),x, algorithm="giac")

[Out] integrate((b\*sec(d\*sqrt(x) + c) + a)^2/x^(5/2), x)

**maple** [A] time = 1.32, size = 0, normalized size = 0.00

$$\int \frac{(a + b \sec(c + d\sqrt{x}))^2}{x^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*sec(c+d\*x^(1/2)))^2/x^(5/2),x)

[Out] int((a+b\*sec(c+d\*x^(1/2)))^2/x^(5/2),x)

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sec(c+d\*x^(1/2)))^2/x^(5/2),x, algorithm="maxima")

[Out] Timed out

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a + \frac{b}{\cos(c+d\sqrt{x})}\right)^2}{x^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b/cos(c + d*x^(1/2)))^2/x^(5/2), x)`

[Out] `int((a + b/cos(c + d*x^(1/2)))^2/x^(5/2), x)`

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \sec(c + d\sqrt{x}))^2}{x^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sec(c+d*x**(1/2)))**2/x**(5/2), x)`

[Out] `Integral((a + b*sec(c + d*sqrt(x)))**2/x**(5/2), x)`

$$3.61 \quad \int \frac{x^{3/2}}{a+b \sec(c+d\sqrt{x})} dx$$

Optimal. Leaf size=653

$$\frac{48ib\text{Li}_5\left(-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)}{ad^5\sqrt{b^2-a^2}} + \frac{48ib\text{Li}_5\left(-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)}{ad^5\sqrt{b^2-a^2}} - \frac{48b\sqrt{x}\text{Li}_4\left(-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)}{ad^4\sqrt{b^2-a^2}} + \frac{48b\sqrt{x}\text{Li}_4\left(-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)}{ad^4\sqrt{b^2-a^2}} + \frac{24ibx\text{Li}_3\left(-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}} - \frac{24ibx\text{Li}_3\left(-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}}$$

[Out]  $2/5*x^{(5/2)}/a+2*I*b*x^2*\ln(1+a*\exp(I*(c+d*x^{(1/2)})))/(b-(-a^2+b^2)^{(1/2)})/a$   
 $/d/(-a^2+b^2)^{(1/2)}-2*I*b*x^2*\ln(1+a*\exp(I*(c+d*x^{(1/2)})))/(b+(-a^2+b^2)^{(1/2)})/a$   
 $/d/(-a^2+b^2)^{(1/2)}+8*b*x^{(3/2)}*\text{polylog}(2,-a*\exp(I*(c+d*x^{(1/2)})))/(b-(-a^2+b^2)^{(1/2)})/a$   
 $/d^2/(-a^2+b^2)^{(1/2)}-8*b*x^{(3/2)}*\text{polylog}(2,-a*\exp(I*(c+d*x^{(1/2)})))/(b+(-a^2+b^2)^{(1/2)})/a$   
 $/d^2/(-a^2+b^2)^{(1/2)}+24*I*b*x*\text{polylog}(3,-a*\exp(I*(c+d*x^{(1/2)})))/(b-(-a^2+b^2)^{(1/2)})/a$   
 $/d^3/(-a^2+b^2)^{(1/2)}-24*I*b*x*\text{polylog}(3,-a*\exp(I*(c+d*x^{(1/2)})))/(b+(-a^2+b^2)^{(1/2)})/a$   
 $/d^3/(-a^2+b^2)^{(1/2)}-48*I*b*\text{polylog}(5,-a*\exp(I*(c+d*x^{(1/2)})))/(b-(-a^2+b^2)^{(1/2)})/a$   
 $/d^5/(-a^2+b^2)^{(1/2)}+48*I*b*\text{polylog}(5,-a*\exp(I*(c+d*x^{(1/2)})))/(b+(-a^2+b^2)^{(1/2)})/a$   
 $/d^5/(-a^2+b^2)^{(1/2)}-48*b*\text{polylog}(4,-a*\exp(I*(c+d*x^{(1/2)})))/(b-(-a^2+b^2)^{(1/2)})$   
 $*x^{(1/2)}/a/d^4/(-a^2+b^2)^{(1/2)}+48*b*\text{polylog}(4,-a*\exp(I*(c+d*x^{(1/2)})))/(b+(-a^2+b^2)^{(1/2)})$   
 $*x^{(1/2)}/a/d^4/(-a^2+b^2)^{(1/2)}$

**Rubi [A]** time = 1.05, antiderivative size = 653, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$ , Rules used = {4204, 4191, 3321, 2264, 2190, 2531, 6609, 2282, 6589}

$$\frac{8bx^{3/2}\text{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}} - \frac{8bx^{3/2}\text{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{\sqrt{b^2-a^2}+b}\right)}{ad^2\sqrt{b^2-a^2}} + \frac{24ibx\text{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}} - \frac{24ibx\text{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{\sqrt{b^2-a^2}+b}\right)}{ad^3\sqrt{b^2-a^2}}$$

Antiderivative was successfully verified.

[In] Int[x^(3/2)/(a + b\*Sec[c + d\*Sqrt[x]]), x]

[Out]  $(2*x^{(5/2)})/(5*a) + ((2*I)*b*x^2*\text{Log}[1 + (a*E^{(I*(c + d*Sqrt[x]))})]/(b - \text{Sqrt}[-a^2 + b^2]))/(a*\text{Sqrt}[-a^2 + b^2]*d) - ((2*I)*b*x^2*\text{Log}[1 + (a*E^{(I*(c + d*Sqrt[x]))})]/(b + \text{Sqrt}[-a^2 + b^2]))/(a*\text{Sqrt}[-a^2 + b^2]*d) + (8*b*x^{(3/2)})*\text{PolyLog}[2, -((a*E^{(I*(c + d*Sqrt[x]))})/(b - \text{Sqrt}[-a^2 + b^2]))]/(a*\text{Sqrt}[-a^2 + b^2]*d^2) - (8*b*x^{(3/2)})*\text{PolyLog}[2, -((a*E^{(I*(c + d*Sqrt[x]))})/(b + \text{Sqrt}[-a^2 + b^2]))]/(a*\text{Sqrt}[-a^2 + b^2]*d^2) + ((24*I)*b*x*\text{PolyLog}[3, -((a*E^{(I*(c + d*Sqrt[x]))})/(b - \text{Sqrt}[-a^2 + b^2]))]/(a*\text{Sqrt}[-a^2 + b^2]*d^3) - ((24*I)*b*x*\text{PolyLog}[3, -((a*E^{(I*(c + d*Sqrt[x]))})/(b + \text{Sqrt}[-a^2 + b^2]))]/(a*\text{Sqrt}[-a^2 + b^2]*d^3) - (48*b*\text{Sqrt}[x]*\text{PolyLog}[4, -((a*E^{(I*(c + d*Sqrt[x]))})/(b - \text{Sqrt}[-a^2 + b^2]))]/(a*\text{Sqrt}[-a^2 + b^2]*d^4) + (48*b*\text{Sqrt}[x]*\text{PolyLog}[4, -((a*E^{(I*(c + d*Sqrt[x]))})/(b + \text{Sqrt}[-a^2 + b^2]))]/(a*\text{Sqrt}[-a^2 + b^2]*d^4)$

```
] * PolyLog[4, -((a * E^(I * (c + d * Sqrt[x]))) / (b + Sqrt[-a^2 + b^2]))] / (a * Sqrt[-a^2 + b^2] * d^4) - ((48 * I) * b * PolyLog[5, -((a * E^(I * (c + d * Sqrt[x]))) / (b - Sqrt[-a^2 + b^2]))] / (a * Sqrt[-a^2 + b^2] * d^5) + ((48 * I) * b * PolyLog[5, -((a * E^(I * (c + d * Sqrt[x]))) / (b + Sqrt[-a^2 + b^2]))] / (a * Sqrt[-a^2 + b^2] * d^5)
```

### Rule 2190

```
Int[(((F_)^((g_)*((e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)) / ((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_)))^(n_))), x_Symbol] := Simp[(((c + d*x)^m * Log[1 + (b*(F^(g*(e + f*x)))^n)/a]) / (b*f*g*n*Log[F]), x] - Dist[(d*m) / (b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1) * Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2264

```
Int[((F_)^(u_)*((f_) + (g_)*(x_))^(m_)) / ((a_) + (b_)*(F_)^(u_) + (c_)*(F_)^(v_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[(f + g*x)^m * F^u / (b - q + 2*c * F^u), x], x] - Dist[(2*c)/q, Int[(f + g*x)^m * F^u / (b + q + 2*c * F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 2531

```
Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_)))^(n_))]*((f_) + (g_)*(x_))^(m_), x_Symbol] := -Simp[((f + g*x)^m * PolyLog[2, -(e*(F^(c*(a + b*x)))^n)] / (b*c*n*Log[F]), x] + Dist[(g*m) / (b*c*n*Log[F]), Int[(f + g*x)^(m - 1) * PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

### Rule 3321

```
Int[((c_) + (d_)*(x_))^(m_) / ((a_) + (b_)*sin[(e_) + Pi*(k_) + (f_)*(x_)]), x_Symbol] := Dist[2, Int[(((c + d*x)^m * E^(I*Pi*(k - 1/2)) * E^(I*(e + f*x))) / (b + 2*a * E^(I*Pi*(k - 1/2)) * E^(I*(e + f*x)) - b * E^(2*I*k*Pi) * E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```



Rule 4191

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(n_.)*((c_.) + (d_.)*(x_.))^(m_.)
, x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 4204

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_.)])^(p_.), x_Symbol
] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_.))^(p_.)]/((d_.) + (e_.)*(x_.)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_.))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_.)))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{3/2}}{a + b \sec(c + d\sqrt{x})} dx &= 2 \operatorname{Subst} \left( \int \frac{x^4}{a + b \sec(c + dx)} dx, x, \sqrt{x} \right) \\
&= 2 \operatorname{Subst} \left( \int \left( \frac{x^4}{a} - \frac{bx^4}{a(b + a \cos(c + dx))} \right) dx, x, \sqrt{x} \right) \\
&= \frac{2x^{5/2}}{5a} - \frac{(2b) \operatorname{Subst} \left( \int \frac{x^4}{b + a \cos(c + dx)} dx, x, \sqrt{x} \right)}{a} \\
&= \frac{2x^{5/2}}{5a} - \frac{(4b) \operatorname{Subst} \left( \int \frac{e^{i(c+dx)} x^4}{a + 2be^{i(c+dx)} + ae^{2i(c+dx)}} dx, x, \sqrt{x} \right)}{a} \\
&= \frac{2x^{5/2}}{5a} - \frac{(4b) \operatorname{Subst} \left( \int \frac{e^{i(c+dx)} x^4}{2b - 2\sqrt{-a^2 + b^2} + 2ae^{i(c+dx)}} dx, x, \sqrt{x} \right)}{\sqrt{-a^2 + b^2}} + \frac{(4b) \operatorname{Subst} \left( \int \frac{e^{i(c+dx)} x^4}{2b + 2\sqrt{-a^2 + b^2} + 2ae^{i(c+dx)}} dx, x, \sqrt{x} \right)}{\sqrt{-a^2 + b^2}} \\
&= \frac{2x^{5/2}}{5a} + \frac{2ibx^2 \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} - \frac{2ibx^2 \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b + \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} - \frac{(8ib) \operatorname{Subst} \left( \int x^3 \frac{e^{i(c+dx)}}{b - a \cos(c + dx)} dx, x, \sqrt{x} \right)}{a\sqrt{-a^2 + b^2} d} \\
&= \frac{2x^{5/2}}{5a} + \frac{2ibx^2 \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} - \frac{2ibx^2 \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b + \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} + \frac{8bx^{3/2} \operatorname{Li}_2 \left( -\frac{ae^{i(c+d\sqrt{x})}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} \\
&= \frac{2x^{5/2}}{5a} + \frac{2ibx^2 \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} - \frac{2ibx^2 \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b + \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} + \frac{8bx^{3/2} \operatorname{Li}_2 \left( -\frac{ae^{i(c+d\sqrt{x})}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} \\
&= \frac{2x^{5/2}}{5a} + \frac{2ibx^2 \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} - \frac{2ibx^2 \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b + \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} + \frac{8bx^{3/2} \operatorname{Li}_2 \left( -\frac{ae^{i(c+d\sqrt{x})}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} \\
&= \frac{2x^{5/2}}{5a} + \frac{2ibx^2 \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} - \frac{2ibx^2 \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b + \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} + \frac{8bx^{3/2} \operatorname{Li}_2 \left( -\frac{ae^{i(c+d\sqrt{x})}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} \\
&= \frac{2x^{5/2}}{5a} + \frac{2ibx^2 \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} - \frac{2ibx^2 \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b + \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} + \frac{8bx^{3/2} \operatorname{Li}_2 \left( -\frac{ae^{i(c+d\sqrt{x})}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d}
\end{aligned}$$

**Mathematica [A]** time = 2.49, size = 725, normalized size = 1.11

$$2 \sec(c + d\sqrt{x}) (a \cos(c + d\sqrt{x}) + b) \left( x^{5/2} + \frac{5ib^{e^{ic}} \left( d^4 x^2 \log \left( 1 + \frac{ae^{i(2c+d\sqrt{x})}}{b^{e^{ic}} - \sqrt{e^{2ic}(b^2 - a^2)}} \right) - d^4 x^2 \log \left( 1 + \frac{ae^{i(2c+d\sqrt{x})}}{\sqrt{e^{2ic}(b^2 - a^2)} + b^{e^{ic}}} \right) - 4id^3 x^{3/2} \text{Li}_2 \left( -\frac{a + b \sec(c + d\sqrt{x})}{b} \right)}{b} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^(3/2)/(a + b\*Sec[c + d\*Sqrt[x]]), x]

[Out] (2\*(b + a\*Cos[c + d\*Sqrt[x]])\*(x^(5/2) + ((5\*I)\*b\*E^(I\*c)\*(d^4\*x^2\*Log[1 + (a\*E^(I\*(2\*c + d\*Sqrt[x])))/(b\*E^(I\*c) - Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)])] - d^4\*x^2\*Log[1 + (a\*E^(I\*(2\*c + d\*Sqrt[x])))/(b\*E^(I\*c) + Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)])] - (4\*I)\*d^3\*x^(3/2)\*PolyLog[2, -((a\*E^(I\*(2\*c + d\*Sqrt[x])))/(b\*E^(I\*c) - Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)])])/(b\*E^(I\*c) - Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)])]) + (4\*I)\*d^3\*x^(3/2)\*PolyLog[2, -((a\*E^(I\*(2\*c + d\*Sqrt[x])))/(b\*E^(I\*c) + Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)])]) + 12\*d^2\*x\*PolyLog[3, -((a\*E^(I\*(2\*c + d\*Sqrt[x])))/(b\*E^(I\*c) - Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)])]) - 12\*d^2\*x\*PolyLog[3, -((a\*E^(I\*(2\*c + d\*Sqrt[x])))/(b\*E^(I\*c) + Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)])]) + (24\*I)\*d\*Sqrt[x]\*PolyLog[4, -((a\*E^(I\*(2\*c + d\*Sqrt[x])))/(b\*E^(I\*c) - Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)])]) - (24\*I)\*d\*Sqrt[x]\*PolyLog[4, -((a\*E^(I\*(2\*c + d\*Sqrt[x])))/(b\*E^(I\*c) + Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)])]) - 24\*PolyLog[5, -((a\*E^(I\*(2\*c + d\*Sqrt[x])))/(b\*E^(I\*c) - Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)])]) + 24\*PolyLog[5, -((a\*E^(I\*(2\*c + d\*Sqrt[x])))/(b\*E^(I\*c) + Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)])])])/(d^5\*Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)])\*Sec[c + d\*Sqrt[x]]/(5\*a\*(a + b\*Sec[c + d\*Sqrt[x]]))

**fricas [F]** time = 0.53, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{x^{\frac{3}{2}}}{b \sec(d\sqrt{x} + c) + a}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)/(a+b\*sec(c+d\*x^(1/2))), x, algorithm="fricas")

[Out] integral(x^(3/2)/(b\*sec(d\*sqrt(x) + c) + a), x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}}}{b \sec(d\sqrt{x} + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)/(a+b\*sec(c+d\*x^(1/2))),x, algorithm="giac")

[Out] integrate(x^(3/2)/(b\*sec(d\*sqrt(x) + c) + a), x)

maple [F] time = 1.07, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}}}{a + b \sec(c + d\sqrt{x})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)/(a+b\*sec(c+d\*x^(1/2))),x)

[Out] int(x^(3/2)/(a+b\*sec(c+d\*x^(1/2))),x)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)/(a+b\*sec(c+d\*x^(1/2))),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a^2-4\*b^2>0)', see 'assume?' for more details)Is 4\*a^2-4\*b^2 positive or negative?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{3/2}}{a + \frac{b}{\cos(c+d\sqrt{x})}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)/(a + b/cos(c + d\*x^(1/2))),x)

[Out] int(x^(3/2)/(a + b/cos(c + d\*x^(1/2))), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}}}{a + b \sec(c + d\sqrt{x})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(a+b*sec(c+d*x**(1/2))),x)
```

```
[Out] Integral(x**(3/2)/(a + b*sec(c + d*sqrt(x))), x)
```

$$3.62 \quad \int \frac{\sqrt{x}}{a+b \sec(c+d\sqrt{x})} dx$$

**Optimal.** Leaf size=393

$$\frac{4ib\text{Li}_3\left(-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}} - \frac{4ib\text{Li}_3\left(-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}} + \frac{4b\sqrt{x}\text{Li}_2\left(-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}} - \frac{4b\sqrt{x}\text{Li}_2\left(-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}} + \frac{2ibx \log\left(1 + \frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)}{ad\sqrt{b^2-a^2}}$$

[Out]  $\frac{2/3*x^{(3/2)}/a+2*I*b*x*\ln(1+a*\exp(I*(c+d*x^{(1/2)})))/(b-(-a^2+b^2)^{(1/2)})}{(-a^2+b^2)^{(1/2)}-2*I*b*x*\ln(1+a*\exp(I*(c+d*x^{(1/2)})))/(b+(-a^2+b^2)^{(1/2)})} / a/d/(-a^2+b^2)^{(1/2)}+4*I*b*polylog(3,-a*\exp(I*(c+d*x^{(1/2)})))/(b-(-a^2+b^2)^{(1/2)})} / a/d^3/(-a^2+b^2)^{(1/2)}-4*I*b*polylog(3,-a*\exp(I*(c+d*x^{(1/2)})))/(b+(-a^2+b^2)^{(1/2)})} / a/d^3/(-a^2+b^2)^{(1/2)}+4*b*polylog(2,-a*\exp(I*(c+d*x^{(1/2)})))/(b-(-a^2+b^2)^{(1/2)})} *x^{(1/2)}/a/d^2/(-a^2+b^2)^{(1/2)}-4*b*polylog(2,-a*\exp(I*(c+d*x^{(1/2)})))/(b+(-a^2+b^2)^{(1/2)})} *x^{(1/2)}/a/d^2/(-a^2+b^2)^{(1/2)}$

**Rubi [A]** time = 0.83, antiderivative size = 393, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$ , Rules used = {4204, 4191, 3321, 2264, 2190, 2531, 2282, 6589}

$$\frac{4b\sqrt{x}\text{PolyLog}\left(2, -\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}} - \frac{4b\sqrt{x}\text{PolyLog}\left(2, -\frac{ae^{i(c+d\sqrt{x})}}{\sqrt{b^2-a^2}+b}\right)}{ad^2\sqrt{b^2-a^2}} + \frac{4ib\text{PolyLog}\left(3, -\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}} - \frac{4ib\text{PolyLog}\left(3, -\frac{ae^{i(c+d\sqrt{x})}}{\sqrt{b^2-a^2}+b}\right)}{ad^3\sqrt{b^2-a^2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x]/(a + b\*Sec[c + d\*Sqrt[x]]), x]

[Out]  $\frac{(2*x^{(3/2)})/(3*a) + ((2*I)*b*x*\text{Log}[1 + (a*E^{(I*(c + d*\text{Sqrt}[x]))})]/(b - \text{Sqrt}[-a^2 + b^2]))/(a*\text{Sqrt}[-a^2 + b^2]*d) - ((2*I)*b*x*\text{Log}[1 + (a*E^{(I*(c + d*\text{Sqrt}[x]))})]/(b + \text{Sqrt}[-a^2 + b^2]))/(a*\text{Sqrt}[-a^2 + b^2]*d) + (4*b*\text{Sqrt}[x]*\text{PolyLog}[2, -((a*E^{(I*(c + d*\text{Sqrt}[x]))})/(b - \text{Sqrt}[-a^2 + b^2]))]/(a*\text{Sqrt}[-a^2 + b^2]*d^2) - (4*b*\text{Sqrt}[x]*\text{PolyLog}[2, -((a*E^{(I*(c + d*\text{Sqrt}[x]))})/(b + \text{Sqrt}[-a^2 + b^2]))]/(a*\text{Sqrt}[-a^2 + b^2]*d^2) + ((4*I)*b*\text{PolyLog}[3, -((a*E^{(I*(c + d*\text{Sqrt}[x]))})/(b - \text{Sqrt}[-a^2 + b^2]))]/(a*\text{Sqrt}[-a^2 + b^2]*d^3) - ((4*I)*b*\text{PolyLog}[3, -((a*E^{(I*(c + d*\text{Sqrt}[x]))})/(b + \text{Sqrt}[-a^2 + b^2]))]/(a*\text{Sqrt}[-a^2 + b^2]*d^3))}{(a*\text{Sqrt}[-a^2 + b^2]*d^3) - ((4*I)*b*\text{PolyLog}[3, -((a*E^{(I*(c + d*\text{Sqrt}[x]))})/(b + \text{Sqrt}[-a^2 + b^2]))]/(a*\text{Sqrt}[-a^2 + b^2]*d^3))}$

**Rule 2190**

Int[(((F\_)^(g\_)\*((e\_) + (f\_)\*(x\_)))^(n\_)\*((c\_) + (d\_)\*(x\_))^(m\_))/((a\_) + (b\_)\*((F\_)^(g\_)\*((e\_) + (f\_)\*(x\_)))^(n\_)), x\_Symbol] := Simp[(((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a]], x]

))<sup>n</sup>)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2264

Int[((F\_)^(u\_)\*((f\_.) + (g\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*(F\_)^(u\_) + (c\_.)\*(F\_)^(v\_)), x\_Symbol] := With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[(2\*c)/q, Int[(f + g\*x)^m\*F^u/(b - q + 2\*c\*F^u), x], x] - Dist[(2\*c)/q, Int[(f + g\*x)^m\*F^u/(b + q + 2\*c\*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2\*u] && LinearQ[u, x] && NeQ[b^2 - 4\*a\*c, 0] && IGtQ[m, 0]

#### Rule 2282

Int[u\_, x\_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_.)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_.)\*((a\_.) + (b\_.)\*x))\*(F\_) [v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

#### Rule 2531

Int[Log[1 + (e\_.)\*((F\_)^(c\_.)\*((a\_.) + (b\_.)\*(x\_)))^(n\_.)]\*((f\_.) + (g\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x))))^n])/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m - 1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

#### Rule 3321

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)/((a\_) + (b\_.)\*sin[(e\_.) + Pi\*(k\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Dist[2, Int[((c + d\*x)^m\*E^(I\*Pi\*(k - 1/2))\*E^(I\*(e + f\*x)))/(b + 2\*a\*E^(I\*Pi\*(k - 1/2))\*E^(I\*(e + f\*x)) - b\*E^(2\*I\*k\*Pi)\*E^(2\*I\*(e + f\*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2\*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]

#### Rule 4191

Int[(csc[(e\_.) + (f\_.)\*(x\_)]\*(b\_.) + (a\_))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Int[ExpandIntegrand[(c + d\*x)^m, 1/(Sin[e + f\*x]^n/(b + a\*Sin[e + f\*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGtQ[m, 0]

#### Rule 4204

Int[(x\_)^(m\_.)\*((a\_.) + (b\_.)\*Sec[(c\_.) + (d\_.)\*(x\_)^(n\_)])^(p\_.), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*Sec[c + d\*x])^p

, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt{x}}{a + b \sec(c + d\sqrt{x})} dx &= 2 \operatorname{Subst} \left( \int \frac{x^2}{a + b \sec(c + dx)} dx, x, \sqrt{x} \right) \\
 &= 2 \operatorname{Subst} \left( \int \left( \frac{x^2}{a} - \frac{bx^2}{a(b + a \cos(c + dx))} \right) dx, x, \sqrt{x} \right) \\
 &= \frac{2x^{3/2}}{3a} - \frac{(2b) \operatorname{Subst} \left( \int \frac{x^2}{b + a \cos(c + dx)} dx, x, \sqrt{x} \right)}{a} \\
 &= \frac{2x^{3/2}}{3a} - \frac{(4b) \operatorname{Subst} \left( \int \frac{e^{i(c+dx)} x^2}{a + 2be^{i(c+dx)} + ae^{2i(c+dx)}} dx, x, \sqrt{x} \right)}{a} \\
 &= \frac{2x^{3/2}}{3a} - \frac{(4b) \operatorname{Subst} \left( \int \frac{e^{i(c+dx)} x^2}{2b - 2\sqrt{-a^2 + b^2} + 2ae^{i(c+dx)}} dx, x, \sqrt{x} \right)}{\sqrt{-a^2 + b^2}} + \frac{(4b) \operatorname{Subst} \left( \int \frac{e^{i(c+dx)} x^2}{2b + 2\sqrt{-a^2 + b^2} + 2ae^{i(c+dx)}} dx, x, \sqrt{x} \right)}{\sqrt{-a^2 + b^2}} \\
 &= \frac{2x^{3/2}}{3a} + \frac{2ibx \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} - \frac{2ibx \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b + \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} - \frac{(4ib) \operatorname{Subst} \left( \int x \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b - \sqrt{-a^2 + b^2}} \right) dx, x, \sqrt{x} \right)}{a} \\
 &= \frac{2x^{3/2}}{3a} + \frac{2ibx \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} - \frac{2ibx \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b + \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} + \frac{4b\sqrt{x} \operatorname{Li}_2 \left( -\frac{ae^{i(c+d\sqrt{x})}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d^2} \\
 &= \frac{2x^{3/2}}{3a} + \frac{2ibx \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} - \frac{2ibx \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b + \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} + \frac{4b\sqrt{x} \operatorname{Li}_2 \left( -\frac{ae^{i(c+d\sqrt{x})}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d^2} \\
 &= \frac{2x^{3/2}}{3a} + \frac{2ibx \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} - \frac{2ibx \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b + \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} + \frac{4b\sqrt{x} \operatorname{Li}_2 \left( -\frac{ae^{i(c+d\sqrt{x})}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d^2}
 \end{aligned}$$



**Mathematica [A]** time = 6.70, size = 486, normalized size = 1.24

$$2 \left( d^3 x^{3/2} \sqrt{e^{2ic} (b^2 - a^2)} + 3ibe^{ic} d^2 x \log \left( 1 + \frac{ae^{i(2c+d\sqrt{x})}}{be^{ic} - \sqrt{e^{2ic} (b^2 - a^2)}} \right) - 3ibe^{ic} d^2 x \log \left( 1 + \frac{ae^{i(2c+d\sqrt{x})}}{\sqrt{e^{2ic} (b^2 - a^2)} + be^{ic}} \right) + 6be^{ic} d\sqrt{x} L \right)$$

$$3ad^3 \sqrt{e^{2ic} (b^2 - a^2)}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x]/(a + b\*Sec[c + d\*Sqrt[x]]), x]

[Out]  $(2*(d^3*\text{Sqrt}[(-a^2 + b^2)*E^((2*I)*c)])*x^{(3/2)} + (3*I)*b*d^2*E^(I*c)*x*\text{Log}[1 + (a*E^(I*(2*c + d*\text{Sqrt}[x])))/(b*E^(I*c) - \text{Sqrt}[(-a^2 + b^2)*E^((2*I)*c)])] - (3*I)*b*d^2*E^(I*c)*x*\text{Log}[1 + (a*E^(I*(2*c + d*\text{Sqrt}[x])))/(b*E^(I*c) + \text{Sqrt}[(-a^2 + b^2)*E^((2*I)*c)])] + 6*b*d*E^(I*c)*\text{Sqrt}[x]*\text{PolyLog}[2, -((a*E^(I*(2*c + d*\text{Sqrt}[x])))/(b*E^(I*c) - \text{Sqrt}[(-a^2 + b^2)*E^((2*I)*c)])]) - 6*b*d*E^(I*c)*\text{Sqrt}[x]*\text{PolyLog}[2, -((a*E^(I*(2*c + d*\text{Sqrt}[x])))/(b*E^(I*c) + \text{Sqrt}[(-a^2 + b^2)*E^((2*I)*c)])]) + (6*I)*b*E^(I*c)*\text{PolyLog}[3, -((a*E^(I*(2*c + d*\text{Sqrt}[x])))/(b*E^(I*c) - \text{Sqrt}[(-a^2 + b^2)*E^((2*I)*c)])]) - (6*I)*b*E^(I*c)*\text{PolyLog}[3, -((a*E^(I*(2*c + d*\text{Sqrt}[x])))/(b*E^(I*c) + \text{Sqrt}[(-a^2 + b^2)*E^((2*I)*c)])])]/(3*a*d^3*\text{Sqrt}[(-a^2 + b^2)*E^((2*I)*c)])$

**fricas [F]** time = 0.62, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{\sqrt{x}}{b \sec(d\sqrt{x} + c) + a}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(a+b\*sec(c+d\*x^(1/2))), x, algorithm="fricas")

[Out] integral(sqrt(x)/(b\*sec(d\*sqrt(x) + c) + a), x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{b \sec(d\sqrt{x} + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(a+b\*sec(c+d\*x^(1/2))), x, algorithm="giac")

[Out] integrate(sqrt(x)/(b\*sec(d\*sqrt(x) + c) + a), x)

**maple [F]** time = 1.01, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{a + b \sec(c + d\sqrt{x})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(1/2)/(a+b*sec(c+d*x^(1/2))),x)`

[Out] `int(x^(1/2)/(a+b*sec(c+d*x^(1/2))),x)`

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(1/2)/(a+b*sec(c+d*x^(1/2))),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a^2-4\*b^2>0)', see 'assume?' for more details)Is 4\*a^2-4\*b^2 positive or negative?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{x}}{a + \frac{b}{\cos(c+d\sqrt{x})}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(1/2)/(a + b/cos(c + d*x^(1/2))),x)`

[Out] `int(x^(1/2)/(a + b/cos(c + d*x^(1/2))), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{a + b \sec(c + d\sqrt{x})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(1/2)/(a+b*sec(c+d*x**(1/2))),x)`

[Out] `Integral(sqrt(x)/(a + b*sec(c + d*sqrt(x))), x)`

$$3.63 \quad \int \frac{1}{\sqrt{x}(a+b \sec(c+d\sqrt{x}))} dx$$

Optimal. Leaf size=68

$$\frac{2\sqrt{x}}{a} - \frac{4b \tanh^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+d\sqrt{x})\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}$$

[Out]  $-4*b*\operatorname{arctanh}((a-b)^{(1/2)}*\tan(1/2*c+1/2*d*x^{(1/2)})/(a+b)^{(1/2)})/a/d/(a-b)^{(1/2)}/(a+b)^{(1/2)+2*x^{(1/2)}/a$

**Rubi [A]** time = 0.08, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {4204, 3783, 2659, 208}

$$\frac{2\sqrt{x}}{a} - \frac{4b \tanh^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+d\sqrt{x})\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[x]\*(a + b\*Sec[c + d\*Sqrt[x]])),x]

[Out]  $(2*\operatorname{Sqrt}[x])/a - (4*b*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a - b]*\operatorname{Tan}[(c + d*\operatorname{Sqrt}[x])/2])/(\operatorname{Sqrt}[a + b])]/(a*\operatorname{Sqrt}[a - b]*\operatorname{Sqrt}[a + b]*d)$

#### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

#### Rule 2659

Int[((a\_) + (b\_.)\*sin[Pi/2 + (c\_.) + (d\_.)\*(x\_)])^(-1), x\_Symbol] := With[{e = FreeFactors[Tan[(c + d\*x)/2], x]}, Dist[(2\*e)/d, Subst[Int[1/(a + b + (a - b)\*e^2\*x^2), x], x, Tan[(c + d\*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

#### Rule 3783

Int[(csc[(c\_.) + (d\_.)\*(x\_)])\*(b\_.) + (a\_)^(-1), x\_Symbol] := Simp[x/a, x] - Dist[1/a, Int[1/(1 + (a\*Sin[c + d\*x])/b), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 4204

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{x} (a + b \sec(c + d\sqrt{x}))} dx &= 2 \operatorname{Subst} \left( \int \frac{1}{a + b \sec(c + dx)} dx, x, \sqrt{x} \right) \\
&= \frac{2\sqrt{x}}{a} - \frac{2 \operatorname{Subst} \left( \int \frac{1}{1 + \frac{a \cos(c+dx)}{b}} dx, x, \sqrt{x} \right)}{a} \\
&= \frac{2\sqrt{x}}{a} - \frac{4 \operatorname{Subst} \left( \int \frac{1}{1 + \frac{a}{b} + (1 - \frac{a}{b})x^2} dx, x, \tan \left( \frac{1}{2} (c + d\sqrt{x}) \right) \right)}{ad} \\
&= \frac{2\sqrt{x}}{a} - \frac{4b \tanh^{-1} \left( \frac{\sqrt{a-b} \tan \left( \frac{1}{2} (c + d\sqrt{x}) \right)}{\sqrt{a+b}} \right)}{a\sqrt{a-b}\sqrt{a+b}d}
\end{aligned}$$

Mathematica [A]    time = 0.26, size = 69, normalized size = 1.01

$$\frac{2 \left( \frac{2b \tanh^{-1} \left( \frac{(b-a) \tan \left( \frac{1}{2} (c + d\sqrt{x}) \right)}{\sqrt{a^2 - b^2}} \right)}{d\sqrt{a^2 - b^2}} + \frac{c}{d} + \sqrt{x} \right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a + b*Sec[c + d*Sqrt[x]])),x]
```

```
[Out] (2*(c/d + Sqrt[x] + (2*b*ArcTanh[(-a + b)*Tan[(c + d*Sqrt[x])/2]]/Sqrt[a^2
- b^2]))/(Sqrt[a^2 - b^2]*d))/a
```

**fricas** [A] time = 0.55, size = 274, normalized size = 4.03

$$\frac{2(a^2 - b^2)d\sqrt{x} + \sqrt{a^2 - b^2} b \log\left(\frac{2ab \cos(d\sqrt{x}+c) - (a^2 - 2b^2) \cos(d\sqrt{x}+c)^2 + 2a^2 - b^2 - 2(\sqrt{a^2 - b^2} b \cos(d\sqrt{x}+c) + \sqrt{a^2 - b^2} a) \sin(d\sqrt{x}+c)}{a^2 \cos(d\sqrt{x}+c)^2 + 2ab \cos(d\sqrt{x}+c) + b^2}\right)}{(a^3 - ab^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*sec(c+d\*x^(1/2)))/x^(1/2),x, algorithm="fricas")

[Out] [(2\*(a^2 - b^2)\*d\*sqrt(x) + sqrt(a^2 - b^2)\*b\*log((2\*a\*b\*cos(d\*sqrt(x) + c) - (a^2 - 2\*b^2)\*cos(d\*sqrt(x) + c)^2 + 2\*a^2 - b^2 - 2\*(sqrt(a^2 - b^2)\*b\*cos(d\*sqrt(x) + c) + sqrt(a^2 - b^2)\*a)\*sin(d\*sqrt(x) + c))/(a^2\*cos(d\*sqrt(x) + c)^2 + 2\*a\*b\*cos(d\*sqrt(x) + c) + b^2)))/((a^3 - a\*b^2)\*d), 2\*((a^2 - b^2)\*d\*sqrt(x) - sqrt(-a^2 + b^2)\*b\*arctan(-(sqrt(-a^2 + b^2)\*b\*cos(d\*sqrt(x) + c) + sqrt(-a^2 + b^2)\*a)/((a^2 - b^2)\*sin(d\*sqrt(x) + c)))/((a^3 - a\*b^2)\*d)]

**giac** [B] time = 0.54, size = 278, normalized size = 4.09

$$\frac{2\left(\sqrt{-a^2 + b^2}(a - 2b)d|-a + b| - \sqrt{-a^2 + b^2}|a||-a + b||d\right)\left(\pi\left[\frac{d\sqrt{x}+c}{2\pi} + \frac{1}{2}\right] + \arctan\left(\frac{\tan\left(\frac{1}{2}d\sqrt{x} + \frac{1}{2}c\right)}{\sqrt{\frac{bd + \sqrt{b^2d^2 + (ad+bd)(ad-bd)}}{ad-bd}}}\right)\right)}{(a^2 - 2ab + b^2)a^2d^2 + (a^2b - 2ab^2 + b^3)d|a||d|}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*sec(c+d\*x^(1/2)))/x^(1/2),x, algorithm="giac")

[Out] 2\*(sqrt(-a^2 + b^2)\*(a - 2\*b)\*d\*abs(-a + b) - sqrt(-a^2 + b^2)\*abs(a)\*abs(-a + b)\*abs(d))\*(pi\*floor(1/2\*(d\*sqrt(x) + c)/pi + 1/2) + arctan(tan(1/2\*d\*sqrt(x) + 1/2\*c)/sqrt(-(b\*d + sqrt(b^2\*d^2 + (a\*d + b\*d)\*(a\*d - b\*d)))/(a\*d - b\*d))))/((a^2 - 2\*a\*b + b^2)\*a^2\*d^2 + (a^2\*b - 2\*a\*b^2 + b^3)\*d\*abs(a)\*abs(d)) + 2\*(a\*d - 2\*b\*d + abs(a)\*abs(d))\*(pi\*floor(1/2\*(d\*sqrt(x) + c)/pi + 1/2) + arctan(tan(1/2\*d\*sqrt(x) + 1/2\*c)/sqrt(-(b\*d - sqrt(b^2\*d^2 + (a\*d + b\*d)\*(a\*d - b\*d)))/(a\*d - b\*d))))/((a^2\*d^2 - b\*d\*abs(a)\*abs(d))

**maple** [A] time = 0.71, size = 71, normalized size = 1.04

$$-\frac{4b \operatorname{arctanh}\left(\frac{(a-b) \tan\left(\frac{c}{2} + \frac{d\sqrt{x}}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{da\sqrt{(a-b)(a+b)}} + \frac{4 \operatorname{arctan}\left(\tan\left(\frac{c}{2} + \frac{d\sqrt{x}}{2}\right)\right)}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a+b*sec(c+d*x^(1/2)))/x^(1/2),x)`

[Out] `-4/d/a*b/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*c+1/2*d*x^(1/2))/((a-b)*(a+b))^(1/2))+4/d/a*arctan(tan(1/2*c+1/2*d*x^(1/2)))`

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*sec(c+d*x^(1/2)))/x^(1/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a^2-4\*b^2>0)', see `assume?` for more details) Is 4\*a^2-4\*b^2 positive or negative?

**mupad** [B] time = 2.75, size = 153, normalized size = 2.25

$$\frac{2\sqrt{x}}{a} + \frac{2b \ln\left(2b e^{d\sqrt{x}} e^{ci} - \frac{b(a+b e^{d\sqrt{x}} e^{ci})^{2i}}{\sqrt{a+b}\sqrt{a-b}}\right)}{ad\sqrt{a+b}\sqrt{a-b}} - \frac{2b \ln\left(2b e^{d\sqrt{x}} e^{ci} + \frac{b(a+b e^{d\sqrt{x}} e^{ci})^{2i}}{\sqrt{a+b}\sqrt{a-b}}\right)}{ad\sqrt{a+b}\sqrt{a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^(1/2)*(a + b/cos(c + d*x^(1/2))))),x)`

[Out] `(2*x^(1/2))/a + (2*b*log(2*b*exp(d*x^(1/2)*1i)*exp(c*1i) - (b*(a + b*exp(d*x^(1/2)*1i)*exp(c*1i))*2i)/((a + b)^(1/2)*(a - b)^(1/2)))/(a*d*(a + b)^(1/2)*(a - b)^(1/2)) - (2*b*log(2*b*exp(d*x^(1/2)*1i)*exp(c*1i) + (b*(a + b*exp(d*x^(1/2)*1i)*exp(c*1i))*2i)/((a + b)^(1/2)*(a - b)^(1/2)))/(a*d*(a + b)^(1/2)*(a - b)^(1/2))`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x} (a + b \sec(c + d\sqrt{x}))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*sec(c+d*x**(1/2)))/x**(1/2),x)`

[Out] `Integral(1/(sqrt(x)*(a + b*sec(c + d*sqrt(x))))), x)`

$$3.64 \quad \int \frac{1}{x^{3/2}(a+b \sec(c+d\sqrt{x}))} dx$$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{1}{x^{3/2}(a+b \sec(c+d\sqrt{x}))}, x\right)$$

[Out] Unintegrable(1/x^(3/2)/(a+b\*sec(c+d\*x^(1/2))), x)

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{x^{3/2}(a+b \sec(c+d\sqrt{x}))} dx$$

Verification is Not applicable to the result.

[In] Int[1/(x^(3/2)\*(a + b\*Sec[c + d\*Sqrt[x]])), x]

[Out] Defer[Int][1/(x^(3/2)\*(a + b\*Sec[c + d\*Sqrt[x]])), x]

Rubi steps

$$\int \frac{1}{x^{3/2}(a+b \sec(c+d\sqrt{x}))} dx = \int \frac{1}{x^{3/2}(a+b \sec(c+d\sqrt{x}))} dx$$

Mathematica [A] time = 4.40, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3/2}(a+b \sec(c+d\sqrt{x}))} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x^(3/2)\*(a + b\*Sec[c + d\*Sqrt[x]])), x]

[Out] Integrate[1/(x^(3/2)\*(a + b\*Sec[c + d\*Sqrt[x]])), x]

fricas [A] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x}}{bx^2 \sec(d\sqrt{x} + c) + ax^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(3/2)/(a+b\*sec(c+d\*x^(1/2))),x, algorithm="fricas")

[Out] integral(sqrt(x)/(b\*x^2\*sec(d\*sqrt(x) + c) + a\*x^2), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \sec(d\sqrt{x} + c) + a)x^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(3/2)/(a+b\*sec(c+d\*x^(1/2))),x, algorithm="giac")

[Out] integrate(1/((b\*sec(d\*sqrt(x) + c) + a)\*x^(3/2)), x)

**maple** [A] time = 1.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{3}{2}}(a + b \sec(c + d\sqrt{x}))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^(3/2)/(a+b\*sec(c+d\*x^(1/2))),x)

[Out] int(1/x^(3/2)/(a+b\*sec(c+d\*x^(1/2))),x)

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(3/2)/(a+b\*sec(c+d\*x^(1/2))),x, algorithm="maxima")

[Out] Timed out

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x^{3/2} \left( a + \frac{b}{\cos(c+d\sqrt{x})} \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^(3/2)\*(a + b/cos(c + d\*x^(1/2))))),x)

[Out] int(1/(x^(3/2)\*(a + b/cos(c + d\*x^(1/2))))), x)



sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{3}{2}} (a + b \sec(c + d\sqrt{x}))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*(3/2)/(a+b\*sec(c+d\*x\*\*(1/2))), x)

[Out] Integral(1/(x\*\*(3/2)\*(a + b\*sec(c + d\*sqrt(x)))), x)

$$3.65 \quad \int \frac{1}{x^{5/2}(a+b \sec(c+d\sqrt{x}))} dx$$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{1}{x^{5/2}(a+b \sec(c+d\sqrt{x}))}, x\right)$$

[Out] Unintegrable(1/x^(5/2)/(a+b\*sec(c+d\*x^(1/2))), x)

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{x^{5/2}(a+b \sec(c+d\sqrt{x}))} dx$$

Verification is Not applicable to the result.

[In] Int[1/(x^(5/2)\*(a + b\*Sec[c + d\*Sqrt[x]])), x]

[Out] Defer[Int][1/(x^(5/2)\*(a + b\*Sec[c + d\*Sqrt[x]])), x]

Rubi steps

$$\int \frac{1}{x^{5/2}(a+b \sec(c+d\sqrt{x}))} dx = \int \frac{1}{x^{5/2}(a+b \sec(c+d\sqrt{x}))} dx$$

Mathematica [A] time = 3.96, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{5/2}(a+b \sec(c+d\sqrt{x}))} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x^(5/2)\*(a + b\*Sec[c + d\*Sqrt[x]])), x]

[Out] Integrate[1/(x^(5/2)\*(a + b\*Sec[c + d\*Sqrt[x]])), x]

fricas [A] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x}}{bx^3 \sec(d\sqrt{x} + c) + ax^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(5/2)/(a+b\*sec(c+d\*x^(1/2))),x, algorithm="fricas")

[Out] integral(sqrt(x)/(b\*x^3\*sec(d\*sqrt(x) + c) + a\*x^3), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \sec(d\sqrt{x} + c) + a)x^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(5/2)/(a+b\*sec(c+d\*x^(1/2))),x, algorithm="giac")

[Out] integrate(1/((b\*sec(d\*sqrt(x) + c) + a)\*x^(5/2)), x)

**maple** [A] time = 0.99, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{5}{2}} (a + b \sec(c + d\sqrt{x}))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^(5/2)/(a+b\*sec(c+d\*x^(1/2))),x)

[Out] int(1/x^(5/2)/(a+b\*sec(c+d\*x^(1/2))),x)

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(5/2)/(a+b\*sec(c+d\*x^(1/2))),x, algorithm="maxima")

[Out] Timed out

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x^{5/2} \left( a + \frac{b}{\cos(c+d\sqrt{x})} \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^(5/2)\*(a + b/cos(c + d\*x^(1/2))))),x)

[Out] int(1/(x^(5/2)\*(a + b/cos(c + d\*x^(1/2))))), x)

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{5}{2}} (a + b \sec(c + d\sqrt{x}))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*(5/2)/(a+b\*sec(c+d\*x\*\*(1/2))),x)

[Out] Integral(1/(x\*\*(5/2)\*(a + b\*sec(c + d\*sqrt(x)))), x)





$$\begin{aligned} & t[-a^2 + b^2]))]/(a^2 \sqrt{-a^2 + b^2} d^4) + ((48I) b^3 \text{PolyLog}[5, -(a * \\ & E^{(I(c + d \sqrt{x}))})/(b - \sqrt{-a^2 + b^2})])/(a^2 (-a^2 + b^2)^{(3/2)} d^5) - ((96I) b * \text{PolyLog}[5, -(a * E^{(I(c + d \sqrt{x}))})/(b - \sqrt{-a^2 + b^2}) \\ & )])/(a^2 \sqrt{-a^2 + b^2} d^5) - ((48I) b^3 \text{PolyLog}[5, -(a * E^{(I(c + d \sqrt{x}))})/(b + \sqrt{-a^2 + b^2})])/(a^2 (-a^2 + b^2)^{(3/2)} d^5) + ((96I) b * \text{PolyLog}[5, -(a * E^{(I(c + d \sqrt{x}))})/(b + \sqrt{-a^2 + b^2})])/(a^2 \sqrt{-a^2 + b^2} d^5) + (2 b^2 x^2 \text{Sin}[c + d \sqrt{x}])/(a (a^2 - b^2) d (b + a * \text{Cos}[c + d \sqrt{x}])) \end{aligned}$$

### Rule 2190

$$\begin{aligned} & \text{Int}[(((F_)^{((g_)*(e_) + (f_)*(x_))})^{(n_)*((c_) + (d_)*(x_))^{(m_))}/ \\ & ((a_) + (b_)*((F_)^{((g_)*(e_) + (f_)*(x_))})^{(n_)}), x\_Symbol] \text{:>} \text{Simp} \\ & [((c + d*x)^m * \text{Log}[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*\text{Log}[F]), x] - \text{Dist} \\ & [(d*m)/(b*f*g*n*\text{Log}[F]), \text{Int}[(c + d*x)^{(m-1)} * \text{Log}[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x\} \&\& \text{IGtQ}[m, 0] \end{aligned}$$

### Rule 2264

$$\begin{aligned} & \text{Int}[((F_)^{(u_)*((f_) + (g_)*(x_))^{(m_))}/((a_) + (b_)*(F_)^{(u_)} + (c_)) \\ & *(F_)^{(v_)}), x\_Symbol] \text{:>} \text{With}\{q = \text{Rt}[b^2 - 4*a*c, 2]\}, \text{Dist}[(2*c)/q, \text{Int} \\ & [(f + g*x)^m * F^u/(b - q + 2*c * F^u), x], x] - \text{Dist}[(2*c)/q, \text{Int}[(f + g*x)^ \\ & m * F^u/(b + q + 2*c * F^u), x], x] /; \text{FreeQ}\{F, a, b, c, f, g\}, x\} \&\& \text{EqQ}[v, \\ & 2*u] \&\& \text{LinearQ}[u, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{IGtQ}[m, 0] \end{aligned}$$

### Rule 2282

$$\begin{aligned} & \text{Int}[u_, x\_Symbol] \text{:>} \text{With}\{v = \text{FunctionOfExponential}[u, x]\}, \text{Dist}[v/D[v, x] \\ & , \text{Subst}[\text{Int}[\text{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x] /; \text{FunctionOfExponentialQ}[u, x] \&\& !\text{MatchQ}[u, (w_)*((a_)*(v_)^{(n_)})^{(m_)} /; \text{FreeQ}\{a, m, n\}, x] \&\& \text{IntegerQ}[m*n] \&\& !\text{MatchQ}[u, E^{((c_)*((a_) + (b_)*x))* (F_)[v_]} /; \text{FreeQ}\{a, b, c\}, x] \&\& \text{InverseFunctionQ}[F[x]] \end{aligned}$$

### Rule 2531

$$\begin{aligned} & \text{Int}[\text{Log}[1 + (e_)*((F_)^{((c_)*((a_) + (b_)*(x_))})^{(n_)}]*((f_) + (g_)) \\ & *(x_))^{(m_)}), x\_Symbol] \text{:>} -\text{Simp}[(f + g*x)^m * \text{PolyLog}[2, -(e*(F^(c*(a + b*x)))^n)]/(b*c*n*\text{Log}[F]), x] + \text{Dist}[(g*m)/(b*c*n*\text{Log}[F]), \text{Int}[(f + g*x)^{(m-1)} * \text{PolyLog}[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; \text{FreeQ}\{F, a, b, c, e, f, g, n\}, x] \&\& \text{GtQ}[m, 0] \end{aligned}$$

### Rule 3321

$$\begin{aligned} & \text{Int}[(((c_) + (d_)*(x_))^{(m_)})/((a_) + (b_)*\text{sin}[(e_) + \text{Pi}*(k_) + (f_)*(x_)]), x\_Symbol] \text{:>} \text{Dist}[2, \text{Int}[(c + d*x)^m * E^{(I*Pi*(k - 1/2))} * E^{(I*(e + f * x))})/(b + 2*a * E^{(I*Pi*(k - 1/2))} * E^{(I*(e + f * x))} - b * E^{(2*I*k*Pi)} * E^{(2*I*( \end{aligned}$$

$e + f*x))$ ),  $x]$ ,  $x]$  /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2\*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]

### Rule 3324

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)/((a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^2, x\_Symbol] := Simp[(b\*(c + d\*x)^m\*cos[e + f\*x])/(f\*(a^2 - b^2)\*(a + b\*sin[e + f\*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d\*x)^m/(a + b\*sin[e + f\*x]), x], x] - Dist[(b\*d\*m)/(f\*(a^2 - b^2)), Int[((c + d\*x)^(m - 1)\*cos[e + f\*x])/(a + b\*sin[e + f\*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]

### Rule 4191

Int[(csc[(e\_.) + (f\_.)\*(x\_)]\*(b\_.) + (a\_.))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Int[ExpandIntegrand[(c + d\*x)^m, 1/(Sin[e + f\*x]^n/(b + a\*sin[e + f\*x]^n)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGtQ[m, 0]

### Rule 4204

Int[(x\_)^((m\_.)\*((a\_.) + (b\_.)\*Sec[(c\_.) + (d\_.)\*(x\_)])^(p\_.), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*Sec[c + d\*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]

### Rule 4522

Int[((e\_.) + (f\_.)\*(x\_))^(m\_.)\*Sin[(c\_.) + (d\_.)\*(x\_)])/(Cos[(c\_.) + (d\_.)\*(x\_)]\*(b\_.) + (a\_.), x\_Symbol] := Simp[(I\*(e + f\*x)^(m + 1))/(b\*f\*(m + 1)), x] + (Int[((e + f\*x)^m\*E^(I\*(c + d\*x)))/(I\*a - Rt[-a^2 + b^2, 2] + I\*b\*E^(I\*(c + d\*x))), x] + Int[((e + f\*x)^m\*E^(I\*(c + d\*x)))/(I\*a + Rt[-a^2 + b^2, 2] + I\*b\*E^(I\*(c + d\*x))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NegQ[a^2 - b^2]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rule 6609

Int[((e\_.) + (f\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (d\_.)\*((F\_)^(c\_.)\*((a\_.) + (b\_.)\*(x\_)))]^(p\_.), x\_Symbol] := Simp[((e + f\*x)^m\*PolyLog[n + 1, d\*(F^(c\*(a



```

+ b*x)))^p]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]

```

Rubi steps



**Mathematica [A]** time = 13.83, size = 2254, normalized size = 1.17

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[x^(3/2)/(a + b\*Sec[c + d\*Sqrt[x]])^2,x]

[Out]  $(2x^{5/2}(b + a\cos[c + d\sqrt{x}])^2\sec[c + d\sqrt{x}]^2)/(5a^2(a + b\sec[c + d\sqrt{x}])^2) + (2bE^{(Ic)}(b + a\cos[c + d\sqrt{x}])^2((-2I)bE^{(Ic)}x^2 + ((1 + E^{(2I)c})\sqrt{-a^2 + b^2}E^{(2I)c})x^{3/2}\log[1 + (aE^{(I(2c + d\sqrt{x}))})/(bE^{(Ic)} - \sqrt{-a^2 + b^2}E^{(2I)c})]) + (2I)a^2d^4E^{(Ic)}x^2\log[1 + (aE^{(I(2c + d\sqrt{x}))})/(bE^{(Ic)} - \sqrt{-a^2 + b^2}E^{(2I)c})]) - Ib^2d^4E^{(Ic)}x^2\log[1 + (aE^{(I(2c + d\sqrt{x}))})/(bE^{(Ic)} - \sqrt{-a^2 + b^2}E^{(2I)c})]) + 4bd^3\sqrt{-a^2 + b^2}E^{(2I)c}x^{3/2}\log[1 + (aE^{(I(2c + d\sqrt{x}))})/(bE^{(Ic)} + \sqrt{-a^2 + b^2}E^{(2I)c})]) - (2I)a^2d^4E^{(Ic)}x^2\log[1 + (aE^{(I(2c + d\sqrt{x}))})/(bE^{(Ic)} + \sqrt{-a^2 + b^2}E^{(2I)c})]) + Ib^2d^4E^{(Ic)}x^2\log[1 + (aE^{(I(2c + d\sqrt{x}))})/(bE^{(Ic)} + \sqrt{-a^2 + b^2}E^{(2I)c})]) - 4d^2((3I)b\sqrt{-a^2 + b^2}E^{(2I)c} - 2a^2dE^{(Ic)}\sqrt{x} + b^2dE^{(Ic)}\sqrt{x})x\text{PolyLog}[2, -(aE^{(I(2c + d\sqrt{x}))})/(bE^{(Ic)} - \sqrt{-a^2 + b^2}E^{(2I)c})]) + 4d^2((-3I)b\sqrt{-a^2 + b^2}E^{(2I)c} - 2a^2dE^{(Ic)}\sqrt{x} + b^2dE^{(Ic)}\sqrt{x})x\text{PolyLog}[2, -(aE^{(I(2c + d\sqrt{x}))})/(bE^{(Ic)} + \sqrt{-a^2 + b^2}E^{(2I)c})]) + 24bd\sqrt{-a^2 + b^2}E^{(2I)c}\sqrt{x}\text{PolyLog}[3, -(aE^{(I(2c + d\sqrt{x}))})/(bE^{(Ic)} - \sqrt{-a^2 + b^2}E^{(2I)c})]) + (24I)a^2d^2E^{(Ic)}x\text{PolyLog}[3, -(aE^{(I(2c + d\sqrt{x}))})/(bE^{(Ic)} - \sqrt{-a^2 + b^2}E^{(2I)c})]) - (12I)b^2d^2E^{(Ic)}x\text{PolyLog}[3, -(aE^{(I(2c + d\sqrt{x}))})/(bE^{(Ic)} - \sqrt{-a^2 + b^2}E^{(2I)c})]) + 24bd\sqrt{-a^2 + b^2}E^{(2I)c}\sqrt{x}\text{PolyLog}[3, -(aE^{(I(2c + d\sqrt{x}))})/(bE^{(Ic)} + \sqrt{-a^2 + b^2}E^{(2I)c})]) - (24I)a^2d^2E^{(Ic)}x\text{PolyLog}[3, -(aE^{(I(2c + d\sqrt{x}))})/(bE^{(Ic)} + \sqrt{-a^2 + b^2}E^{(2I)c})]) + (12I)b^2d^2E^{(Ic)}x\text{PolyLog}[3, -(aE^{(I(2c + d\sqrt{x}))})/(bE^{(Ic)} + \sqrt{-a^2 + b^2}E^{(2I)c})]) + (24I)b\sqrt{-a^2 + b^2}E^{(2I)c}\text{PolyLog}[4, -(aE^{(I(2c + d\sqrt{x}))})/(bE^{(Ic)} - \sqrt{-a^2 + b^2}E^{(2I)c})]) - 48a^2dE^{(Ic)}\sqrt{x}\text{PolyLog}[4, -(aE^{(I(2c + d\sqrt{x}))})/(bE^{(Ic)} - \sqrt{-a^2 + b^2}E^{(2I)c})]) + 24b^2dE^{(Ic)}\sqrt{x}\text{PolyLog}[4, -(aE^{(I(2c + d\sqrt{x}))})/(bE^{(Ic)} - \sqrt{-a^2 + b^2}E^{(2I)c})]) + (24I)b\sqrt{-a^2 + b^2}E^{(2I)c}\text{PolyLog}[4, -(aE^{(I(2c + d\sqrt{x}))})/(bE^{(Ic)} + \sqrt{-a^2 + b^2}E^{(2I)c})]) + 48a^2dE^{(Ic)}\sqrt{x}\text{PolyLog}[4, -(aE^{(I(2c + d\sqrt{x}))})/(bE^{(Ic)} + \sqrt{-a^2 + b^2}E^{(2I)c})]) - 24b^2dE^{(Ic)}\sqrt{x}\text{PolyLog}[4, -(aE^{(I(2c + d\sqrt{x}))})/(bE^{(Ic)} + \sqrt{-a^2 + b^2}E^{(2I)c})]) - (48I)a^2E^{(Ic)}\text{PolyLog}[5, -(aE^{(I(2c + d\sqrt{x}))})/(bE^{(Ic)} - \sqrt{-a^2 + b^2}E^{(2I)c})]) + (48I)a^2E^{(Ic)}\text{PolyLog}[5, -(aE^{(I(2c + d\sqrt{x}))})/(bE^{(Ic)} + \sqrt{-a^2 + b^2}E^{(2I)c})])$

$E^{(I*c)} - \text{Sqrt}[(-a^2 + b^2)*E^{((2*I)*c)}]] + (24*I)*b^2*E^{(I*c)}*PolyLog[5,$   
 $-((a*E^{(I*(2*c + d*Sqrt[x]))})/(b*E^{(I*c)} - \text{Sqrt}[(-a^2 + b^2)*E^{((2*I)*c)}]))$   
 $] + (48*I)*a^2*E^{(I*c)}*PolyLog[5, -((a*E^{(I*(2*c + d*Sqrt[x]))})/(b*E^{(I*c)} +$   
 $\text{Sqrt}[(-a^2 + b^2)*E^{((2*I)*c)}]))] - (24*I)*b^2*E^{(I*c)}*PolyLog[5, -((a*E^{(I*(2*c + d*Sqrt[x]))})/(b*E^{(I*c)} +$   
 $\text{Sqrt}[(-a^2 + b^2)*E^{((2*I)*c)}])))]/(d^4*E^{(I*c)}*Sqrt[(-a^2 + b^2)*E^{((2*I)*c)}])*Sec[c + d*Sqrt[x]]^2)/(a^2*(a^2 -$   
 $b^2)*d*(1 + E^{((2*I)*c)})*(a + b*Sec[c + d*Sqrt[x]])^2) + (2*(b + a*Cos[c +$   
 $d*Sqrt[x]])*Sec[c + d*Sqrt[x]]^2*(b^3*x^2*Sin[c] - a*b^2*x^2*Sin[d*Sqrt[x]])))/(a^2*(-a + b)*(a + b)*d*(a + b*Sec[c + d*Sqrt[x]])^2*(Cos[c/2] - Sin[c/2]))*(Cos[c/2] + Sin[c/2]))$

**fricas** [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^{\frac{3}{2}}}{b^2 \sec(d\sqrt{x} + c)^2 + 2ab \sec(d\sqrt{x} + c) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)/(a+b\*sec(c+d\*x^(1/2)))^2,x, algorithm="fricas")

[Out] integral(x^(3/2)/(b^2\*sec(d\*sqrt(x) + c)^2 + 2\*a\*b\*sec(d\*sqrt(x) + c) + a^2), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}}}{(b \sec(d\sqrt{x} + c) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)/(a+b\*sec(c+d\*x^(1/2)))^2,x, algorithm="giac")

[Out] integrate(x^(3/2)/(b\*sec(d\*sqrt(x) + c) + a)^2, x)

**maple** [F] time = 1.33, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}}}{(a + b \sec(c + d\sqrt{x}))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)/(a+b\*sec(c+d\*x^(1/2)))^2,x)

[Out] int(x^(3/2)/(a+b\*sec(c+d\*x^(1/2)))^2,x)

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)/(a+b\*sec(c+d\*x^(1/2)))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a^2-4\*b^2>0)', see `assume?` for more details)Is 4\*a^2-4\*b^2 positive or negative?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{3/2}}{\left(a + \frac{b}{\cos(c+d\sqrt{x})}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)/(a + b/cos(c + d\*x^(1/2)))^2,x)

[Out] int(x^(3/2)/(a + b/cos(c + d\*x^(1/2)))^2, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}}}{\left(a + b \sec(c + d\sqrt{x})\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(3/2)/(a+b\*sec(c+d\*x\*\*(1/2)))\*\*2,x)

[Out] Integral(x\*\*(3/2)/(a + b\*sec(c + d\*sqrt(x)))\*\*2, x)

$$3.67 \quad \int \frac{\sqrt{x}}{(a+b \sec(c+d\sqrt{x}))^2} dx$$

**Optimal.** Leaf size=1125

$$\frac{2ix \log\left(\frac{e^{i(c+d\sqrt{x})}a}{b-\sqrt{b^2-a^2}} + 1\right) b^3}{a^2 (b^2 - a^2)^{3/2} d} + \frac{2ix \log\left(\frac{e^{i(c+d\sqrt{x})}a}{b+\sqrt{b^2-a^2}} + 1\right) b^3}{a^2 (b^2 - a^2)^{3/2} d} - \frac{4\sqrt{x} \operatorname{Li}_2\left(-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right) b^3}{a^2 (b^2 - a^2)^{3/2} d^2} + \frac{4\sqrt{x} \operatorname{Li}_2\left(-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right) b^3}{a^2 (b^2 - a^2)^{3/2} d^2} - \frac{4i \operatorname{Li}_3}{a^2 (b^2 - a^2)^{3/2} d^2}$$

[Out]  $-2*I*b^3*x*\ln(1+a*\exp(I*(c+d*x^(1/2))))/(b-(-a^2+b^2)^(1/2))/a^2/(-a^2+b^2)^(3/2)/d+2/3*x^(3/2)/a^2+8*I*b*polylog(3,-a*\exp(I*(c+d*x^(1/2))))/(b-(-a^2+b^2)^(1/2))/a^2/d^3/(-a^2+b^2)^(1/2)-8*I*b*polylog(3,-a*\exp(I*(c+d*x^(1/2))))/(b+(-a^2+b^2)^(1/2))/a^2/d^3/(-a^2+b^2)^(1/2)-2*I*b^2*x/a^2/(a^2-b^2)/d-4*I*b^2*polylog(2,-a*\exp(I*(c+d*x^(1/2))))/(b-I*(a^2-b^2)^(1/2))/a^2/(a^2-b^2)/d^3-4*I*b*x*\ln(1+a*\exp(I*(c+d*x^(1/2))))/(b+(-a^2+b^2)^(1/2))/a^2/d/(-a^2+b^2)^(1/2)-4*I*b^2*polylog(2,-a*\exp(I*(c+d*x^(1/2))))/(b+I*(a^2-b^2)^(1/2))/a^2/(a^2-b^2)/d^3+2*b^2*x*\sin(c+d*x^(1/2))/a/(a^2-b^2)/d/(b+a*\cos(c+d*x^(1/2)))+4*I*b^3*polylog(3,-a*\exp(I*(c+d*x^(1/2))))/(b+(-a^2+b^2)^(1/2))/a^2/(-a^2+b^2)^(3/2)/d^3+4*I*b*x*\ln(1+a*\exp(I*(c+d*x^(1/2))))/(b-(-a^2+b^2)^(1/2))/a^2/d/(-a^2+b^2)^(1/2)-4*I*b^3*polylog(3,-a*\exp(I*(c+d*x^(1/2))))/(b-(-a^2+b^2)^(1/2))/a^2/(-a^2+b^2)^(3/2)/d^3+2*I*b^3*x*\ln(1+a*\exp(I*(c+d*x^(1/2))))/(b+(-a^2+b^2)^(1/2))/a^2/(-a^2+b^2)^(3/2)/d+4*b^2*\ln(1+a*\exp(I*(c+d*x^(1/2))))/(b-I*(a^2-b^2)^(1/2))*x^(1/2)/a^2/(a^2-b^2)/d^2+4*b^2*\ln(1+a*\exp(I*(c+d*x^(1/2))))/(b+I*(a^2-b^2)^(1/2))*x^(1/2)/a^2/(a^2-b^2)/d^2-4*b^3*polylog(2,-a*\exp(I*(c+d*x^(1/2))))/(b-(-a^2+b^2)^(1/2))*x^(1/2)/a^2/(-a^2+b^2)^(3/2)/d^2+4*b^3*polylog(2,-a*\exp(I*(c+d*x^(1/2))))/(b+(-a^2+b^2)^(1/2))*x^(1/2)/a^2/(-a^2+b^2)^(3/2)/d^2+8*b*polylog(2,-a*\exp(I*(c+d*x^(1/2))))/(b-(-a^2+b^2)^(1/2))*x^(1/2)/a^2/d^2/(-a^2+b^2)^(1/2)-8*b*polylog(2,-a*\exp(I*(c+d*x^(1/2))))/(b+(-a^2+b^2)^(1/2))*x^(1/2)/a^2/d^2/(-a^2+b^2)^(1/2)$

**Rubi [A]** time = 2.11, antiderivative size = 1125, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 12, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$ , Rules used = {4204, 4191, 3324, 3321, 2264, 2190, 2531, 2282, 6589, 4522, 2279, 2391}

$$\frac{2ix \log\left(\frac{e^{i(c+d\sqrt{x})}a}{b-\sqrt{b^2-a^2}} + 1\right) b^3}{a^2 (b^2 - a^2)^{3/2} d} + \frac{2ix \log\left(\frac{e^{i(c+d\sqrt{x})}a}{b+\sqrt{b^2-a^2}} + 1\right) b^3}{a^2 (b^2 - a^2)^{3/2} d} - \frac{4\sqrt{x} \operatorname{PolyLog}\left(2, -\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right) b^3}{a^2 (b^2 - a^2)^{3/2} d^2} + \frac{4\sqrt{x} \operatorname{PolyLog}\left(2, -\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right) b^3}{a^2 (b^2 - a^2)^{3/2} d^2}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x]/(a + b\*Sec[c + d\*Sqrt[x]])^2,x]

```
[Out] ((-2*I)*b^2*x)/(a^2*(a^2 - b^2)*d) + (2*x^(3/2))/(3*a^2) + (4*b^2*Sqrt[x]*Log[1 + (a*E^(I*(c + d*Sqrt[x])))/(b - I*Sqrt[a^2 - b^2])])/(a^2*(a^2 - b^2)*d^2) + (4*b^2*Sqrt[x]*Log[1 + (a*E^(I*(c + d*Sqrt[x])))/(b + I*Sqrt[a^2 - b^2])])/(a^2*(a^2 - b^2)*d^2) - ((2*I)*b^3*x*Log[1 + (a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d) + ((4*I)*b*x*Log[1 + (a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d) + ((2*I)*b^3*x*Log[1 + (a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d) - ((4*I)*b*x*Log[1 + (a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d) - ((4*I)*b^2*PolyLog[2, -(a*E^(I*(c + d*Sqrt[x])))/(b - I*Sqrt[a^2 - b^2])])/(a^2*(a^2 - b^2)*d^3) - ((4*I)*b^2*PolyLog[2, -(a*E^(I*(c + d*Sqrt[x])))/(b + I*Sqrt[a^2 - b^2])])/(a^2*(a^2 - b^2)*d^3) - (4*b^3*Sqrt[x]*PolyLog[2, -(a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d^2) + (8*b*Sqrt[x]*PolyLog[2, -(a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d^2) + (4*b^3*Sqrt[x]*PolyLog[2, -(a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d^2) - (8*b*Sqrt[x]*PolyLog[2, -(a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d^2) - ((4*I)*b^3*PolyLog[3, -(a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d^3) + ((8*I)*b*PolyLog[3, -(a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d^3) + ((4*I)*b^3*PolyLog[3, -(a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d^3) - ((8*I)*b*PolyLog[3, -(a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d^3) + (2*b^2*x*Sin[c + d*Sqrt[x]])/(a*(a^2 - b^2)*d*(b + a*Cos[c + d*Sqrt[x]]))
```

### Rule 2190

```
Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2264

```
Int[(F_)^u*((f_) + (g_)*(x_))^(m_)]/((a_) + (b_)*(F_)^u + (c_)*(F_)^v), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[(f + g*x)^m*F^u/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[(f + g*x)^m*F^u/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

### Rule 2279

```
Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_)))^(n_))], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x],
Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_)))^(n_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m-1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n]], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

Rule 3321

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)]), x_Symbol] := Dist[2, Int[((c + d*x)^m*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3324

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2, x_Symbol] := Simp[(b*(c + d*x)^m*Cos[e + f*x])/(f*(a^2 - b^2)*(a + b*Sin[e + f*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x], x] - Dist[(b*d*m)/(f*(a^2 - b^2)), Int[((c + d*x)^(m-1)*Cos[e + f*x])/(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4191

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGtQ[m, 0]
```



$Q[m, 0]$

Rule 4204

$\text{Int}[(x_)^{(m_.)}*((a_.) + (b_.)*\text{Sec}[(c_.) + (d_.)*(x_)^{(n_.)}])^{(p_.)}, x\_Symbol]$   
 $]:> \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)*(a + b*\text{Sec}[c + d*x])^p}$   
 $, x], x, x^n], x] /; \text{FreeQ}\{a, b, c, d, m, n, p\}, x\} \&\& \text{IGtQ}[\text{Simplify}[(m + 1)/n], 0] \&\& \text{IntegerQ}[p]$

Rule 4522

$\text{Int}[(((e_.) + (f_.)*(x_))^{(m_.)}*\text{Sin}[(c_.) + (d_.)*(x_)]) / (\text{Cos}[(c_.) + (d_.)$   
 $*(x_)]*(b_.) + (a_.)), x\_Symbol] :> \text{Simp}[(I*(e + f*x)^{(m + 1)}) / (b*f*(m + 1))$   
 $, x] + (\text{Int}[(e + f*x)^m * E^{(I*(c + d*x))} / (I*a - \text{Rt}[-a^2 + b^2, 2] + I*b * E^{(I*(c + d*x))}$   
 $), x] + \text{Int}[(e + f*x)^m * E^{(I*(c + d*x))} / (I*a + \text{Rt}[-a^2 + b^2, 2] + I*b * E^{(I*(c + d*x))}$   
 $), x]) /; \text{FreeQ}\{a, b, c, d, e, f\}, x\} \&\& \text{IGtQ}[m, 0] \&\& \text{NegQ}[a^2 - b^2]$

Rule 6589

$\text{Int}[\text{PolyLog}[n_, (c_.)*((a_.) + (b_.)*(x_))^{(p_.)}] / ((d_.) + (e_.)*(x_)), x\_Symbol]$   
 $]:> \text{Simp}[\text{PolyLog}[n + 1, c*(a + b*x)^p] / (e*p), x] /; \text{FreeQ}\{a, b, c, d, e, n, p\}, x\} \&\& \text{EqQ}[b*d, a*e]$

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x}}{(a + b \sec(c + d\sqrt{x}))^2} dx &= 2 \operatorname{Subst} \left( \int \frac{x^2}{(a + b \sec(c + dx))^2} dx, x, \sqrt{x} \right) \\
&= 2 \operatorname{Subst} \left( \int \left( \frac{x^2}{a^2} + \frac{b^2 x^2}{a^2 (b + a \cos(c + dx))^2} - \frac{2bx^2}{a^2 (b + a \cos(c + dx))} \right) dx, x, \sqrt{x} \right) \\
&= \frac{2x^{3/2}}{3a^2} - \frac{(4b) \operatorname{Subst} \left( \int \frac{x^2}{b+a \cos(c+dx)} dx, x, \sqrt{x} \right)}{a^2} + \frac{(2b^2) \operatorname{Subst} \left( \int \frac{x^2}{(b+a \cos(c+dx))^2} dx, x, \sqrt{x} \right)}{a^2} \\
&= \frac{2x^{3/2}}{3a^2} + \frac{2b^2 x \sin(c + d\sqrt{x})}{a(a^2 - b^2)d(b + a \cos(c + d\sqrt{x}))} - \frac{(8b) \operatorname{Subst} \left( \int \frac{e^{i(c+dx)} x^2}{a+2be^{i(c+dx)}+ae^{2i(c+dx)}} dx, x, \sqrt{x} \right)}{a^2} \\
&= -\frac{2ib^2 x}{a^2(a^2 - b^2)d} + \frac{2x^{3/2}}{3a^2} + \frac{2b^2 x \sin(c + d\sqrt{x})}{a(a^2 - b^2)d(b + a \cos(c + d\sqrt{x}))} - \frac{(4b^3) \operatorname{Subst} \left( \int \frac{x^2}{(b+a \cos(c+dx))^2} dx, x, \sqrt{x} \right)}{a^2} \\
&= -\frac{2ib^2 x}{a^2(a^2 - b^2)d} + \frac{2x^{3/2}}{3a^2} + \frac{4b^2 \sqrt{x} \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b-i\sqrt{a^2-b^2}} \right)}{a^2(a^2 - b^2)d^2} + \frac{4b^2 \sqrt{x} \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b+i\sqrt{a^2-b^2}} \right)}{a^2(a^2 - b^2)d^2} \\
&= -\frac{2ib^2 x}{a^2(a^2 - b^2)d} + \frac{2x^{3/2}}{3a^2} + \frac{4b^2 \sqrt{x} \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b-i\sqrt{a^2-b^2}} \right)}{a^2(a^2 - b^2)d^2} + \frac{4b^2 \sqrt{x} \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b+i\sqrt{a^2-b^2}} \right)}{a^2(a^2 - b^2)d^2} \\
&= -\frac{2ib^2 x}{a^2(a^2 - b^2)d} + \frac{2x^{3/2}}{3a^2} + \frac{4b^2 \sqrt{x} \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b-i\sqrt{a^2-b^2}} \right)}{a^2(a^2 - b^2)d^2} + \frac{4b^2 \sqrt{x} \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b+i\sqrt{a^2-b^2}} \right)}{a^2(a^2 - b^2)d^2} \\
&= -\frac{2ib^2 x}{a^2(a^2 - b^2)d} + \frac{2x^{3/2}}{3a^2} + \frac{4b^2 \sqrt{x} \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b-i\sqrt{a^2-b^2}} \right)}{a^2(a^2 - b^2)d^2} + \frac{4b^2 \sqrt{x} \log \left( 1 + \frac{ae^{i(c+d\sqrt{x})}}{b+i\sqrt{a^2-b^2}} \right)}{a^2(a^2 - b^2)d^2}
\end{aligned}$$

**Mathematica [A]** time = 9.44, size = 1210, normalized size = 1.08

$$2(b + a \cos(c + d\sqrt{x})) \sec^2(c + d\sqrt{x}) \left( \frac{3x(a \sin(d\sqrt{x}) - b \sin(c))b^2}{(a-b)(a+b)d(\cos(\frac{c}{2}) - \sin(\frac{c}{2}))(\cos(\frac{c}{2}) + \sin(\frac{c}{2}))} + \frac{3(b+a \cos(c+d\sqrt{x}))}{2id^2 e^{ic} x \log\left(\frac{e^{i(2c+d)}}{be^{ic} - \sqrt{b^2}}\right)} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sqrt[x]/(a + b\*Sec[c + d\*Sqrt[x]])^2,x]

[Out] (2\*(b + a\*Cos[c + d\*Sqrt[x]])\*Sec[c + d\*Sqrt[x]]^2\*(x^(3/2)\*(b + a\*Cos[c + d\*Sqrt[x]]) + (3\*b\*(b + a\*Cos[c + d\*Sqrt[x]])\*(((2\*I)\*b\*d^2\*E^((2\*I)\*c)\*x)/(1 + E^((2\*I)\*c)) + (2\*b\*d\*Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)]\*Sqrt[x]\*Log[1 + (a\*E^(I\*(2\*c + d\*Sqrt[x])))]/(b\*E^(I\*c) - Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)]]) + (2\*I)\*a^2\*d^2\*E^(I\*c)\*x\*Log[1 + (a\*E^(I\*(2\*c + d\*Sqrt[x])))]/(b\*E^(I\*c) - Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)]]) - I\*b^2\*d^2\*E^(I\*c)\*x\*Log[1 + (a\*E^(I\*(2\*c + d\*Sqrt[x])))]/(b\*E^(I\*c) - Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)]]) + 2\*b\*d\*Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)]\*Sqrt[x]\*Log[1 + (a\*E^(I\*(2\*c + d\*Sqrt[x])))]/(b\*E^(I\*c) + Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)]]) - (2\*I)\*a^2\*d^2\*E^(I\*c)\*x\*Log[1 + (a\*E^(I\*(2\*c + d\*Sqrt[x])))]/(b\*E^(I\*c) + Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)]]) + I\*b^2\*d^2\*E^(I\*c)\*x\*Log[1 + (a\*E^(I\*(2\*c + d\*Sqrt[x])))]/(b\*E^(I\*c) + Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)]]) + 2\*((-I)\*b\*Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)] + 2\*a^2\*d\*E^(I\*c)\*Sqrt[x] - b^2\*d\*E^(I\*c)\*Sqrt[x])\*PolyLog[2, -((a\*E^(I\*(2\*c + d\*Sqrt[x])))]/(b\*E^(I\*c) - Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)]))] + 2\*((-I)\*b\*Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)] - 2\*a^2\*d\*E^(I\*c)\*Sqrt[x] + b^2\*d\*E^(I\*c)\*Sqrt[x])\*PolyLog[2, -((a\*E^(I\*(2\*c + d\*Sqrt[x])))]/(b\*E^(I\*c) + Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)]))] + (4\*I)\*a^2\*E^(I\*c)\*PolyLog[3, -((a\*E^(I\*(2\*c + d\*Sqrt[x])))]/(b\*E^(I\*c) - Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)]))] - (2\*I)\*b^2\*E^(I\*c)\*PolyLog[3, -((a\*E^(I\*(2\*c + d\*Sqrt[x])))]/(b\*E^(I\*c) - Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)]))] - (4\*I)\*a^2\*E^(I\*c)\*PolyLog[3, -((a\*E^(I\*(2\*c + d\*Sqrt[x])))]/(b\*E^(I\*c) + Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)]))] + (2\*I)\*b^2\*E^(I\*c)\*PolyLog[3, -((a\*E^(I\*(2\*c + d\*Sqrt[x])))]/(b\*E^(I\*c) + Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)]))] / Sqrt[(-a^2 + b^2)\*E^((2\*I)\*c)] / ((a^2 - b^2)\*d^3) + (3\*b^2\*x\*(-(b\*Sin[c]) + a\*Sin[d\*Sqrt[x]])) / ((a - b)\*(a + b)\*d\*(Cos[c/2] - Sin[c/2])\*(Cos[c/2] + Sin[c/2])) / (3\*a^2\*(a + b\*Sec[c + d\*Sqrt[x]])^2)

**fricas** [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x}}{b^2 \sec(d\sqrt{x} + c)^2 + 2ab \sec(d\sqrt{x} + c) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(a+b\*sec(c+d\*x^(1/2)))^2,x, algorithm="fricas")

[Out] integral(sqrt(x)/(b^2\*sec(d\*sqrt(x) + c)^2 + 2\*a\*b\*sec(d\*sqrt(x) + c) + a^2), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{(b \sec(d\sqrt{x} + c) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(a+b\*sec(c+d\*x^(1/2)))^2,x, algorithm="giac")

[Out] integrate(sqrt(x)/(b\*sec(d\*sqrt(x) + c) + a)^2, x)

**maple** [F] time = 1.16, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{(a + b \sec(c + d\sqrt{x}))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)/(a+b\*sec(c+d\*x^(1/2)))^2,x)

[Out] int(x^(1/2)/(a+b\*sec(c+d\*x^(1/2)))^2,x)

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(a+b\*sec(c+d\*x^(1/2)))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a^2-4\*b^2>0)', see `assume?` for more details)Is 4\*a^2-4\*b^2 positive or negative?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{x}}{\left(a + \frac{b}{\cos(c+d\sqrt{x})}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)/(a + b/cos(c + d\*x^(1/2)))^2, x)

[Out] int(x^(1/2)/(a + b/cos(c + d\*x^(1/2)))^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{\left(a + b \sec(c + d\sqrt{x})\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(1/2)/(a+b\*sec(c+d\*x\*\*(1/2)))\*\*2, x)

[Out] Integral(sqrt(x)/(a + b\*sec(c + d\*sqrt(x)))\*\*2, x)

$$3.68 \quad \int \frac{1}{\sqrt{x} (a+b \sec(c+d\sqrt{x}))^2} dx$$

**Optimal.** Leaf size=127

$$-\frac{4b(2a^2 - b^2) \tanh^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+d\sqrt{x})\right)}{\sqrt{a+b}}\right)}{a^2 d(a-b)^{3/2}(a+b)^{3/2}} + \frac{2b^2 \tan(c+d\sqrt{x})}{ad(a^2 - b^2)(a+b \sec(c+d\sqrt{x}))} + \frac{2\sqrt{x}}{a^2}$$

[Out]  $-4*b*(2*a^2-b^2)*\operatorname{arctanh}((a-b)^{(1/2)}*\tan(1/2*c+1/2*d*x^{(1/2)})/(a+b)^{(1/2)})/a^2/(a-b)^{(3/2)/(a+b)^{(3/2)/d+2*x^{(1/2)}/a^2+2*b^2*\tan(c+d*x^{(1/2)})/a/(a^2-b^2)/d/(a+b*\sec(c+d*x^{(1/2)}))$

**Rubi [A]** time = 0.20, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$ , Rules used = {4204, 3785, 3919, 3831, 2659, 208}

$$-\frac{4b(2a^2 - b^2) \tanh^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+d\sqrt{x})\right)}{\sqrt{a+b}}\right)}{a^2 d(a-b)^{3/2}(a+b)^{3/2}} + \frac{2b^2 \tan(c+d\sqrt{x})}{ad(a^2 - b^2)(a+b \sec(c+d\sqrt{x}))} + \frac{2\sqrt{x}}{a^2}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[x]\*(a + b\*Sec[c + d\*Sqrt[x]])^2), x]

[Out]  $(2*\sqrt{x})/a^2 - (4*b*(2*a^2 - b^2)*\operatorname{ArcTanh}[(\sqrt{a-b})*\tan[(c + d*\sqrt{x}))/2])/(\sqrt{a+b})/(a^2*(a-b)^{(3/2)}*(a+b)^{(3/2)*d} + (2*b^2*\tan[c + d*\sqrt{x}])/(a*(a^2 - b^2)*d*(a+b*\sec[c + d*\sqrt{x}])))$

**Rule 208**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

**Rule 2659**

Int[((a\_) + (b\_.)\*sin[Pi/2 + (c\_.) + (d\_.)\*(x\_)])^(-1), x\_Symbol] := With[{e = FreeFactors[Tan[(c + d\*x)/2], x]}, Dist[(2\*e)/d, Subst[Int[1/(a + b + (a - b)\*e^2\*x^2), x], x, Tan[(c + d\*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

**Rule 3785**

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_.))^(n_), x_Symbol] := Simp[(b^2*Cot
[c + d*x]*(a + b*Csc[c + d*x])^(n + 1))/(a*d*(n + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

### Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol]
:= Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

### Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_.)), x_Symbol] := Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
a*d, 0]
```

### Rule 4204

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol]
:= Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

### Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{x} (a + b \sec(c + d\sqrt{x}))^2} dx &= 2 \operatorname{Subst} \left( \int \frac{1}{(a + b \sec(c + dx))^2} dx, x, \sqrt{x} \right) \\
&= \frac{2b^2 \tan(c + d\sqrt{x})}{a(a^2 - b^2)d(a + b \sec(c + d\sqrt{x}))} - \frac{2 \operatorname{Subst} \left( \int \frac{-a^2 + b^2 + ab \sec(c + dx)}{a + b \sec(c + dx)} dx, x, \sqrt{x} \right)}{a(a^2 - b^2)} \\
&= \frac{2\sqrt{x}}{a^2} + \frac{2b^2 \tan(c + d\sqrt{x})}{a(a^2 - b^2)d(a + b \sec(c + d\sqrt{x}))} - \frac{(2b(2a^2 - b^2)) \operatorname{Subst} \left( \int \frac{\sec(c + dx)}{a + b \sec(c + dx)} dx, x, \sqrt{x} \right)}{a^2(a^2 - b^2)} \\
&= \frac{2\sqrt{x}}{a^2} + \frac{2b^2 \tan(c + d\sqrt{x})}{a(a^2 - b^2)d(a + b \sec(c + d\sqrt{x}))} - \frac{(2(2a^2 - b^2)) \operatorname{Subst} \left( \int \frac{1}{1 + \frac{a \cos(c + dx)}{b}} dx, x, \sqrt{x} \right)}{a^2(a^2 - b^2)} \\
&= \frac{2\sqrt{x}}{a^2} + \frac{2b^2 \tan(c + d\sqrt{x})}{a(a^2 - b^2)d(a + b \sec(c + d\sqrt{x}))} - \frac{(4(2a^2 - b^2)) \operatorname{Subst} \left( \int \frac{1}{1 + \frac{a}{b} + \frac{a \cos(c + dx)}{b}} dx, x, \sqrt{x} \right)}{a^2(a^2 - b^2)} \\
&= \frac{2\sqrt{x}}{a^2} - \frac{4b(2a^2 - b^2) \tanh^{-1} \left( \frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c + d\sqrt{x})\right)}{\sqrt{a+b}} \right)}{a^2(a-b)^{3/2}(a+b)^{3/2}d} + \frac{2b^2 \tan(c + d\sqrt{x})}{a(a^2 - b^2)d(a + b \sec(c + d\sqrt{x}))}
\end{aligned}$$

**Mathematica [A]** time = 0.79, size = 163, normalized size = 1.28

$$\frac{2 \left( \frac{b((a^2 - b^2)(c + d\sqrt{x}) + ab \sin(c + d\sqrt{x})) + a(a^2 - b^2)(c + d\sqrt{x}) \cos(c + d\sqrt{x})}{a \cos(c + d\sqrt{x}) + b} - \frac{2b(b^2 - 2a^2) \tanh^{-1} \left( \frac{(b-a) \tan\left(\frac{1}{2}(c + d\sqrt{x})\right)}{\sqrt{a^2 - b^2}} \right)}{\sqrt{a^2 - b^2}} \right)}{a^2 d (a - b)(a + b)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[x]\*(a + b\*Sec[c + d\*Sqrt[x]])^2), x]

[Out] (2\*((-2\*b\*(-2\*a^2 + b^2)\*ArcTanh[(-a + b)\*Tan[(c + d\*Sqrt[x])/2]]/Sqrt[a^2 - b^2]))/Sqrt[a^2 - b^2] + (a\*(a^2 - b^2)\*(c + d\*Sqrt[x])\*Cos[c + d\*Sqrt[x]] + b\*((a^2 - b^2)\*(c + d\*Sqrt[x]) + a\*b\*Sin[c + d\*Sqrt[x]]))/(b + a\*Cos[c + d\*Sqrt[x]]))/(a^2\*(a - b)\*(a + b)\*d)



**fricas [B]** time = 0.58, size = 574, normalized size = 4.52

$$\frac{2(a^5 - 2a^3b^2 + ab^4)d\sqrt{x} \cos(d\sqrt{x} + c) + 2(a^4b - 2a^2b^3 + b^5)d\sqrt{x} + ((2a^3b - ab^3)\sqrt{a^2 - b^2} \cos(d\sqrt{x} + c)}{(a^7 - 2a^5b^2 + a^3b^4)d\cos(d\sqrt{x} + c) + (a^6b - 2a^4b^3 + a^2b^5)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*sec(c+d\*x^(1/2)))^2/x^(1/2),x, algorithm="fricas")

[Out] [(2\*(a^5 - 2\*a^3\*b^2 + a\*b^4)\*d\*sqrt(x)\*cos(d\*sqrt(x) + c) + 2\*(a^4\*b - 2\*a^2\*b^3 + b^5)\*d\*sqrt(x) + ((2\*a^3\*b - a\*b^3)\*sqrt(a^2 - b^2)\*cos(d\*sqrt(x) + c) + (2\*a^2\*b^2 - b^4)\*sqrt(a^2 - b^2))\*log((2\*a\*b\*cos(d\*sqrt(x) + c) - (a^2 - 2\*b^2)\*cos(d\*sqrt(x) + c)^2 + 2\*a^2 - b^2 - 2\*(sqrt(a^2 - b^2)\*b\*cos(d\*sqrt(x) + c) + sqrt(a^2 - b^2)\*a)\*sin(d\*sqrt(x) + c))/(a^2\*cos(d\*sqrt(x) + c)^2 + 2\*a\*b\*cos(d\*sqrt(x) + c) + b^2)) + 2\*(a^3\*b^2 - a\*b^4)\*sin(d\*sqrt(x) + c))/((a^7 - 2\*a^5\*b^2 + a^3\*b^4)\*d\*cos(d\*sqrt(x) + c) + (a^6\*b - 2\*a^4\*b^3 + a^2\*b^5)\*d), 2\*((a^5 - 2\*a^3\*b^2 + a\*b^4)\*d\*sqrt(x)\*cos(d\*sqrt(x) + c) + (a^4\*b - 2\*a^2\*b^3 + b^5)\*d\*sqrt(x) - ((2\*a^3\*b - a\*b^3)\*sqrt(-a^2 + b^2)\*cos(d\*sqrt(x) + c) + (2\*a^2\*b^2 - b^4)\*sqrt(-a^2 + b^2))\*arctan(-(sqrt(-a^2 + b^2)\*b\*cos(d\*sqrt(x) + c) + sqrt(-a^2 + b^2)\*a)/((a^2 - b^2)\*sin(d\*sqrt(x) + c))) + (a^3\*b^2 - a\*b^4)\*sin(d\*sqrt(x) + c))/((a^7 - 2\*a^5\*b^2 + a^3\*b^4)\*d\*cos(d\*sqrt(x) + c) + (a^6\*b - 2\*a^4\*b^3 + a^2\*b^5)\*d)]

**giac [A]** time = 0.74, size = 196, normalized size = 1.54

$$\frac{4b^2 \tan\left(\frac{1}{2}d\sqrt{x} + \frac{1}{2}c\right)}{(a^3d - ab^2d)\left(a \tan\left(\frac{1}{2}d\sqrt{x} + \frac{1}{2}c\right)^2 - b \tan\left(\frac{1}{2}d\sqrt{x} + \frac{1}{2}c\right)^2 - a - b\right)} + \frac{4(2a^2b - b^3)\left(\pi \left[\frac{d\sqrt{x}+c}{2\pi} + \frac{1}{2}\right] \operatorname{sgn}(2a - 2b)\right)}{(a^4d - a^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*sec(c+d\*x^(1/2)))^2/x^(1/2),x, algorithm="giac")

[Out] -4\*b^2\*tan(1/2\*d\*sqrt(x) + 1/2\*c)/((a^3\*d - a\*b^2\*d)\*(a\*tan(1/2\*d\*sqrt(x) + 1/2\*c)^2 - b\*tan(1/2\*d\*sqrt(x) + 1/2\*c)^2 - a - b)) + 4\*(2\*a^2\*b - b^3)\*(pi\*floor(1/2\*(d\*sqrt(x) + c)/pi + 1/2)\*sgn(2\*a - 2\*b) + arctan((a\*tan(1/2\*d\*sqrt(x) + 1/2\*c) - b\*tan(1/2\*d\*sqrt(x) + 1/2\*c))/sqrt(-a^2 + b^2)))/((a^4\*d - a^2\*b^2\*d)\*sqrt(-a^2 + b^2)) + 2\*(d\*sqrt(x) + c)/(a^2\*d)

**maple [A]** time = 0.85, size = 216, normalized size = 1.70

$$\frac{4b^2 \tan\left(\frac{c}{2} + \frac{d\sqrt{x}}{2}\right) - 8b \operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{c}{2} + \frac{d\sqrt{x}}{2}\right)}{\sqrt{(a-b)(a+b)}}\right) + 4b^3 \operatorname{arctan}\left(\frac{(a-b)\tan\left(\frac{c}{2} + \frac{d\sqrt{x}}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{da(a^2 - b^2)\left(a\left(\tan^2\left(\frac{c}{2} + \frac{d\sqrt{x}}{2}\right)\right) - \left(\tan^2\left(\frac{c}{2} + \frac{d\sqrt{x}}{2}\right)\right)b - a - b\right) - d(a-b)(a+b)\sqrt{(a-b)(a+b)} + da^2(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a+b*sec(c+d*x^(1/2)))^2/x^(1/2),x)`

[Out] `-4/d*b^2/a/(a^2-b^2)*tan(1/2*c+1/2*d*x^(1/2))/(a*tan(1/2*c+1/2*d*x^(1/2))^2 - tan(1/2*c+1/2*d*x^(1/2))^2*b-a-b)-8/d*b/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*c+1/2*d*x^(1/2))/((a-b)*(a+b))^(1/2))+4/d*b^3/a^2/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*c+1/2*d*x^(1/2))/((a-b)*(a+b))^(1/2))+4/d/a^2*arctan(tan(1/2*c+1/2*d*x^(1/2)))`

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*sec(c+d*x^(1/2)))^2/x^(1/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a^2-4\*b^2>0)', see 'assume?' for more details)Is 4\*a^2-4\*b^2 positive or negative?

**mupad [B]** time = 5.25, size = 330, normalized size = 2.60

$$\frac{\frac{b^2 4i}{ad(a^2-b^2)} + \frac{b^3 e^{c 1i+d \sqrt{x} 1i} 4i}{a^2 d(a^2-b^2)}}{a + a e^{c 2i+d \sqrt{x} 2i} + 2 b e^{c 1i+d \sqrt{x} 1i}} + \frac{2 \sqrt{x}}{a^2} + \frac{\ln\left(e^{c 1i+d \sqrt{x} 1i} \left(4 a^2 b - 2 b^3\right) - \frac{(4 a^2 b - 2 b^3)(a^2 - b^2)(a + b e^{c 1i+d \sqrt{x} 1i}) 1i}{(a+b)^{3/2}(a-b)^{3/2}}\right)}{a^2 d(a+b)^{3/2}(a-b)^{3/2}} (4 a^2 b - 2 b^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^(1/2)*(a + b/cos(c + d*x^(1/2))))^2,x)`

[Out] `((b^2*4i)/(a*d*(a^2 - b^2)) + (b^3*exp(c*1i + d*x^(1/2)*1i)*4i)/(a^2*d*(a^2 - b^2)))/(a + a*exp(c*2i + d*x^(1/2)*2i) + 2*b*exp(c*1i + d*x^(1/2)*1i)) + (2*x^(1/2))/a^2 + (log(exp(c*1i + d*x^(1/2)*1i)*(4*a^2*b - 2*b^3) - ((4*a^2*b - 2*b^3)*(a^2 - b^2)*(a + b*exp(c*1i + d*x^(1/2)*1i))*1i)/((a + b)^(3/2)*(a - b)^(3/2)))*(4*a^2*b - 2*b^3))/(a^2*d*(a + b)^(3/2)*(a - b)^(3/2)) -`

```
(2*b*log(exp(c*1i + d*x^(1/2)*1i))*(4*a^2*b - 2*b^3) + (b*(a^2 - b^2)*(2*a^2
- b^2)*(a + b*exp(c*1i + d*x^(1/2)*1i))*2i)/((a + b)^(3/2)*(a - b)^(3/2)))
*(2*a^2 - b^2))/(a^2*d*(a + b)^(3/2)*(a - b)^(3/2))
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x} (a + b \sec(c + d\sqrt{x}))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(c+d*x**(1/2)))*2/x**(1/2), x)
```

```
[Out] Integral(1/(sqrt(x)*(a + b*sec(c + d*sqrt(x)))*2), x)
```

$$3.69 \quad \int \frac{1}{x^{3/2} (a + b \sec(c + d\sqrt{x}))^2} dx$$

Optimal. Leaf size=25

$$\text{Int} \left( \frac{1}{x^{3/2} (a + b \sec(c + d\sqrt{x}))^2}, x \right)$$

[Out] Unintegrable(1/x^(3/2)/(a+b\*sec(c+d\*x^(1/2)))^2,x)

**Rubi [A]** time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{x^{3/2} (a + b \sec(c + d\sqrt{x}))^2} dx$$

Verification is Not applicable to the result.

[In] Int[1/(x^(3/2)\*(a + b\*Sec[c + d\*Sqrt[x]]))^2],x]

[Out] Defer[Int][1/(x^(3/2)\*(a + b\*Sec[c + d\*Sqrt[x]]))^2], x]

Rubi steps

$$\int \frac{1}{x^{3/2} (a + b \sec(c + d\sqrt{x}))^2} dx = \int \frac{1}{x^{3/2} (a + b \sec(c + d\sqrt{x}))^2} dx$$

**Mathematica [A]** time = 29.08, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3/2} (a + b \sec(c + d\sqrt{x}))^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x^(3/2)\*(a + b\*Sec[c + d\*Sqrt[x]]))^2],x]

[Out] Integrate[1/(x^(3/2)\*(a + b\*Sec[c + d\*Sqrt[x]]))^2], x]

**fricas [A]** time = 0.51, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{\sqrt{x}}{b^2 x^2 \sec(d\sqrt{x} + c)^2 + 2 ab x^2 \sec(d\sqrt{x} + c) + a^2 x^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(3/2)/(a+b\*sec(c+d\*x^(1/2)))^2,x, algorithm="fricas")

[Out] integral(sqrt(x)/(b^2\*x^2\*sec(d\*sqrt(x) + c)^2 + 2\*a\*b\*x^2\*sec(d\*sqrt(x) + c) + a^2\*x^2), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \sec(d\sqrt{x} + c) + a)^2 x^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(3/2)/(a+b\*sec(c+d\*x^(1/2)))^2,x, algorithm="giac")

[Out] integrate(1/((b\*sec(d\*sqrt(x) + c) + a)^2\*x^(3/2)), x)

**maple** [A] time = 1.31, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{3}{2}} (a + b \sec(c + d\sqrt{x}))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^(3/2)/(a+b\*sec(c+d\*x^(1/2)))^2,x)

[Out] int(1/x^(3/2)/(a+b\*sec(c+d\*x^(1/2)))^2,x)

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(3/2)/(a+b\*sec(c+d\*x^(1/2)))^2,x, algorithm="maxima")

[Out] Timed out

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x^{3/2} \left( a + \frac{b}{\cos(c+d\sqrt{x})} \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^(3/2)*(a + b/cos(c + d*x^(1/2))))^2), x)`

[Out] `int(1/(x^(3/2)*(a + b/cos(c + d*x^(1/2))))^2), x)`

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{3}{2}} (a + b \sec(c + d\sqrt{x}))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**(3/2)/(a+b*sec(c+d*x**(1/2)))**2,x)`

[Out] `Integral(1/(x**(3/2)*(a + b*sec(c + d*sqrt(x))))**2), x)`

$$3.70 \quad \int \frac{1}{x^{5/2}(a+b \sec(c+d\sqrt{x}))^2} dx$$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{1}{x^{5/2}(a+b \sec(c+d\sqrt{x}))^2}, x\right)$$

[Out] Unintegrable(1/x^(5/2)/(a+b\*sec(c+d\*x^(1/2)))^2, x)

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{x^{5/2}(a+b \sec(c+d\sqrt{x}))^2} dx$$

Verification is Not applicable to the result.

[In] Int[1/(x^(5/2)\*(a + b\*Sec[c + d\*Sqrt[x]]))^2], x]

[Out] Defer[Int][1/(x^(5/2)\*(a + b\*Sec[c + d\*Sqrt[x]]))^2], x]

Rubi steps

$$\int \frac{1}{x^{5/2}(a+b \sec(c+d\sqrt{x}))^2} dx = \int \frac{1}{x^{5/2}(a+b \sec(c+d\sqrt{x}))^2} dx$$

Mathematica [A] time = 29.21, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{5/2}(a+b \sec(c+d\sqrt{x}))^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x^(5/2)\*(a + b\*Sec[c + d\*Sqrt[x]]))^2], x]

[Out] Integrate[1/(x^(5/2)\*(a + b\*Sec[c + d\*Sqrt[x]]))^2], x]

fricas [A] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x}}{b^2x^3 \sec(d\sqrt{x} + c)^2 + 2abx^3 \sec(d\sqrt{x} + c) + a^2x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(5/2)/(a+b\*sec(c+d\*x^(1/2)))^2,x, algorithm="fricas")

[Out] integral(sqrt(x)/(b^2\*x^3\*sec(d\*sqrt(x) + c)^2 + 2\*a\*b\*x^3\*sec(d\*sqrt(x) + c) + a^2\*x^3), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \sec(d\sqrt{x} + c) + a)^2 x^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(5/2)/(a+b\*sec(c+d\*x^(1/2)))^2,x, algorithm="giac")

[Out] integrate(1/((b\*sec(d\*sqrt(x) + c) + a)^2\*x^(5/2)), x)

maple [A] time = 1.24, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{5}{2}} (a + b \sec(c + d\sqrt{x}))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^(5/2)/(a+b\*sec(c+d\*x^(1/2)))^2,x)

[Out] int(1/x^(5/2)/(a+b\*sec(c+d\*x^(1/2)))^2,x)

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(5/2)/(a+b\*sec(c+d\*x^(1/2)))^2,x, algorithm="maxima")

[Out] Timed out

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x^{5/2} \left( a + \frac{b}{\cos(c+d\sqrt{x})} \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.



[In] `int(1/(x^(5/2)*(a + b/cos(c + d*x^(1/2))))^2), x)`

[Out] `int(1/(x^(5/2)*(a + b/cos(c + d*x^(1/2))))^2), x)`

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{5}{2}} (a + b \sec(c + d\sqrt{x}))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**(5/2)/(a+b*sec(c+d*x**(1/2)))**2, x)`

[Out] `Integral(1/(x**(5/2)*(a + b*sec(c + d*sqrt(x)))**2), x)`

### 3.71 $\int (ex)^m (a + b \sec(c + dx^n))^p dx$

Optimal. Leaf size=32

$$x^{-m}(ex)^m \text{Int}(x^m (a + b \sec(c + dx^n))^p, x)$$

[Out]  $(e*x)^m \text{Unintegrable}(x^m*(a+b*\sec(c+d*x^n))^p, x)/(x^m)$

**Rubi [A]** time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int (ex)^m (a + b \sec(c + dx^n))^p dx$$

Verification is Not applicable to the result.

[In]  $\text{Int}[(e*x)^m*(a + b*\text{Sec}[c + d*x^n])^p, x]$

[Out]  $((e*x)^m*\text{Defer}[\text{Int}][x^m*(a + b*\text{Sec}[c + d*x^n])^p, x])/x^m$

Rubi steps

$$\int (ex)^m (a + b \sec(c + dx^n))^p dx = (x^{-m}(ex)^m) \int x^m (a + b \sec(c + dx^n))^p dx$$

**Mathematica [A]** time = 3.09, size = 0, normalized size = 0.00

$$\int (ex)^m (a + b \sec(c + dx^n))^p dx$$

Verification is Not applicable to the result.

[In]  $\text{Integrate}[(e*x)^m*(a + b*\text{Sec}[c + d*x^n])^p, x]$

[Out]  $\text{Integrate}[(e*x)^m*(a + b*\text{Sec}[c + d*x^n])^p, x]$

**fricas [A]** time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}((ex)^m (b \sec(dx^n + c) + a)^p, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((e*x)^m*(a+b*\sec(c+d*x^n))^p, x, \text{algorithm}="fricas")$

[Out]  $\text{integral}((e*x)^m*(b*\sec(d*x^n + c) + a)^p, x)$

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m (b \sec(dx^n + c) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^m\*(a+b\*sec(c+d\*x^n))^p,x, algorithm="giac")

[Out] integrate((e\*x)^m\*(b\*sec(d\*x^n + c) + a)^p, x)

**maple** [A] time = 1.51, size = 0, normalized size = 0.00

$$\int (ex)^m (a + b \sec(c + dx^n))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x)^m\*(a+b\*sec(c+d\*x^n))^p,x)

[Out] int((e\*x)^m\*(a+b\*sec(c+d\*x^n))^p,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m (b \sec(dx^n + c) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^m\*(a+b\*sec(c+d\*x^n))^p,x, algorithm="maxima")

[Out] integrate((e\*x)^m\*(b\*sec(d\*x^n + c) + a)^p, x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \left( a + \frac{b}{\cos(c + dx^n)} \right)^p (ex)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cos(c + d\*x^n))^p\*(e\*x)^m,x)

[Out] int((a + b/cos(c + d\*x^n))^p\*(e\*x)^m, x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m (a + b \sec(c + dx^n))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)\*\*m\*(a+b\*sec(c+d\*x\*\*n))\*\*p,x)

[Out] Integral((e\*x)\*\*m\*(a + b\*sec(c + d\*x\*\*n))\*\*p, x)

### 3.72 $\int (ex)^{-1+n} (a + b \sec(c + dx^n)) dx$

Optimal. Leaf size=44

$$\frac{a(ex)^n}{en} + \frac{bx^{-n}(ex)^n \tanh^{-1}(\sin(c + dx^n))}{den}$$

[Out] a\*(e\*x)^n/e/n+b\*(e\*x)^n\*arctanh(sin(c+d\*x^n))/d/e/n/(x^n)

**Rubi [A]** time = 0.05, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {14, 4208, 4204, 3770}

$$\frac{a(ex)^n}{en} + \frac{bx^{-n}(ex)^n \tanh^{-1}(\sin(c + dx^n))}{den}$$

Antiderivative was successfully verified.

[In] Int[(e\*x)^(-1 + n)\*(a + b\*Sec[c + d\*x^n]),x]

[Out] (a\*(e\*x)^n)/(e\*n) + (b\*(e\*x)^n\*ArcTanh[Sin[c + d\*x^n]])/(d\*e\*n\*x^n)

#### Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

#### Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]
```

#### Rule 4204

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]
```

#### Rule 4208

```
Int[((e_)*(x_))^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Dist[(e^IntPart[m]*(e*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a + b*Sec[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\int (ex)^{-1+n} (a + b \sec(c + dx^n)) dx &= \int (a(ex)^{-1+n} + b(ex)^{-1+n} \sec(c + dx^n)) dx \\
&= \frac{a(ex)^n}{en} + b \int (ex)^{-1+n} \sec(c + dx^n) dx \\
&= \frac{a(ex)^n}{en} + \frac{(bx^{-n}(ex)^n) \int x^{-1+n} \sec(c + dx^n) dx}{e} \\
&= \frac{a(ex)^n}{en} + \frac{(bx^{-n}(ex)^n) \text{Subst}\left(\int \sec(c + dx) dx, x, x^n\right)}{en} \\
&= \frac{a(ex)^n}{en} + \frac{bx^{-n}(ex)^n \tanh^{-1}(\sin(c + dx^n))}{den}
\end{aligned}$$

**Mathematica [A]** time = 0.12, size = 38, normalized size = 0.86

$$\frac{x^{-n}(ex)^n (adx^n + b \tanh^{-1}(\sin(c + dx^n)))}{den}$$

Antiderivative was successfully verified.

[In] Integrate[(e\*x)^(-1 + n)\*(a + b\*Sec[c + d\*x^n]),x]

[Out] ((e\*x)^n\*(a\*d\*x^n + b\*ArcTanh[Sin[c + d\*x^n]]))/(d\*e\*n\*x^n)

**fricas [A]** time = 0.75, size = 60, normalized size = 1.36

$$\frac{2ade^{n-1}x^n + be^{n-1} \log(\sin(dx^n + c) + 1) - be^{n-1} \log(-\sin(dx^n + c) + 1)}{2dn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^(-1+n)\*(a+b\*sec(c+d\*x^n)),x, algorithm="fricas")

[Out] 1/2\*(2\*a\*d\*e^(n - 1)\*x^n + b\*e^(n - 1)\*log(sin(d\*x^n + c) + 1) - b\*e^(n - 1)\*log(-sin(d\*x^n + c) + 1))/(d\*n)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sec(dx^n + c) + a) (ex)^{n-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^(-1+n)\*(a+b\*sec(c+d\*x^n)),x, algorithm="giac")

[Out] integrate((b\*sec(d\*x^n + c) + a)\*(e\*x)^(n - 1), x)

**maple [C]** time = 1.58, size = 159, normalized size = 3.61

$$\frac{ax e^{\frac{(-1+n)\left(-i\pi \operatorname{csgn}(ie)\operatorname{csgn}(ix)\operatorname{csgn}(iex)+i\pi \operatorname{csgn}(ie)\operatorname{csgn}(iex)^2+i\pi \operatorname{csgn}(ix)\operatorname{csgn}(iex)^2-i\pi \operatorname{csgn}(iex)^3+2\ln(x)+2\ln(e)\right)}{2}}}{n} - \frac{2i \arctan\left(e^{i(c+dx^n)}\right) e^n b e^{\frac{i\pi \operatorname{csgn}(iex)(-1+n)}{2}}}{den}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x)^(-1+n)\*(a+b\*sec(c+d\*x^n)), x)

[Out] a/n\*x\*exp(1/2\*(-1+n)\*(-I\*Pi\*csgn(I\*e)\*csgn(I\*x)\*csgn(I\*e\*x)+I\*Pi\*csgn(I\*e)\*csgn(I\*e\*x)^2+I\*Pi\*csgn(I\*x)\*csgn(I\*e\*x)^2-I\*Pi\*csgn(I\*e\*x)^3+2\*ln(x)+2\*ln(e)))-2\*I\*arctan(exp(I\*(c+d\*x^n)))/d/e\*e^n/n\*b\*exp(1/2\*I\*Pi\*csgn(I\*e\*x)\*(-1+n)\*(csgn(I\*e\*x)-csgn(I\*x))\*(-csgn(I\*e\*x)+csgn(I\*e)))

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$2be^n \int \frac{x^n \cos(2dx^n + 2c) \cos(dx^n + c) + x^n \sin(2dx^n + 2c) \sin(dx^n + c) + x^n \cos(dx^n + c)}{ex \cos(2dx^n + 2c)^2 + ex \sin(2dx^n + 2c)^2 + 2ex \cos(2dx^n + 2c) + ex} dx + \frac{(ex)^n a}{en}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^(-1+n)\*(a+b\*sec(c+d\*x^n)), x, algorithm="maxima")

[Out] 2\*b\*e^n\*integrate((x^n\*cos(2\*d\*x^n + 2\*c)\*cos(d\*x^n + c) + x^n\*sin(2\*d\*x^n + 2\*c)\*sin(d\*x^n + c) + x^n\*cos(d\*x^n + c))/(e\*x\*cos(2\*d\*x^n + 2\*c)^2 + e\*x\*sin(2\*d\*x^n + 2\*c)^2 + 2\*e\*x\*cos(2\*d\*x^n + 2\*c) + e\*x), x) + (e\*x)^n\*a/(e\*n)

**mupad [B]** time = 2.66, size = 104, normalized size = 2.36

$$\frac{(ex)^n \left( b \ln\left(-b(ex)^{n-1} 2i - 2be^{c1i} e^{dx^n 1i} (ex)^{n-1}\right) - b \ln\left(b(ex)^{n-1} 2i - 2be^{c1i} e^{dx^n 1i} (ex)^{n-1}\right) + a dx^n \right)}{den x^n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cos(c + d\*x^n))\*(e\*x)^(n - 1), x)

[Out] ((e\*x)^n\*(b\*log(-b\*(e\*x)^(n-1)\*2i - 2\*b\*exp(c\*1i)\*exp(d\*x^n\*1i)\*(e\*x)^(n-1)) - b\*log(b\*(e\*x)^(n-1)\*2i - 2\*b\*exp(c\*1i)\*exp(d\*x^n\*1i)\*(e\*x)^(n-1)) + a\*d\*x^n)/(d\*e\*n\*x^n)

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^{n-1} (a + b \sec(c + dx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(-1+n)*(a+b*sec(c+d*x**n)),x)
```

```
[Out] Integral((e*x)**(n - 1)*(a + b*sec(c + d*x**n)), x)
```

### 3.73 $\int (ex)^{-1+2n} (a + b \sec(c + dx^n)) dx$

**Optimal.** Leaf size=149

$$\frac{a(ex)^{2n}}{2en} + \frac{ibx^{-2n}(ex)^{2n}\text{Li}_2(-ie^{i(dx^n+c)})}{d^2en} - \frac{ibx^{-2n}(ex)^{2n}\text{Li}_2(ie^{i(dx^n+c)})}{d^2en} - \frac{2ibx^{-n}(ex)^{2n}\tan^{-1}(e^{i(c+dx^n)})}{den}$$

[Out]  $1/2*a*(e*x)^{(2*n)}/e/n-2*I*b*(e*x)^{(2*n)}*\arctan(\exp(I*(c+d*x^n)))/d/e/n/(x^n)+I*b*(e*x)^{(2*n)}*\text{polylog}(2,-I*\exp(I*(c+d*x^n)))/d^2/e/n/(x^{(2*n)})-I*b*(e*x)^{(2*n)}*\text{polylog}(2,I*\exp(I*(c+d*x^n)))/d^2/e/n/(x^{(2*n)})$

**Rubi [A]** time = 0.11, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$ , Rules used = {14, 4208, 4204, 4181, 2279, 2391}

$$\frac{ibx^{-2n}(ex)^{2n}\text{PolyLog}(2, -ie^{i(c+dx^n)})}{d^2en} - \frac{ibx^{-2n}(ex)^{2n}\text{PolyLog}(2, ie^{i(c+dx^n)})}{d^2en} + \frac{a(ex)^{2n}}{2en} - \frac{2ibx^{-n}(ex)^{2n}\tan^{-1}(e^{i(c+dx^n)})}{den}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(e*x)^{-1+2*n}*(a + b*\text{Sec}[c + d*x^n]), x]$

[Out]  $(a*(e*x)^{(2*n)})/(2*e*n) - ((2*I)*b*(e*x)^{(2*n)}*\text{ArcTan}[E^{(I*(c + d*x^n))}])/(d*e*n*x^n) + (I*b*(e*x)^{(2*n)}*\text{PolyLog}[2, (-I)*E^{(I*(c + d*x^n))}])/(d^2*e*n*x^{(2*n)}) - (I*b*(e*x)^{(2*n)}*\text{PolyLog}[2, I*E^{(I*(c + d*x^n))}])/(d^2*e*n*x^{(2*n)})$

#### Rule 14

$\text{Int}[(u_)*((c_)*(x_))^{(m_)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(c*x)^m*u, x], x] /; \text{FreeQ}[\{c, m\}, x] \ \&\& \ \text{SumQ}[u] \ \&\& \ !\text{LinearQ}[u, x] \ \&\& \ !\text{MatchQ}[u, (a_ + (b_)*(v_)) /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{InverseFunctionQ}[v]]$

#### Rule 2279

$\text{Int}[\text{Log}[(a_ + (b_)*((F_)^{((e_)*((c_)) + (d_)*(x_)))^{(n_)}], x\_Symbol] \rightarrow \text{Dist}[1/(d*e*n*\text{Log}[F]), \text{Subst}[\text{Int}[\text{Log}[a + b*x]/x, x], x, (F^{(e*(c + d*x))})^n], x] /; \text{FreeQ}[\{F, a, b, c, d, e, n\}, x] \ \&\& \ \text{GtQ}[a, 0]$

#### Rule 2391

$\text{Int}[\text{Log}[(c_)*((d_)) + (e_)*(x_)^{(n_)}]/(x_), x\_Symbol] \rightarrow -\text{Simp}[\text{PolyLog}[2, -(c*e*x^n)]/n, x] /; \text{FreeQ}[\{c, d, e, n\}, x] \ \&\& \ \text{EqQ}[c*d, 1]$

#### Rule 4181



```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Dist
[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))],
x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

#### Rule 4204

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol]
:> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p,
x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

#### Rule 4208

```
Int[((e_)*(x_))^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x
_Symbol] :> Dist[(e^IntPart[m]*(e*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a
+ b*Sec[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

#### Rubi steps

$$\begin{aligned}
\int (ex)^{-1+2n} (a + b \sec(c + dx^n)) dx &= \int (a(ex)^{-1+2n} + b(ex)^{-1+2n} \sec(c + dx^n)) dx \\
&= \frac{a(ex)^{2n}}{2en} + b \int (ex)^{-1+2n} \sec(c + dx^n) dx \\
&= \frac{a(ex)^{2n}}{2en} + \frac{(bx^{-2n}(ex)^{2n}) \int x^{-1+2n} \sec(c + dx^n) dx}{e} \\
&= \frac{a(ex)^{2n}}{2en} + \frac{(bx^{-2n}(ex)^{2n}) \text{Subst}\left(\int x \sec(c + dx) dx, x, x^n\right)}{en} \\
&= \frac{a(ex)^{2n}}{2en} - \frac{2ibx^{-n}(ex)^{2n} \tan^{-1}\left(e^{i(c+dx^n)}\right)}{den} - \frac{(bx^{-2n}(ex)^{2n}) \text{Subst}\left(\int \log(1 - ix) dx, x, x^n\right)}{den} \\
&= \frac{a(ex)^{2n}}{2en} - \frac{2ibx^{-n}(ex)^{2n} \tan^{-1}\left(e^{i(c+dx^n)}\right)}{den} + \frac{(ibx^{-2n}(ex)^{2n}) \text{Subst}\left(\int \frac{\log(1-ix)}{x} dx, x, x^n\right)}{d^2en} \\
&= \frac{a(ex)^{2n}}{2en} - \frac{2ibx^{-n}(ex)^{2n} \tan^{-1}\left(e^{i(c+dx^n)}\right)}{den} + \frac{ibx^{-2n}(ex)^{2n} \text{Li}_2\left(-ie^{i(c+dx^n)}\right)}{d^2en} - \dots
\end{aligned}$$

**Mathematica [A]** time = 0.65, size = 188, normalized size = 1.26

$$\frac{(ex)^{2n} \cos(c + dx^n) (a + b \sec(c + dx^n)) \left( a + \frac{bx^{-2n} \left( 2i \left( \operatorname{Li}_2 \left( -ie^{-i(dx^n+c)} \right) - \operatorname{Li}_2 \left( ie^{-i(dx^n+c)} \right) \right) + (-2c - 2dx^n + \pi) \left( \log \left( 1 - ie^{-i(c+dx^n)} \right) - \log \left( 1 + ie^{-i(c+dx^n)} \right) \right) \right)}{d^2}}{2en (a \cos(c + dx^n) + b)}$$

Antiderivative was successfully verified.

[In] Integrate[(e\*x)^(-1 + 2\*n)\*(a + b\*Sec[c + d\*x^n]), x]

[Out] ((e\*x)^(2\*n)\*Cos[c + d\*x^n]\*(a + (b\*((-2\*c + Pi - 2\*d\*x^n)\*(Log[1 - I/E^(I\*(c + d\*x^n))] - Log[1 + I/E^(I\*(c + d\*x^n))]) - (-2\*c + Pi)\*Log[Cot[(2\*c + Pi + 2\*d\*x^n)/4]] + (2\*I)\*(PolyLog[2, (-I)/E^(I\*(c + d\*x^n))] - PolyLog[2, I/E^(I\*(c + d\*x^n))])))/(d^2\*x^(2\*n))\*(a + b\*Sec[c + d\*x^n]))/(2\*e\*n\*(b + a\*Cos[c + d\*x^n]))

**fricas [B]** time = 1.48, size = 470, normalized size = 3.15

$$\frac{ad^2e^{2n-1}x^{2n} - bce^{2n-1} \log(\cos(dx^n + c) + i \sin(dx^n + c) + i) + bce^{2n-1} \log(\cos(dx^n + c) - i \sin(dx^n + c) + i)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^(-1+2\*n)\*(a+b\*sec(c+d\*x^n)), x, algorithm="fricas")

[Out] 1/2\*(a\*d^2\*e^(2\*n - 1)\*x^(2\*n) - b\*c\*e^(2\*n - 1)\*log(cos(d\*x^n + c) + I\*sin(d\*x^n + c) + I) + b\*c\*e^(2\*n - 1)\*log(cos(d\*x^n + c) - I\*sin(d\*x^n + c) + I) - b\*c\*e^(2\*n - 1)\*log(-cos(d\*x^n + c) + I\*sin(d\*x^n + c) + I) + b\*c\*e^(2\*n - 1)\*log(-cos(d\*x^n + c) - I\*sin(d\*x^n + c) + I) - I\*b\*e^(2\*n - 1)\*dilog(I\*cos(d\*x^n + c) + sin(d\*x^n + c)) - I\*b\*e^(2\*n - 1)\*dilog(I\*cos(d\*x^n + c) - sin(d\*x^n + c)) + I\*b\*e^(2\*n - 1)\*dilog(-I\*cos(d\*x^n + c) + sin(d\*x^n + c)) + I\*b\*e^(2\*n - 1)\*dilog(-I\*cos(d\*x^n + c) - sin(d\*x^n + c)) + (b\*d\*e^(2\*n - 1)\*x^n + b\*c\*e^(2\*n - 1))\*log(I\*cos(d\*x^n + c) + sin(d\*x^n + c) + 1) - (b\*d\*e^(2\*n - 1)\*x^n + b\*c\*e^(2\*n - 1))\*log(I\*cos(d\*x^n + c) - sin(d\*x^n + c) + 1) + (b\*d\*e^(2\*n - 1)\*x^n + b\*c\*e^(2\*n - 1))\*log(-I\*cos(d\*x^n + c) + sin(d\*x^n + c) + 1) - (b\*d\*e^(2\*n - 1)\*x^n + b\*c\*e^(2\*n - 1))\*log(-I\*cos(d\*x^n + c) - sin(d\*x^n + c) + 1))/(d^2\*n)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sec(dx^n + c) + a) (ex)^{2n-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^(-1+2\*n)\*(a+b\*sec(c+d\*x^n)), x, algorithm="giac")

[Out] integrate((b\*sec(d\*x^n + c) + a)\*(e\*x)^(2\*n - 1), x)

**maple** [C] time = 1.87, size = 873, normalized size = 5.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x)^(-1+2\*n)\*(a+b\*sec(c+d\*x^n)), x)

[Out]  $\frac{1}{2} \frac{a}{n} x \exp\left(\frac{1}{2}(-1+2n)(-i\pi \operatorname{csgn}(Ie) \operatorname{csgn}(Ix) \operatorname{csgn}(Ie*x) + i\pi \operatorname{csgn}(Ie) \operatorname{csgn}(Ie*x)^2 + i\pi \operatorname{csgn}(Ix) \operatorname{csgn}(Ie*x)^2 - i\pi \operatorname{csgn}(Ie*x)^3 + 2 \ln(x) + 2 \ln(e))\right) + \frac{I}{d} \frac{1}{n} e^{(e^n)^{2b} \ln(1 + \exp(Ix^n d))} (-\exp(2Ic))^{1/2} x^n (-\exp(2Ic))^{1/2} (-1)^{1/2 \operatorname{csgn}(Ie) \operatorname{csgn}(Ix) \operatorname{csgn}(Ie*x)} (-1)^{-1/2 \operatorname{csgn}(Ix) \operatorname{csgn}(Ie*x)^2} (-1)^{-1/2 \operatorname{csgn}(Ie) \operatorname{csgn}(Ie*x)^2} \exp(-i\pi n \operatorname{csgn}(Ie*x)^3) \exp(i\pi n \operatorname{csgn}(Ie) \operatorname{csgn}(Ie*x)^2) \exp(i\pi n \operatorname{csgn}(Ix) \operatorname{csgn}(Ie*x)^2) \exp(-i\pi n \operatorname{csgn}(Ie) \operatorname{csgn}(Ix) \operatorname{csgn}(Ie*x)) \exp(1/2 i\pi \operatorname{csgn}(Ie*x)^3) \exp(-Ic) - \frac{I}{d} \frac{1}{n} e^{(e^n)^{2b} \ln(1 - \exp(Ix^n d))} (-\exp(2Ic))^{1/2} x^n (-\exp(2Ic))^{1/2} (-1)^{1/2 \operatorname{csgn}(Ie) \operatorname{csgn}(Ix) \operatorname{csgn}(Ie*x)} (-1)^{-1/2 \operatorname{csgn}(Ix) \operatorname{csgn}(Ie*x)^2} (-1)^{-1/2 \operatorname{csgn}(Ie) \operatorname{csgn}(Ie*x)^2} \exp(-i\pi n \operatorname{csgn}(Ie*x)^3) \exp(i\pi n \operatorname{csgn}(Ie) \operatorname{csgn}(Ie*x)^2) \exp(i\pi n \operatorname{csgn}(Ix) \operatorname{csgn}(Ie*x)^2) \exp(-i\pi n \operatorname{csgn}(Ie) \operatorname{csgn}(Ix) \operatorname{csgn}(Ie*x)) \exp(1/2 i\pi \operatorname{csgn}(Ie*x)^3) \exp(-Ic) + \frac{1}{d^2} \frac{1}{n} e^{(e^n)^{2b} \operatorname{dilog}(1 + \exp(Ix^n d))} (-\exp(2Ic))^{1/2} (-\exp(2Ic))^{1/2} (-1)^{1/2 \operatorname{csgn}(Ie) \operatorname{csgn}(Ix) \operatorname{csgn}(Ie*x)} (-1)^{-1/2 \operatorname{csgn}(Ix) \operatorname{csgn}(Ie*x)^2} (-1)^{-1/2 \operatorname{csgn}(Ie) \operatorname{csgn}(Ie*x)^2} \exp(-i\pi n \operatorname{csgn}(Ie*x)^3) \exp(i\pi n \operatorname{csgn}(Ie) \operatorname{csgn}(Ie*x)^2) \exp(i\pi n \operatorname{csgn}(Ix) \operatorname{csgn}(Ie*x)^2) \exp(-i\pi n \operatorname{csgn}(Ie) \operatorname{csgn}(Ix) \operatorname{csgn}(Ie*x)) \exp(1/2 i\pi \operatorname{csgn}(Ie*x)^3) \exp(-Ic) - \frac{1}{d^2} \frac{1}{n} e^{(e^n)^{2b} \operatorname{dilog}(1 - \exp(Ix^n d))} (-\exp(2Ic))^{1/2} (-\exp(2Ic))^{1/2} (-1)^{1/2 \operatorname{csgn}(Ie) \operatorname{csgn}(Ix) \operatorname{csgn}(Ie*x)} (-1)^{-1/2 \operatorname{csgn}(Ix) \operatorname{csgn}(Ie*x)^2} (-1)^{-1/2 \operatorname{csgn}(Ie) \operatorname{csgn}(Ie*x)^2} \exp(-i\pi n \operatorname{csgn}(Ie*x)^3) \exp(i\pi n \operatorname{csgn}(Ie) \operatorname{csgn}(Ie*x)^2) \exp(i\pi n \operatorname{csgn}(Ix) \operatorname{csgn}(Ie*x)^2) \exp(-i\pi n \operatorname{csgn}(Ie) \operatorname{csgn}(Ix) \operatorname{csgn}(Ie*x)) \exp(1/2 i\pi \operatorname{csgn}(Ie*x)^3) \exp(-Ic)$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$2be^{2n} \int \frac{x^{2n} \cos(2dx^n + 2c) \cos(dx^n + c) + x^{2n} \sin(2dx^n + 2c) \sin(dx^n + c) + x^{2n} \cos(dx^n + c)}{ex \cos(2dx^n + 2c)^2 + ex \sin(2dx^n + 2c)^2 + 2ex \cos(2dx^n + 2c) + ex} dx + \frac{(ex)^{2n}}{2en}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^(-1+2\*n)\*(a+b\*sec(c+d\*x^n)), x, algorithm="maxima")

[Out]  $2b e^{(2n)} \operatorname{integrate}\left(\frac{x^{(2n)} \cos(2d*x^n + 2c) \cos(dx^n + c) + x^{(2n)} \sin(2d*x^n + 2c) \sin(dx^n + c) + x^{(2n)} \cos(dx^n + c)}{e*x*\cos(2d*x^n + 2c)^2 + e*x*\sin(2d*x^n + 2c)^2 + 2e*x*\cos(2d*x^n + 2c) + e*x}, x\right) + \frac{1}{2} (e*x)^{(2n)} a / (e^n)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left( a + \frac{b}{\cos(c + dx^n)} \right) (ex)^{2n-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b/cos(c + d*x^n))*(e*x)^(2*n - 1), x)`

[Out] `int((a + b/cos(c + d*x^n))*(e*x)^(2*n - 1), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^{2n-1} (a + b \sec(c + dx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)**(-1+2*n)*(a+b*sec(c+d*x**n)), x)`

[Out] `Integral((e*x)**(2*n - 1)*(a + b*sec(c + d*x**n)), x)`

### 3.74 $\int (ex)^{-1+3n} (a + b \sec(c + dx^n)) dx$

**Optimal.** Leaf size=235

$$\frac{a(ex)^{3n}}{3en} - \frac{2bx^{-3n}(ex)^{3n}\text{Li}_3(-ie^{i(dx^n+c)})}{d^3en} + \frac{2bx^{-3n}(ex)^{3n}\text{Li}_3(ie^{i(dx^n+c)})}{d^3en} + \frac{2ibx^{-2n}(ex)^{3n}\text{Li}_2(-ie^{i(dx^n+c)})}{d^2en} - \frac{2ibx^{-2n}(ex)^{3n}\text{Li}_2(ie^{i(dx^n+c)})}{d^2en}$$

[Out]  $1/3*a*(e*x)^{(3*n)}/e/n-2*I*b*(e*x)^{(3*n)}*\arctan(\exp(I*(c+d*x^n)))/d/e/n/(x^n)+2*I*b*(e*x)^{(3*n)}*\text{polylog}(2,-I*\exp(I*(c+d*x^n)))/d^2/e/n/(x^{(2*n)})-2*I*b*(e*x)^{(3*n)}*\text{polylog}(2,I*\exp(I*(c+d*x^n)))/d^2/e/n/(x^{(2*n)})-2*b*(e*x)^{(3*n)}*\text{polylog}(3,-I*\exp(I*(c+d*x^n)))/d^3/e/n/(x^{(3*n)})+2*b*(e*x)^{(3*n)}*\text{polylog}(3,I*\exp(I*(c+d*x^n)))/d^3/e/n/(x^{(3*n)})$

**Rubi [A]** time = 0.19, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$ , Rules used = {14, 4208, 4204, 4181, 2531, 2282, 6589}

$$-\frac{2bx^{-3n}(ex)^{3n}\text{PolyLog}(3,-ie^{i(c+dx^n)})}{d^3en} + \frac{2bx^{-3n}(ex)^{3n}\text{PolyLog}(3,ie^{i(c+dx^n)})}{d^3en} + \frac{2ibx^{-2n}(ex)^{3n}\text{PolyLog}(2,-ie^{i(c+dx^n)})}{d^2en} - \frac{2ibx^{-2n}(ex)^{3n}\text{PolyLog}(2,ie^{i(c+dx^n)})}{d^2en}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(e*x)^{-1+3*n}*(a + b*\text{Sec}[c + d*x^n]), x]$

[Out]  $(a*(e*x)^{(3*n)})/(3*e*n) - ((2*I)*b*(e*x)^{(3*n)}*\text{ArcTan}[E^{(I*(c + d*x^n))}])/(d*e*n*x^n) + ((2*I)*b*(e*x)^{(3*n)}*\text{PolyLog}[2, (-I)*E^{(I*(c + d*x^n))}])/(d^2*e*n*x^{(2*n)}) - ((2*I)*b*(e*x)^{(3*n)}*\text{PolyLog}[2, I*E^{(I*(c + d*x^n))}])/(d^2*e*n*x^{(2*n)}) - (2*b*(e*x)^{(3*n)}*\text{PolyLog}[3, (-I)*E^{(I*(c + d*x^n))}])/(d^3*e*n*x^{(3*n)}) + (2*b*(e*x)^{(3*n)}*\text{PolyLog}[3, I*E^{(I*(c + d*x^n))}])/(d^3*e*n*x^{(3*n)})$

#### Rule 14

$\text{Int}[(u_*)*((c_*)*(x_*)^m), x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(c*x)^m*u, x], x] /; \text{FreeQ}\{c, m, x\} \ \&\& \ \text{SumQ}[u] \ \&\& \ !\text{LinearQ}[u, x] \ \&\& \ !\text{MatchQ}[u, (a_*) + (b_*)*(v_*)] /; \text{FreeQ}\{a, b, x\} \ \&\& \ \text{InverseFunctionQ}[v]$

#### Rule 2282

$\text{Int}[u, x\_Symbol] \rightarrow \text{With}\{v = \text{FunctionOfExponential}[u, x]\}, \text{Dist}[v/D[v, x], \text{Subst}[\text{Int}[\text{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x] /; \text{FunctionOfExponentialQ}[u, x] \ \&\& \ !\text{MatchQ}[u, (w_*)*((a_*)*(v_*)^n)^m] /; \text{FreeQ}\{a, m, n, x\} \ \&\& \ \text{IntegerQ}[m*n] \ \&\& \ !\text{MatchQ}[u, E^{((c_*)*((a_*) + (b_*)*x))}*(F_)] [v_] /; \text{FreeQ}\{a, b, c, x\} \ \&\& \ \text{InverseFunctionQ}[F[x]]$

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_)))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
))^n]])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x)
)], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4204

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 4208

```
Int[((e_)*(x_))^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x
_Symbol] := Dist[(e^IntPart[m]*(e*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a
+ b*Sec[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int (ex)^{-1+3n} (a + b \sec(c + dx^n)) dx &= \int (a(ex)^{-1+3n} + b(ex)^{-1+3n} \sec(c + dx^n)) dx \\
&= \frac{a(ex)^{3n}}{3en} + b \int (ex)^{-1+3n} \sec(c + dx^n) dx \\
&= \frac{a(ex)^{3n}}{3en} + \frac{(bx^{-3n}(ex)^{3n}) \int x^{-1+3n} \sec(c + dx^n) dx}{e} \\
&= \frac{a(ex)^{3n}}{3en} + \frac{(bx^{-3n}(ex)^{3n}) \text{Subst}\left(\int x^2 \sec(c + dx) dx, x, x^n\right)}{en} \\
&= \frac{a(ex)^{3n}}{3en} - \frac{2ibx^{-n}(ex)^{3n} \tan^{-1}\left(e^{i(c+dx^n)}\right)}{den} - \frac{(2bx^{-3n}(ex)^{3n}) \text{Subst}\left(\int x \log\left(\frac{den}{den}\right) dx, x, x^n\right)}{den} \\
&= \frac{a(ex)^{3n}}{3en} - \frac{2ibx^{-n}(ex)^{3n} \tan^{-1}\left(e^{i(c+dx^n)}\right)}{den} + \frac{2ibx^{-2n}(ex)^{3n} \text{Li}_2\left(-ie^{i(c+dx^n)}\right)}{d^2en} \\
&= \frac{a(ex)^{3n}}{3en} - \frac{2ibx^{-n}(ex)^{3n} \tan^{-1}\left(e^{i(c+dx^n)}\right)}{den} + \frac{2ibx^{-2n}(ex)^{3n} \text{Li}_2\left(-ie^{i(c+dx^n)}\right)}{d^2en} \\
&= \frac{a(ex)^{3n}}{3en} - \frac{2ibx^{-n}(ex)^{3n} \tan^{-1}\left(e^{i(c+dx^n)}\right)}{den} + \frac{2ibx^{-2n}(ex)^{3n} \text{Li}_2\left(-ie^{i(c+dx^n)}\right)}{d^2en}
\end{aligned}$$

**Mathematica** [F] time = 1.32, size = 0, normalized size = 0.00

$$\int (ex)^{-1+3n} (a + b \sec(c + dx^n)) dx$$

Verification is Not applicable to the result.

[In] Integrate[(e\*x)^(-1 + 3\*n)\*(a + b\*Sec[c + d\*x^n]), x]

[Out] Integrate[(e\*x)^(-1 + 3\*n)\*(a + b\*Sec[c + d\*x^n]), x]

**fricas** [C] time = 0.83, size = 655, normalized size = 2.79

$$\frac{2ad^3e^{3n-1}x^{3n} - 6ibde^{3n-1}x^n \text{Li}_2(i \cos(dx^n + c) + \sin(dx^n + c)) - 6ibde^{3n-1}x^n \text{Li}_2(i \cos(dx^n + c) - \sin(dx^n + c))}{d^2en}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^(-1+3\*n)\*(a+b\*sec(c+d\*x^n)), x, algorithm="fricas")

[Out] 1/6\*(2\*a\*d^3\*e^(3\*n - 1)\*x^(3\*n) - 6\*I\*b\*d\*e^(3\*n - 1)\*x^n\*dilog(I\*cos(d\*x^n + c) + sin(d\*x^n + c)) - 6\*I\*b\*d\*e^(3\*n - 1)\*x^n\*dilog(I\*cos(d\*x^n + c) - sin(d\*x^n + c)) + 6\*I\*b\*d\*e^(3\*n - 1)\*x^n\*dilog(-I\*cos(d\*x^n + c) + sin(d\*x^n + c)))

$x^n + c)) + 6*I*b*d*e^{(3*n - 1)*x^n}*dilog(-I*\cos(d*x^n + c) - \sin(d*x^n + c)) + 3*b*c^2*e^{(3*n - 1)*\log(\cos(d*x^n + c) + I*\sin(d*x^n + c) + I) - 3*b*c^2*e^{(3*n - 1)*\log(\cos(d*x^n + c) - I*\sin(d*x^n + c) + I) + 3*b*c^2*e^{(3*n - 1)*\log(-\cos(d*x^n + c) + I*\sin(d*x^n + c) + I) - 3*b*c^2*e^{(3*n - 1)*\log(-\cos(d*x^n + c) - I*\sin(d*x^n + c) + I) - 6*b*e^{(3*n - 1)*polylog(3, I*\cos(d*x^n + c) + \sin(d*x^n + c)) + 6*b*e^{(3*n - 1)*polylog(3, I*\cos(d*x^n + c) - \sin(d*x^n + c)) - 6*b*e^{(3*n - 1)*polylog(3, -I*\cos(d*x^n + c) + \sin(d*x^n + c)) + 6*b*e^{(3*n - 1)*polylog(3, -I*\cos(d*x^n + c) - \sin(d*x^n + c))} + 3*(b*d^2*e^{(3*n - 1)*x^{(2*n)} - b*c^2*e^{(3*n - 1)})*\log(I*\cos(d*x^n + c) + \sin(d*x^n + c) + 1) - 3*(b*d^2*e^{(3*n - 1)*x^{(2*n)} - b*c^2*e^{(3*n - 1)})*\log(I*\cos(d*x^n + c) - \sin(d*x^n + c) + 1) + 3*(b*d^2*e^{(3*n - 1)*x^{(2*n)} - b*c^2*e^{(3*n - 1)})*\log(-I*\cos(d*x^n + c) + \sin(d*x^n + c) + 1) - 3*(b*d^2*e^{(3*n - 1)*x^{(2*n)} - b*c^2*e^{(3*n - 1)})*\log(-I*\cos(d*x^n + c) - \sin(d*x^n + c) + 1))/(d^{3*n})$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sec(dx^n + c) + a)(ex)^{3n-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^(-1+3\*n)\*(a+b\*sec(c+d\*x^n)),x, algorithm="giac")

[Out] integrate((b\*sec(d\*x^n + c) + a)\*(e\*x)^(3\*n - 1), x)

**maple** [F] time = 2.27, size = 0, normalized size = 0.00

$$\int (ex)^{-1+3n} (a + b \sec(c + dx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x)^(-1+3\*n)\*(a+b\*sec(c+d\*x^n)),x)

[Out] int((e\*x)^(-1+3\*n)\*(a+b\*sec(c+d\*x^n)),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$2be^{3n} \int \frac{x^{3n} \cos(2dx^n + 2c) \cos(dx^n + c) + x^{3n} \sin(2dx^n + 2c) \sin(dx^n + c) + x^{3n} \cos(dx^n + c)}{ex \cos(2dx^n + 2c)^2 + ex \sin(2dx^n + 2c)^2 + 2ex \cos(2dx^n + 2c) + ex} dx + \frac{(ex)^{3n} a}{3en}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^(-1+3\*n)\*(a+b\*sec(c+d\*x^n)),x, algorithm="maxima")

[Out] 2\*b\*e^{(3\*n)\*integrate((x^{(3\*n)}\*cos(2\*d\*x^n + 2\*c)\*cos(d\*x^n + c) + x^{(3\*n)}\*sin(2\*d\*x^n + 2\*c)\*sin(d\*x^n + c) + x^{(3\*n)}\*cos(d\*x^n + c))/(e\*x\*cos(2\*d\*x^n



$n + 2*c)^2 + e*x*\sin(2*d*x^n + 2*c)^2 + 2*e*x*\cos(2*d*x^n + 2*c) + e*x), x)$   
 $+ 1/3*(e*x)^(3*n)*a/(e*n)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left( a + \frac{b}{\cos(c + dx^n)} \right) (ex)^{3n-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b/cos(c + d*x^n))*(e*x)^(3*n - 1), x)`

[Out] `int((a + b/cos(c + d*x^n))*(e*x)^(3*n - 1), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^{3n-1} (a + b \sec(c + dx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)**(-1+3*n)*(a+b*sec(c+d*x**n)), x)`

[Out] `Integral((e*x)**(3*n - 1)*(a + b*sec(c + d*x**n)), x)`

### 3.75 $\int (ex)^{-1+n} (a + b \sec(c + dx^n))^2 dx$

Optimal. Leaf size=79

$$\frac{a^2(ex)^n}{en} + \frac{2abx^{-n}(ex)^n \tanh^{-1}(\sin(c + dx^n))}{den} + \frac{b^2x^{-n}(ex)^n \tan(c + dx^n)}{den}$$

[Out]  $a^2*(e*x)^n/e/n+2*a*b*(e*x)^n*\operatorname{arctanh}(\sin(c+d*x^n))/d/e/n/(x^n)+b^2*(e*x)^n*\tan(c+d*x^n)/d/e/n/(x^n)$

**Rubi [A]** time = 0.09, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$ , Rules used = {4208, 4204, 3773, 3770, 3767, 8}

$$\frac{a^2(ex)^n}{en} + \frac{2abx^{-n}(ex)^n \tanh^{-1}(\sin(c + dx^n))}{den} + \frac{b^2x^{-n}(ex)^n \tan(c + dx^n)}{den}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(e*x)^{-1+n}*(a + b*\operatorname{Sec}[c + d*x^n])^2,x]$

[Out]  $(a^2*(e*x)^n)/(e*n) + (2*a*b*(e*x)^n*\operatorname{ArcTanh}[\operatorname{Sin}[c + d*x^n]])/(d*e*n*x^n) + (b^2*(e*x)^n*\operatorname{Tan}[c + d*x^n])/(d*e*n*x^n)$

#### Rule 8

$\operatorname{Int}[a_, x\_Symbol] \rightarrow \operatorname{Simp}[a*x, x] /; \operatorname{FreeQ}[a, x]$

#### Rule 3767

$\operatorname{Int}[\operatorname{csc}[(c_.) + (d_.)*(x_)]^{(n_)}, x\_Symbol] \rightarrow -\operatorname{Dist}[d^{(-1)}, \operatorname{Subst}[\operatorname{Int}[\operatorname{ExpandIntegrand}[(1 + x^2)^{(n/2 - 1)}, x], x], x, \operatorname{Cot}[c + d*x]], x] /; \operatorname{FreeQ}[\{c, d\}, x] \ \&\& \operatorname{IGtQ}[n/2, 0]$

#### Rule 3770

$\operatorname{Int}[\operatorname{csc}[(c_.) + (d_.)*(x_)], x\_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{ArcTanh}[\operatorname{Cos}[c + d*x]]/d, x] /; \operatorname{FreeQ}[\{c, d\}, x]$

#### Rule 3773

$\operatorname{Int}[(\operatorname{csc}[(c_.) + (d_.)*(x_)]*(b_.) + (a_.))^2, x\_Symbol] \rightarrow \operatorname{Simp}[a^2*x, x] + (\operatorname{Dist}[2*a*b, \operatorname{Int}[\operatorname{Csc}[c + d*x], x], x] + \operatorname{Dist}[b^2, \operatorname{Int}[\operatorname{Csc}[c + d*x]^2, x], x]) /; \operatorname{FreeQ}[\{a, b, c, d\}, x]$

Rule 4204

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol]
  := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p, x], x, x^n], x]
  /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]
```

Rule 4208

```
Int[((e_)*(x_))^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol]
  := Dist[(e^IntPart[m]*(e*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a + b*Sec[c + d*x^n])^p, x], x]
  /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
 \int (ex)^{-1+n} (a + b \sec(c + dx^n))^2 dx &= \frac{(x^{-n}(ex)^n) \int x^{-1+n} (a + b \sec(c + dx^n))^2 dx}{e} \\
 &= \frac{(x^{-n}(ex)^n) \text{Subst}\left(\int (a + b \sec(c + dx))^2 dx, x, x^n\right)}{en} \\
 &= \frac{a^2(ex)^n}{en} + \frac{(2abx^{-n}(ex)^n) \text{Subst}\left(\int \sec(c + dx) dx, x, x^n\right)}{en} + \frac{(b^2x^{-n}(ex)^n) \text{Subst}\left(\int 1 dx, x, x^n\right)}{en} \\
 &= \frac{a^2(ex)^n}{en} + \frac{2abx^{-n}(ex)^n \tanh^{-1}(\sin(c + dx^n))}{den} - \frac{(b^2x^{-n}(ex)^n) \text{Subst}\left(\int 1 dx, x, x^n\right)}{den} \\
 &= \frac{a^2(ex)^n}{en} + \frac{2abx^{-n}(ex)^n \tanh^{-1}(\sin(c + dx^n))}{den} + \frac{b^2x^{-n}(ex)^n \tan(c + dx^n)}{den}
 \end{aligned}$$

**Mathematica** [A] time = 0.42, size = 54, normalized size = 0.68

$$\frac{x^{-n}(ex)^n (a^2 dx^n + 2ab \tanh^{-1}(\sin(c + dx^n)) + b^2 \tan(c + dx^n))}{den}$$

Antiderivative was successfully verified.

[In] Integrate[(e\*x)^(-1 + n)\*(a + b\*Sec[c + d\*x^n])^2, x]

[Out] ((e\*x)^n\*(a^2\*d\*x^n + 2\*a\*b\*ArcTanh[Sin[c + d\*x^n]] + b^2\*Tan[c + d\*x^n]))/(d\*e\*n\*x^n)

**fricas** [A] time = 0.58, size = 113, normalized size = 1.43

$$\frac{a^2 de^{n-1} x^n \cos(dx^n + c) + abe^{n-1} \cos(dx^n + c) \log(\sin(dx^n + c) + 1) - abe^{n-1} \cos(dx^n + c) \log(-\sin(dx^n + c))}{dn \cos(dx^n + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^(-1+n)\*(a+b\*sec(c+d\*x^n))^2,x, algorithm="fricas")

[Out] (a^2\*d\*e^(n - 1)\*x^n\*cos(d\*x^n + c) + a\*b\*e^(n - 1)\*cos(d\*x^n + c)\*log(sin(d\*x^n + c) + 1) - a\*b\*e^(n - 1)\*cos(d\*x^n + c)\*log(-sin(d\*x^n + c) + 1) + b^2\*e^(n - 1)\*sin(d\*x^n + c))/(d\*n\*cos(d\*x^n + c))

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sec(dx^n + c) + a)^2 (ex)^{n-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^(-1+n)\*(a+b\*sec(c+d\*x^n))^2,x, algorithm="giac")

[Out] integrate((b\*sec(d\*x^n + c) + a)^2\*(e\*x)^(n - 1), x)

**maple** [C] time = 1.19, size = 276, normalized size = 3.49

$$\frac{a^2 x e^{\frac{(-1+n)(-i\pi \operatorname{csgn}(ie) \operatorname{csgn}(ix) \operatorname{csgn}(iex) + i\pi \operatorname{csgn}(ie) \operatorname{csgn}(iex)^2 + i\pi \operatorname{csgn}(ix) \operatorname{csgn}(iex)^2 - i\pi \operatorname{csgn}(iex)^3 + 2 \ln(x) + 2 \ln(e))}{2}}}{n} + \frac{2ix b^2 e^{\frac{(-1+n)(-i\pi \operatorname{csgn}(ie) \operatorname{csgn}(ix) \operatorname{csgn}(iex) + i\pi \operatorname{csgn}(ie) \operatorname{csgn}(iex)^2 + i\pi \operatorname{csgn}(ix) \operatorname{csgn}(iex)^2 - i\pi \operatorname{csgn}(iex)^3 + 2 \ln(x) + 2 \ln(e))}{2}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x)^(-1+n)\*(a+b\*sec(c+d\*x^n))^2,x)

[Out] a^2/n\*x\*exp(1/2\*(-1+n)\*(-I\*Pi\*csgn(I\*e)\*csgn(I\*x)\*csgn(I\*e\*x)+I\*Pi\*csgn(I\*e)\*csgn(I\*e\*x)^2+I\*Pi\*csgn(I\*x)\*csgn(I\*e\*x)^2-I\*Pi\*csgn(I\*e\*x)^3+2\*ln(x)+2\*ln(e))+2\*I\*x\*b^2\*exp(1/2\*(-1+n)\*(-I\*Pi\*csgn(I\*e)\*csgn(I\*x)\*csgn(I\*e\*x)+I\*Pi\*csgn(I\*e)\*csgn(I\*e\*x)^2+I\*Pi\*csgn(I\*x)\*csgn(I\*e\*x)^2-I\*Pi\*csgn(I\*e\*x)^3+2\*ln(x)+2\*ln(e)))/d/n/(x^n)/(1+exp(2\*I\*(c+d\*x^n)))-4\*I\*arctan(exp(I\*(c+d\*x^n)))/d/e\*e^n/n\*a\*b\*exp(1/2\*I\*Pi\*csgn(I\*e\*x)\*(-1+n)\*(csgn(I\*e\*x)-csgn(I\*x))\*(-csgn(I\*e\*x)+csgn(I\*e)))

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^(-1+n)\*(a+b\*sec(c+d\*x^n))^2,x, algorithm="maxima")

[Out] Timed out

**mupad [B]** time = 2.71, size = 180, normalized size = 2.28

$$\frac{a^2 x (ex)^{n-1}}{n} + \frac{b^2 x (ex)^{n-1} 2i}{dn x^n (e^{c 2i + d x^n 2i} + 1)} + \frac{2 a b x \ln(-a b (ex)^{n-1} 4i - 4 a b e^{c 1i} e^{d x^n 1i} (ex)^{n-1})}{dn x^n} - \frac{2 a b x \ln(a b (ex)^{n-1} 4i + 4 a b e^{c 1i} e^{d x^n 1i} (ex)^{n-1})}{dn x^n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cos(c + d\*x^n))^2\*(e\*x)^(n - 1), x)

[Out] (a^2\*x\*(e\*x)^(n - 1))/n + (b^2\*x\*(e\*x)^(n - 1)\*2i)/(d\*n\*x^n\*(exp(c\*2i + d\*x^n\*2i) + 1)) + (2\*a\*b\*x\*log(- a\*b\*(e\*x)^(n - 1)\*4i - 4\*a\*b\*exp(c\*1i)\*exp(d\*x^n\*1i)\*(e\*x)^(n - 1)))/(d\*n\*x^n) - (2\*a\*b\*x\*log(a\*b\*(e\*x)^(n - 1)\*4i + 4\*a\*b\*exp(c\*1i)\*exp(d\*x^n\*1i)\*(e\*x)^(n - 1)))/(d\*n\*x^n)

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^{n-1} (a + b \sec(c + dx^n))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)\*\*(-1+n)\*(a+b\*sec(c+d\*x\*\*n))\*\*2, x)

[Out] Integral((e\*x)\*\*(n - 1)\*(a + b\*sec(c + d\*x\*\*n))\*\*2, x)

### 3.76 $\int (ex)^{-1+2n} (a + b \sec(c + dx^n))^2 dx$

**Optimal.** Leaf size=221

$$\frac{a^2(ex)^{2n}}{2en} + \frac{2iabx^{-2n}(ex)^{2n}\text{Li}_2(-ie^{i(dx^n+c)})}{d^2en} - \frac{2iabx^{-2n}(ex)^{2n}\text{Li}_2(ie^{i(dx^n+c)})}{d^2en} - \frac{4iabx^{-n}(ex)^{2n}\tan^{-1}(e^{i(c+dx^n)})}{den} + \frac{b^2x^{-2n}(ex)^{2n}}{den}$$

[Out]  $1/2*a^2*(e*x)^{(2*n)}/e/n-4*I*a*b*(e*x)^{(2*n)}*\arctan(\exp(I*(c+d*x^n)))/d/e/n/(x^n)+b^2*(e*x)^{(2*n)}*\ln(\cos(c+d*x^n))/d^2/e/n/(x^{(2*n)})+2*I*a*b*(e*x)^{(2*n)}*\text{polylog}(2,-I*\exp(I*(c+d*x^n)))/d^2/e/n/(x^{(2*n)})-2*I*a*b*(e*x)^{(2*n)}*\text{polylog}(2,I*\exp(I*(c+d*x^n)))/d^2/e/n/(x^{(2*n)})+b^2*(e*x)^{(2*n)}*\tan(c+d*x^n)/d/e/n/(x^n)$

**Rubi [A]** time = 0.20, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {4208, 4204, 4190, 4181, 2279, 2391, 4184, 3475}

$$\frac{2iabx^{-2n}(ex)^{2n}\text{PolyLog}(2, -ie^{i(c+dx^n)})}{d^2en} - \frac{2iabx^{-2n}(ex)^{2n}\text{PolyLog}(2, ie^{i(c+dx^n)})}{d^2en} + \frac{a^2(ex)^{2n}}{2en} - \frac{4iabx^{-n}(ex)^{2n}\tan^{-1}(e^{i(c+dx^n)})}{den}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(e*x)^{-1+2*n}*(a + b*\text{Sec}[c + d*x^n])^2, x]$

[Out]  $(a^2*(e*x)^{(2*n)})/(2*e*n) - ((4*I)*a*b*(e*x)^{(2*n)}*\text{ArcTan}[E^{(I*(c + d*x^n))}])/d^2*e*n*x^n + (b^2*(e*x)^{(2*n)}*\text{Log}[\text{Cos}[c + d*x^n]])/d^2*e*n*x^{(2*n)} + ((2*I)*a*b*(e*x)^{(2*n)}*\text{PolyLog}[2, (-I)*E^{(I*(c + d*x^n))}])/d^2*e*n*x^{(2*n)} - ((2*I)*a*b*(e*x)^{(2*n)}*\text{PolyLog}[2, I*E^{(I*(c + d*x^n))}])/d^2*e*n*x^{(2*n)} + (b^2*(e*x)^{(2*n)}*\text{Tan}[c + d*x^n])/d^2*e*n*x^n$

#### Rule 2279

$\text{Int}[\text{Log}[(a_) + (b_)*((F_)^{((e_)*((c_) + (d_)*(x_)))})^{(n_)}], x\_Symbol]$   
 $\text{:> Dist}[1/(d*e*n*\text{Log}[F]), \text{Subst}[\text{Int}[\text{Log}[a + b*x]/x, x], x, (F^{(e*(c + d*x))})^n], x] /;$   $\text{FreeQ}\{F, a, b, c, d, e, n\}, x \ \&\& \ \text{GtQ}[a, 0]$

#### Rule 2391

$\text{Int}[\text{Log}[(c_)*((d_) + (e_)*(x_)^{(n_)})]/(x_), x\_Symbol] \text{:> -Simp}[\text{PolyLog}[2, -(c*e*x^n)]/n, x] /;$   $\text{FreeQ}\{c, d, e, n\}, x \ \&\& \ \text{EqQ}[c*d, 1]$

#### Rule 3475

$\text{Int}[\tan[(c_) + (d_)*(x_)], x\_Symbol] \text{:> -Simp}[\text{Log}[\text{RemoveContent}[\text{Cos}[c + d*x], x]]/d, x] /;$   $\text{FreeQ}\{c, d\}, x$

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol]
  := Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))], x], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := -Simp[(((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4190

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^((n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 4204

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]
```

Rule 4208

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Dist[(e^IntPart[m]*(e*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a + b*Sec[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\int (ex)^{-1+2n} (a + b \sec(c + dx^n))^2 dx &= \frac{(x^{-2n}(ex)^{2n}) \int x^{-1+2n} (a + b \sec(c + dx^n))^2 dx}{e} \\
&= \frac{(x^{-2n}(ex)^{2n}) \text{Subst} \left( \int x(a + b \sec(c + dx))^2 dx, x, x^n \right)}{en} \\
&= \frac{(x^{-2n}(ex)^{2n}) \text{Subst} \left( \int (a^2x + 2abx \sec(c + dx) + b^2x \sec^2(c + dx)) dx, x, x^n \right)}{en} \\
&= \frac{a^2(ex)^{2n}}{2en} + \frac{(2abx^{-2n}(ex)^{2n}) \text{Subst} \left( \int x \sec(c + dx) dx, x, x^n \right)}{en} + \frac{(b^2x^{-2n}(ex)^{2n}) \text{Subst} \left( \int x \sec^2(c + dx) dx, x, x^n \right)}{en} \\
&= \frac{a^2(ex)^{2n}}{2en} - \frac{4iabx^{-n}(ex)^{2n} \tan^{-1} \left( e^{i(c+dx^n)} \right)}{den} + \frac{b^2x^{-n}(ex)^{2n} \tan(c + dx^n)}{den} - \frac{(b^2x^{-2n}(ex)^{2n}) \text{Subst} \left( \int x \sec^2(c + dx) dx, x, x^n \right)}{en} \\
&= \frac{a^2(ex)^{2n}}{2en} - \frac{4iabx^{-n}(ex)^{2n} \tan^{-1} \left( e^{i(c+dx^n)} \right)}{den} + \frac{b^2x^{-2n}(ex)^{2n} \log(\cos(c + dx^n))}{d^2en} \\
&= \frac{a^2(ex)^{2n}}{2en} - \frac{4iabx^{-n}(ex)^{2n} \tan^{-1} \left( e^{i(c+dx^n)} \right)}{den} + \frac{b^2x^{-2n}(ex)^{2n} \log(\cos(c + dx^n))}{d^2en}
\end{aligned}$$

**Mathematica [A]** time = 5.16, size = 347, normalized size = 1.57

$$x^{-2n}(ex)^{2n} \left( dx^n (a^2 dx^n + 2b^2 \tan(c)) - \frac{4ab \csc(c) \left( i \text{Li}_2 \left( -e^{i(dx^n - \tan^{-1}(\cot(c)))} \right) - i \text{Li}_2 \left( e^{i(dx^n - \tan^{-1}(\cot(c)))} \right) + (dx^n - \tan^{-1}(\cot(c))) \left( \log \left( 1 - e^{i(dx^n - \tan^{-1}(\cot(c)))} \right) \right) \right)}{\sqrt{\csc^2(c)}} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(e\*x)^(-1 + 2\*n)\*(a + b\*Sec[c + d\*x^n])^2,x]

[Out] ((e\*x)^(2\*n)\*(8\*a\*b\*ArcTan[Cot[c]]\*ArcTanh[Sin[c] + Cos[c]\*Tan[(d\*x^n)/2]] - (4\*a\*b\*Csc[c]\*((d\*x^n - ArcTan[Cot[c]])\*(Log[1 - E^(I\*(d\*x^n - ArcTan[Cot[c]])]) - Log[1 + E^(I\*(d\*x^n - ArcTan[Cot[c]])]) + I\*PolyLog[2, -E^(I\*(d\*x^n - ArcTan[Cot[c]])]) - I\*PolyLog[2, E^(I\*(d\*x^n - ArcTan[Cot[c]])])]) / Sqrt[Csc[c]^2 + (2\*b^2\*d\*x^n\*Sin[(d\*x^n)/2]) / ((Cos[c/2] - Sin[c/2])\*(Cos[(c + d\*x^n)/2] - Sin[(c + d\*x^n)/2])) + (2\*b^2\*d\*x^n\*Sin[(d\*x^n)/2]) / ((Cos[c/2] + Sin[c/2])\*(Cos[(c + d\*x^n)/2] + Sin[(c + d\*x^n)/2])) - 2\*b^2\*d\*x^n\*Tan[c] + d\*x^n\*(a^2\*d\*x^n + 2\*b^2\*Tan[c]) + 2\*b^2\*(Log[Cos[c + d\*x^n]] + d\*x^n\*Tan[c])) / (2\*d^2\*e\*n\*x^(2\*n))

**fricas [B]** time = 0.94, size = 656, normalized size = 2.97

$$a^2 d^2 e^{2n-1} x^{2n} \cos(dx^n + c) + 2 b^2 d e^{2n-1} x^n \sin(dx^n + c) - 2 i a b e^{2n-1} \cos(dx^n + c) \text{Li}_2(i \cos(dx^n + c) + \sin(dx^n + c))$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^(-1+2\*n)\*(a+b\*sec(c+d\*x^n))^2,x, algorithm="fricas")

[Out]  $\frac{1}{2}(a^2d^2e^{(2n-1)x^{2n}}\cos(dx^n+c) + 2b^2de^{(2n-1)x^n}\sin(dx^n+c) - 2Iab^2e^{(2n-1)\cos(dx^n+c)}\operatorname{dilog}(I\cos(dx^n+c) + \sin(dx^n+c)) - 2Iab^2e^{(2n-1)\cos(dx^n+c)}\operatorname{dilog}(I\cos(dx^n+c) - \sin(dx^n+c)) + 2Iab^2e^{(2n-1)\cos(dx^n+c)}\operatorname{dilog}(-I\cos(dx^n+c) + \sin(dx^n+c)) + 2Iab^2e^{(2n-1)\cos(dx^n+c)}\operatorname{dilog}(-I\cos(dx^n+c) - \sin(dx^n+c)) - (2ab^2c - b^2)e^{(2n-1)\cos(dx^n+c)}\log(\cos(dx^n+c) + I\sin(dx^n+c) + I) + (2ab^2c + b^2)e^{(2n-1)\cos(dx^n+c)}\log(\cos(dx^n+c) - I\sin(dx^n+c) + I) - (2ab^2c - b^2)e^{(2n-1)\cos(dx^n+c)}\log(-\cos(dx^n+c) + I\sin(dx^n+c) + I) + (2ab^2c + b^2)e^{(2n-1)\cos(dx^n+c)}\log(-\cos(dx^n+c) - I\sin(dx^n+c) + I) + 2(ab^2de^{(2n-1)x^n} + ab^2ce^{(2n-1)\cos(dx^n+c)})\log(I\cos(dx^n+c) + \sin(dx^n+c) + 1) - 2(ab^2de^{(2n-1)x^n} + ab^2ce^{(2n-1)\cos(dx^n+c)})\log(I\cos(dx^n+c) - \sin(dx^n+c) + 1) + 2(ab^2de^{(2n-1)x^n} + ab^2ce^{(2n-1)\cos(dx^n+c)})\log(-I\cos(dx^n+c) + \sin(dx^n+c) + 1) - 2(ab^2de^{(2n-1)x^n} + ab^2ce^{(2n-1)\cos(dx^n+c)})\log(-I\cos(dx^n+c) - \sin(dx^n+c) + 1)) / (d^{2n}\cos(dx^n+c))$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sec(dx^n + c) + a)^2 (ex)^{2n-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^(-1+2\*n)\*(a+b\*sec(c+d\*x^n))^2,x, algorithm="giac")

[Out] integrate((b\*sec(d\*x^n + c) + a)^2\*(e\*x)^(2\*n - 1), x)

maple [C] time = 1.20, size = 1096, normalized size = 4.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x)^(-1+2\*n)\*(a+b\*sec(c+d\*x^n))^2,x)

[Out]  $\frac{1}{2}a^2/n*x*\exp(1/2*(-1+2*n)*(-I*Pi*csgn(I*e)*csgn(I*x)*csgn(I*e*x)+I*Pi*csgn(I*e)*csgn(I*e*x)^2+I*Pi*csgn(I*x)*csgn(I*e*x)^2-I*Pi*csgn(I*e*x)^3+2*\ln(x)+2*\ln(e)))+2*I*x*b^2*\exp(1/2*(-1+2*n)*(-I*Pi*csgn(I*e)*csgn(I*x)*csgn(I*e*x)+I*Pi*csgn(I*e)*csgn(I*e*x)^2+I*Pi*csgn(I*x)*csgn(I*e*x)^2-I*Pi*csgn(I*e*x)^3+2*\ln(x)+2*\ln(e)))/d/n/(x^n)/(1+\exp(2*I*(c+d*x^n)))+1/d^2*b^2*\ln(1+\exp(2*I*(c+d*x^n)))/n/e*(e^n)^2*\exp(1/2*I*csgn(I*e*x)*Pi*(-1+2*n)*(csgn(I*e*x)$

```

-csgn(I*x))*(-csgn(I*e*x)+csgn(I*e)))-2/d^2*b^2*ln(exp(I*x^n*d))/n/e*(e^n)^
2*exp(1/2*I*csgn(I*e*x)*Pi*(-1+2*n))*(csgn(I*e*x)-csgn(I*x))*(-csgn(I*e*x)+c
sgn(I*e)))+2*I/d*b*ln(1+exp(I*x^n*d)*(-exp(2*I*c))^(1/2))*x^n*(-exp(2*I*c))
^(1/2)/n*a/e*(e^n)^2*exp(-1/2*I*(2*Pi*n*csgn(I*e*x)^3-2*Pi*n*csgn(I*e)*csgn
(I*e*x)^2-2*Pi*n*csgn(I*x)*csgn(I*e*x)^2+2*Pi*n*csgn(I*e)*csgn(I*x)*csgn(I*
e*x)-Pi*csgn(I*e*x)^3+Pi*csgn(I*e)*csgn(I*e*x)^2+Pi*csgn(I*x)*csgn(I*e*x)^2
-Pi*csgn(I*e*x)*csgn(I*e)*csgn(I*x)+2*c))-2*I/d*b*ln(1-exp(I*x^n*d)*(-exp(2
*I*c))^(1/2))*x^n*(-exp(2*I*c))^(1/2)/n*a/e*(e^n)^2*exp(-1/2*I*(2*Pi*n*csgn
(I*e*x)^3-2*Pi*n*csgn(I*e)*csgn(I*e*x)^2-2*Pi*n*csgn(I*x)*csgn(I*e*x)^2+2*P
i*n*csgn(I*e)*csgn(I*x)*csgn(I*e*x)-Pi*csgn(I*e*x)^3+Pi*csgn(I*e)*csgn(I*e*
x)^2+Pi*csgn(I*x)*csgn(I*e*x)^2-Pi*csgn(I*e*x)*csgn(I*e)*csgn(I*x)+2*c))+2/
d^2*b*dilog(1+exp(I*x^n*d)*(-exp(2*I*c))^(1/2))*(-exp(2*I*c))^(1/2)/n*a/e*(
e^n)^2*exp(-1/2*I*(2*Pi*n*csgn(I*e*x)^3-2*Pi*n*csgn(I*e)*csgn(I*e*x)^2-2*Pi
*n*csgn(I*x)*csgn(I*e*x)^2+2*Pi*n*csgn(I*e)*csgn(I*x)*csgn(I*e*x)-Pi*csgn(I
*e*x)^3+Pi*csgn(I*e)*csgn(I*e*x)^2+Pi*csgn(I*x)*csgn(I*e*x)^2-Pi*csgn(I*e*x
)*csgn(I*e)*csgn(I*x)+2*c))-2/d^2*b*dilog(1-exp(I*x^n*d)*(-exp(2*I*c))^(1/2
))*(-exp(2*I*c))^(1/2)/n*a/e*(e^n)^2*exp(-1/2*I*(2*Pi*n*csgn(I*e*x)^3-2*Pi*
n*csgn(I*e)*csgn(I*e*x)^2-2*Pi*n*csgn(I*x)*csgn(I*e*x)^2+2*Pi*n*csgn(I*e)*c
sgn(I*x)*csgn(I*e*x)-Pi*csgn(I*e*x)^3+Pi*csgn(I*e)*csgn(I*e*x)^2+Pi*csgn(I*
x)*csgn(I*e*x)^2-Pi*csgn(I*e*x)*csgn(I*e)*csgn(I*x)+2*c))

```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(ex)^{2n} a^2}{2en} + \frac{2b^2 e^{2n} x^n \sin(2dx^n + 2c) + 2 \left( den \cos(2dx^n + 2c)^2 + den \sin(2dx^n + 2c)^2 + 2 den \cos(2dx^n + 2c) \right)}{den \cos(2dx^n + 2c)^2 + den \sin(2dx^n + 2c)^2 + 2 den \cos(2dx^n + 2c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+2*n)*(a+b*sec(c+d*x^n))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(e*x)^(2*n)*a^2/(e*n) + (2*b^2*e^(2*n)*x^n*sin(2*d*x^n + 2*c) + (d*e*n*
cos(2*d*x^n + 2*c)^2 + d*e*n*sin(2*d*x^n + 2*c)^2 + 2*d*e*n*cos(2*d*x^n + 2
*c) + d*e*n)*integrate(2*(2*a*b*d*e^(2*n)*x^(2*n)*cos(2*d*x^n + 2*c)*cos(d*
x^n + c) + 2*a*b*d*e^(2*n)*x^(2*n)*cos(d*x^n + c) + (2*a*b*d*e^(2*n)*x^(2*n
)*sin(d*x^n + c) - b^2*e^(2*n)*x^n*sin(2*d*x^n + 2*c)))/(d*e*x*cos(2*d*x^n
+ 2*c)^2 + d*e*x*sin(2*d*x^n + 2*c)^2 + 2*d*e*x*cos(2*d*x^n + 2*c) + d*e*x)
, x))/(d*e*n*cos(2*d*x^n + 2*c)^2 + d*e*n*sin(2*d*x^n + 2*c)^2 + 2*d*e*n*co
s(2*d*x^n + 2*c) + d*e*n)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left( a + \frac{b}{\cos(c + dx^n)} \right)^2 (ex)^{2n-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b/cos(c + d*x^n))^2*(e*x)^(2*n - 1), x)`

[Out] `int((a + b/cos(c + d*x^n))^2*(e*x)^(2*n - 1), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^{2n-1} (a + b \sec(c + dx^n))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)**(-1+2*n)*(a+b*sec(c+d*x**n))**2, x)`

[Out] `Integral((e*x)**(2*n - 1)*(a + b*sec(c + d*x**n))**2, x)`

### 3.77 $\int (ex)^{-1+3n} (a + b \sec(c + dx^n))^2 dx$

**Optimal.** Leaf size=390

$$\frac{a^2(ex)^{3n}}{3en} - \frac{4abx^{-3n}(ex)^{3n}\text{Li}_3(-ie^{i(dx^n+c)})}{d^3en} + \frac{4abx^{-3n}(ex)^{3n}\text{Li}_3(ie^{i(dx^n+c)})}{d^3en} + \frac{4iabx^{-2n}(ex)^{3n}\text{Li}_2(-ie^{i(dx^n+c)})}{d^2en} - \frac{4iabx^{-2n}(ex)^{3n}\text{Li}_2(ie^{i(dx^n+c)})}{d^2en}$$

[Out]  $\frac{1}{3}a^2(e*x)^{(3*n)}/e/n - I*b^2*(e*x)^{(3*n)}/d/e/n/(x^n) - 4*I*a*b*(e*x)^{(3*n)*\arctan(\exp(I*(c+d*x^n)))/d/e/n/(x^n) + 2*b^2*(e*x)^{(3*n)*\ln(1+\exp(2*I*(c+d*x^n)))}/d^2/e/n/(x^{(2*n)}) + 4*I*a*b*(e*x)^{(3*n)*\text{polylog}(2, -I*\exp(I*(c+d*x^n)))/d^2/e/n/(x^{(2*n)}) - 4*I*a*b*(e*x)^{(3*n)*\text{polylog}(2, I*\exp(I*(c+d*x^n)))/d^2/e/n/(x^{(2*n)}) - I*b^2*(e*x)^{(3*n)*\text{polylog}(2, -\exp(2*I*(c+d*x^n)))/d^3/e/n/(x^{(3*n)}) - 4*a*b*(e*x)^{(3*n)*\text{polylog}(3, -I*\exp(I*(c+d*x^n)))/d^3/e/n/(x^{(3*n)}) + 4*a*b*(e*x)^{(3*n)*\text{polylog}(3, I*\exp(I*(c+d*x^n)))/d^3/e/n/(x^{(3*n)}) + b^2*(e*x)^{(3*n)*\tan(c+d*x^n)/d/e/n/(x^n)}$

**Rubi [A]** time = 0.40, antiderivative size = 390, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {4208, 4204, 4190, 4181, 2531, 2282, 6589, 4184, 3719, 2190, 2279, 2391}

$$-\frac{4abx^{-3n}(ex)^{3n}\text{PolyLog}(3, -ie^{i(c+dx^n)})}{d^3en} + \frac{4abx^{-3n}(ex)^{3n}\text{PolyLog}(3, ie^{i(c+dx^n)})}{d^3en} + \frac{4iabx^{-2n}(ex)^{3n}\text{PolyLog}(2, -ie^{i(c+dx^n)})}{d^2en} - \frac{4iabx^{-2n}(ex)^{3n}\text{PolyLog}(2, ie^{i(c+dx^n)})}{d^2en}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(e*x)^{-1 + 3*n}*(a + b*\text{Sec}[c + d*x^n])^2, x]$

[Out]  $\frac{a^2*(e*x)^{(3*n)}}{(3*e*n)} - \frac{(I*b^2*(e*x)^{(3*n)})}{(d*e*n*x^n)} - \frac{((4*I)*a*b*(e*x)^{(3*n)*\text{ArcTan}[E^{(I*(c + d*x^n))}]})}{(d*e*n*x^n)} + \frac{(2*b^2*(e*x)^{(3*n)*\text{Log}[1 + E^{((2*I)*(c + d*x^n))}]})}{(d^2*e*n*x^{(2*n)})} + \frac{((4*I)*a*b*(e*x)^{(3*n)*\text{PolyLog}[2, (-I)*E^{(I*(c + d*x^n))}]})}{(d^2*e*n*x^{(2*n)})} - \frac{((4*I)*a*b*(e*x)^{(3*n)*\text{PolyLog}[2, I*E^{(I*(c + d*x^n))}]})}{(d^2*e*n*x^{(2*n)})} - \frac{(I*b^2*(e*x)^{(3*n)*\text{PolyLog}[3, (-I)*E^{(I*(c + d*x^n))}]})}{(d^3*e*n*x^{(3*n)})} - \frac{(4*a*b*(e*x)^{(3*n)*\text{PolyLog}[3, (-I)*E^{(I*(c + d*x^n))}]})}{(d^3*e*n*x^{(3*n)})} + \frac{(4*a*b*(e*x)^{(3*n)*\text{PolyLog}[3, I*E^{(I*(c + d*x^n))}]})}{(d^3*e*n*x^{(3*n)})} + \frac{(b^2*(e*x)^{(3*n)*\text{Tan}[c + d*x^n])}{(d*e*n*x^n)}$

**Rule 2190**

$\text{Int}[(((F_)^\wedge((g_) * ((e_) + (f_) * (x_)))^\wedge(n_) * ((c_) + (d_) * (x_))^\wedge(m_)) / ((a_) + (b_) * ((F_)^\wedge((g_) * ((e_) + (f_) * (x_)))^\wedge(n_)), x\_Symbol] :> \text{Simp} [((c + d*x)^\wedge m * \text{Log}[1 + (b*(F^\wedge(g*(e + f*x)))^\wedge n] / a)] / (b*f*g*n * \text{Log}[F]), x] - \text{Dist} [(d*m) / (b*f*g*n * \text{Log}[F]), \text{Int} [(c + d*x)^\wedge (m - 1) * \text{Log}[1 + (b*(F^\wedge(g*(e + f*x)))^\wedge n] / a], x], x] /; \text{FreeQ} [\{F, a, b, c, d, e, f, g, n\}, x] \&\& \text{IGtQ} [m, 0]$

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_)))]^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)^v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_)))]^(n_.)]*((f_.) + (g_.)
*(x_)^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
))))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
+ f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := -Sim
```

$p[((c + d*x)^m * \cot[e + f*x])/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)} * \cot[e + f*x], x], x] /;$  FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

#### Rule 4190

$\text{Int}[(\csc[e] + (f*x)*b) + (a)^{(n)}*((c) + (d)*(x))^m], x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(c + d*x)^m, (a + b*\csc[e + f*x])^n, x], x] /;$  FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]

#### Rule 4204

$\text{Int}[(x)^m*((a) + (b)*\sec[(c) + (d)*(x)^n])^p], x\_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m+1)/n) - 1}*(a + b*\sec[c + d*x])^p}, x], x, x^n], x] /;$  FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m+1)/n], 0] && IntegerQ[p]

#### Rule 4208

$\text{Int}[(e*(x))^m*((a) + (b)*\sec[(c) + (d)*(x)^n])^p], x\_Symbol] \rightarrow \text{Dist}[(e^{\text{IntPart}[m]}*(e*x)^{\text{FracPart}[m]})/x^{\text{FracPart}[m]}, \text{Int}[x^m*(a + b*\sec[c + d*x^n])^p, x], x] /;$  FreeQ[{a, b, c, d, e, m, n, p}, x]

#### Rule 6589

$\text{Int}[\text{PolyLog}[n, (c)*((a) + (b)*(x))^p]/((d) + (e)*(x)), x\_Symbol] \rightarrow \text{Simp}[\text{PolyLog}[n + 1, c*(a + b*x)^p]/(e*p), x] /;$  FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

#### Rubi steps

$$\begin{aligned}
\int (ex)^{-1+3n} (a + b \sec(c + dx^n))^2 dx &= \frac{(x^{-3n}(ex)^{3n}) \int x^{-1+3n} (a + b \sec(c + dx^n))^2 dx}{e} \\
&= \frac{(x^{-3n}(ex)^{3n}) \text{Subst}\left(\int x^2 (a + b \sec(c + dx))^2 dx, x, x^n\right)}{en} \\
&= \frac{(x^{-3n}(ex)^{3n}) \text{Subst}\left(\int (a^2 x^2 + 2abx^2 \sec(c + dx) + b^2 x^2 \sec^2(c + dx)) dx, x, x^n\right)}{en} \\
&= \frac{a^2 (ex)^{3n}}{3en} + \frac{(2abx^{-3n}(ex)^{3n}) \text{Subst}\left(\int x^2 \sec(c + dx) dx, x, x^n\right)}{en} + \frac{(b^2 x^{-3n}(ex)^{3n}) \text{Subst}\left(\int x^2 \sec^2(c + dx) dx, x, x^n\right)}{en} \\
&= \frac{a^2 (ex)^{3n}}{3en} - \frac{4iabx^{-n}(ex)^{3n} \tan^{-1}\left(e^{i(c+dx^n)}\right)}{den} + \frac{b^2 x^{-n}(ex)^{3n} \tan(c + dx^n)}{den} \\
&= \frac{a^2 (ex)^{3n}}{3en} - \frac{ib^2 x^{-n}(ex)^{3n}}{den} - \frac{4iabx^{-n}(ex)^{3n} \tan^{-1}\left(e^{i(c+dx^n)}\right)}{den} + \frac{4iabx^{-2n}(ex)^{3n}}{den} \\
&= \frac{a^2 (ex)^{3n}}{3en} - \frac{ib^2 x^{-n}(ex)^{3n}}{den} - \frac{4iabx^{-n}(ex)^{3n} \tan^{-1}\left(e^{i(c+dx^n)}\right)}{den} + \frac{2b^2 x^{-2n}(ex)^{3n}}{den} \\
&= \frac{a^2 (ex)^{3n}}{3en} - \frac{ib^2 x^{-n}(ex)^{3n}}{den} - \frac{4iabx^{-n}(ex)^{3n} \tan^{-1}\left(e^{i(c+dx^n)}\right)}{den} + \frac{2b^2 x^{-2n}(ex)^{3n}}{den} \\
&= \frac{a^2 (ex)^{3n}}{3en} - \frac{ib^2 x^{-n}(ex)^{3n}}{den} - \frac{4iabx^{-n}(ex)^{3n} \tan^{-1}\left(e^{i(c+dx^n)}\right)}{den} + \frac{2b^2 x^{-2n}(ex)^{3n}}{den}
\end{aligned}$$

**Mathematica** [F] time = 11.66, size = 0, normalized size = 0.00

$$\int (ex)^{-1+3n} (a + b \sec(c + dx^n))^2 dx$$

Verification is Not applicable to the result.

[In] Integrate[(e\*x)^(-1 + 3\*n)\*(a + b\*Sec[c + d\*x^n])^2,x]

[Out] Integrate[(e\*x)^(-1 + 3\*n)\*(a + b\*Sec[c + d\*x^n])^2, x]

**fricas** [C] time = 1.28, size = 1028, normalized size = 2.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^(-1+3\*n)\*(a+b\*sec(c+d\*x^n))^2,x, algorithm="fricas")

```
[Out] 1/3*(a^2*d^3*e^(3*n - 1)*x^(3*n)*cos(d*x^n + c) + 3*b^2*d^2*e^(3*n - 1)*x^(2*n)*sin(d*x^n + c) - 6*a*b*e^(3*n - 1)*cos(d*x^n + c)*polylog(3, I*cos(d*x^n + c) + sin(d*x^n + c)) + 6*a*b*e^(3*n - 1)*cos(d*x^n + c)*polylog(3, I*cos(d*x^n + c) - sin(d*x^n + c)) - 6*a*b*e^(3*n - 1)*cos(d*x^n + c)*polylog(3, -I*cos(d*x^n + c) + sin(d*x^n + c)) + 6*a*b*e^(3*n - 1)*cos(d*x^n + c)*polylog(3, -I*cos(d*x^n + c) - sin(d*x^n + c)) + 3*(a*b*c^2 - b^2*c)*e^(3*n - 1)*cos(d*x^n + c)*log(cos(d*x^n + c) + I*sin(d*x^n + c) + I) - 3*(a*b*c^2 + b^2*c)*e^(3*n - 1)*cos(d*x^n + c)*log(cos(d*x^n + c) - I*sin(d*x^n + c) + I) + 3*(a*b*c^2 - b^2*c)*e^(3*n - 1)*cos(d*x^n + c)*log(-cos(d*x^n + c) + I*sin(d*x^n + c) + I) - 3*(a*b*c^2 + b^2*c)*e^(3*n - 1)*cos(d*x^n + c)*log(-cos(d*x^n + c) - I*sin(d*x^n + c) + I) + (-6*I*a*b*d*e^(3*n - 1)*x^n + 3*I*b^2*e^(3*n - 1))*cos(d*x^n + c)*dilog(I*cos(d*x^n + c) + sin(d*x^n + c)) + (-6*I*a*b*d*e^(3*n - 1)*x^n - 3*I*b^2*e^(3*n - 1))*cos(d*x^n + c)*dilog(I*cos(d*x^n + c) - sin(d*x^n + c)) + (6*I*a*b*d*e^(3*n - 1)*x^n - 3*I*b^2*e^(3*n - 1))*cos(d*x^n + c)*dilog(-I*cos(d*x^n + c) + sin(d*x^n + c)) + (6*I*a*b*d*e^(3*n - 1)*x^n + 3*I*b^2*e^(3*n - 1))*cos(d*x^n + c)*dilog(-I*cos(d*x^n + c) - sin(d*x^n + c)) + 3*(a*b*d^2*e^(3*n - 1)*x^(2*n) + b^2*d*e^(3*n - 1)*x^n - (a*b*c^2 - b^2*c)*e^(3*n - 1))*cos(d*x^n + c)*log(I*cos(d*x^n + c) + sin(d*x^n + c) + 1) - 3*(a*b*d^2*e^(3*n - 1)*x^(2*n) - b^2*d*e^(3*n - 1)*x^n - (a*b*c^2 + b^2*c)*e^(3*n - 1))*cos(d*x^n + c)*log(I*cos(d*x^n + c) - sin(d*x^n + c) + 1) + 3*(a*b*d^2*e^(3*n - 1)*x^(2*n) + b^2*d*e^(3*n - 1)*x^n - (a*b*c^2 - b^2*c)*e^(3*n - 1))*cos(d*x^n + c)*log(-I*cos(d*x^n + c) + sin(d*x^n + c) + 1) - 3*(a*b*d^2*e^(3*n - 1)*x^(2*n) - b^2*d*e^(3*n - 1)*x^n - (a*b*c^2 + b^2*c)*e^(3*n - 1))*cos(d*x^n + c)*log(-I*cos(d*x^n + c) - sin(d*x^n + c) + 1))/(d^3*n*cos(d*x^n + c))
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sec(dx^n + c) + a)^2 (ex)^{3n-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+3*n)*(a+b*sec(c+d*x^n))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x^n + c) + a)^2*(e*x)^(3*n - 1), x)
```

**maple** [F] time = 3.02, size = 0, normalized size = 0.00

$$\int (ex)^{-1+3n} (a + b \sec(c + dx^n))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(-1+3*n)*(a+b*sec(c+d*x^n))^2,x)
```

```
[Out] int((e*x)^(-1+3*n)*(a+b*sec(c+d*x^n))^2,x)
```



**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(ex)^{3n} a^2}{3en} + \frac{2b^2 e^{3n} x^{2n} \sin(2dx^n + 2c) + 4 \left( den \cos(2dx^n + 2c)^2 + den \sin(2dx^n + 2c)^2 + 2 den \cos(2dx^n + 2c) \right)}{den \cos(2dx^n + 2c)^2 + den \sin(2dx^n + 2c)^2 + 2 den \cos(2dx^n + 2c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^(-1+3\*n)\*(a+b\*sec(c+d\*x^n))^2,x, algorithm="maxima")

[Out] 1/3\*(e\*x)^(3\*n)\*a^2/(e\*n) + (2\*b^2\*e^(3\*n)\*x^(2\*n)\*sin(2\*d\*x^n + 2\*c) + (d\*e\*n\*cos(2\*d\*x^n + 2\*c)^2 + d\*e\*n\*sin(2\*d\*x^n + 2\*c)^2 + 2\*d\*e\*n\*cos(2\*d\*x^n + 2\*c) + d\*e\*n)\*integrate(4\*(a\*b\*d\*e^(3\*n)\*x^(3\*n)\*cos(2\*d\*x^n + 2\*c)\*cos(d\*x^n + c) + a\*b\*d\*e^(3\*n)\*x^(3\*n)\*cos(d\*x^n + c) + (a\*b\*d\*e^(3\*n)\*x^(3\*n)\*sin(d\*x^n + c) - b^2\*e^(3\*n)\*x^(2\*n))\*sin(2\*d\*x^n + 2\*c))/(d\*e\*x\*cos(2\*d\*x^n + 2\*c)^2 + d\*e\*x\*sin(2\*d\*x^n + 2\*c)^2 + 2\*d\*e\*x\*cos(2\*d\*x^n + 2\*c) + d\*e\*x), x))/(d\*e\*n\*cos(2\*d\*x^n + 2\*c)^2 + d\*e\*n\*sin(2\*d\*x^n + 2\*c)^2 + 2\*d\*e\*n\*cos(2\*d\*x^n + 2\*c) + d\*e\*n)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left( a + \frac{b}{\cos(c + dx^n)} \right)^2 (ex)^{3n-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cos(c + d\*x^n))^2\*(e\*x)^(3\*n - 1), x)

[Out] int((a + b/cos(c + d\*x^n))^2\*(e\*x)^(3\*n - 1), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^{3n-1} (a + b \sec(c + dx^n))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)\*\*(-1+3\*n)\*(a+b\*sec(c+d\*x\*\*n))\*\*2,x)

[Out] Integral((e\*x)\*\*(3\*n - 1)\*(a + b\*sec(c + d\*x\*\*n))\*\*2, x)

$$3.78 \quad \int \frac{(ex)^{-1+n}}{a+b \sec(c+dx^n)} dx$$

Optimal. Leaf size=87

$$\frac{(ex)^n}{aen} - \frac{2bx^{-n}(ex)^n \tanh^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx^n)\right)}{\sqrt{a+b}}\right)}{aden\sqrt{a-b}\sqrt{a+b}}$$

[Out]  $(e*x)^n/a/e/n-2*b*(e*x)^n*\operatorname{arctanh}((a-b)^{(1/2)}*\tan(1/2*c+1/2*d*x^n)/(a+b)^{(1/2}))/a/d/e/n/(x^n)/(a-b)^{(1/2)}/(a+b)^{(1/2)}$

**Rubi [A]** time = 0.15, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$ , Rules used = {4208, 4204, 3783, 2659, 208}

$$\frac{(ex)^n}{aen} - \frac{2bx^{-n}(ex)^n \tanh^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx^n)\right)}{\sqrt{a+b}}\right)}{aden\sqrt{a-b}\sqrt{a+b}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(e*x)^{-1+n}/(a+b*\operatorname{Sec}[c+d*x^n]),x]$

[Out]  $(e*x)^n/(a*e*n) - (2*b*(e*x)^n*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a-b]*\operatorname{Tan}[(c+d*x^n)/2])/(\operatorname{Sqrt}[a+b])]/(a*\operatorname{Sqrt}[a-b]*\operatorname{Sqrt}[a+b]*d*e*n*x^n)$

### Rule 208

$\operatorname{Int}[(a_+ + (b_+)*(x_+)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-(a/b), 2]*\operatorname{ArcTanh}[x/\operatorname{Rt}[-(a/b), 2]])/a, x] /; \operatorname{FreeQ}\{a, b\}, x] \ \&\& \operatorname{NegQ}[a/b]$

### Rule 2659

$\operatorname{Int}[(a_+ + (b_+)*\sin[\operatorname{Pi}/2 + (c_+) + (d_+)*(x_+)])^{-1}, x\_Symbol] \rightarrow \operatorname{With}\{e = \operatorname{FreeFactors}[\operatorname{Tan}[(c+d*x)/2], x], \operatorname{Dist}[(2*e)/d, \operatorname{Subst}[\operatorname{Int}[1/(a+b+(a-b)*e^2*x^2), x], x, \operatorname{Tan}[(c+d*x)/2]/e], x]\} /; \operatorname{FreeQ}\{a, b, c, d\}, x] \ \&\& \operatorname{NeQ}[a^2 - b^2, 0]$

### Rule 3783

$\operatorname{Int}[(\operatorname{csc}[(c_+) + (d_+)*(x_+)]*(b_+) + (a_+))^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[x/a, x] - \operatorname{Dist}[1/a, \operatorname{Int}[1/(1+(a*\sin[c+d*x])/b), x], x] /; \operatorname{FreeQ}\{a, b, c, d\}, x] \ \&\& \operatorname{NeQ}[a^2 - b^2, 0]$

Rule 4204

Int[(x\_)^(m\_.)\*((a\_.) + (b\_.)\*Sec[(c\_.) + (d\_.)\*(x\_)^(n\_.)])^(p\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*Sec[c + d\*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]

Rule 4208

Int[((e\_)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*Sec[(c\_.) + (d\_.)\*(x\_)^(n\_.)])^(p\_.), x\_Symbol] :> Dist[(e^IntPart[m]\*(e\*x)^FracPart[m])/x^FracPart[m], Int[x^m\*(a + b\*Sec[c + d\*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]

Rubi steps

$$\begin{aligned}
 \int \frac{(ex)^{-1+n}}{a + b \sec(c + dx^n)} dx &= \frac{(x^{-n}(ex)^n) \int \frac{x^{-1+n}}{a+b \sec(c+dx^n)} dx}{e} \\
 &= \frac{(x^{-n}(ex)^n) \text{Subst}\left(\int \frac{1}{a+b \sec(c+dx)} dx, x, x^n\right)}{en} \\
 &= \frac{(ex)^n}{aen} - \frac{(x^{-n}(ex)^n) \text{Subst}\left(\int \frac{1}{1+\frac{a \cos(c+dx)}{b}} dx, x, x^n\right)}{aen} \\
 &= \frac{(ex)^n}{aen} - \frac{(2x^{-n}(ex)^n) \text{Subst}\left(\int \frac{1}{1+\frac{a}{b}+(1-\frac{a}{b})x^2} dx, x, \tan\left(\frac{1}{2}(c + dx^n)\right)\right)}{aden} \\
 &= \frac{(ex)^n}{aen} - \frac{2bx^{-n}(ex)^n \tanh^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx^n)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}den}
 \end{aligned}$$

Mathematica [A] time = 0.29, size = 80, normalized size = 0.92

$$\frac{(ex)^n \left( \frac{2bx^{-n} \tanh^{-1}\left(\frac{(b-a) \tan\left(\frac{1}{2}(c+dx^n)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + cx^{-n} + d \right)}{aden}$$

Antiderivative was successfully verified.

[In] Integrate[(e\*x)^(-1 + n)/(a + b\*Sec[c + d\*x^n]), x]

[Out] ((e\*x)^n\*(d + c/x^n + (2\*b\*ArcTanh[((-a + b)\*Tan[(c + d\*x^n)/2])]/Sqrt[a^2 - b^2]))/(Sqrt[a^2 - b^2]\*x^n))/(a\*d\*e^n)

**fricas** [A] time = 0.64, size = 300, normalized size = 3.45

$$\frac{2(a^2 - b^2)de^{n-1}x^n + \sqrt{a^2 - b^2}be^{n-1} \log\left(\frac{2ab \cos(dx^n+c) - (a^2 - 2b^2) \cos(dx^n+c)^2 + 2a^2 - b^2 - 2(\sqrt{a^2 - b^2}b \cos(dx^n+c) + \sqrt{a^2 - b^2}a) \sin(dx^n+c)}{a^2 \cos(dx^n+c)^2 + 2ab \cos(dx^n+c) + b^2}\right)}{2(a^3 - ab^2)dn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^(-1+n)/(a+b\*sec(c+d\*x^n)), x, algorithm="fricas")

[Out] [1/2\*(2\*(a^2 - b^2)\*d\*e^(n - 1)\*x^n + sqrt(a^2 - b^2)\*b\*e^(n - 1)\*log((2\*a\*b\*cos(d\*x^n + c) - (a^2 - 2\*b^2)\*cos(d\*x^n + c)^2 + 2\*a^2 - b^2 - 2\*(sqrt(a^2 - b^2)\*b\*cos(d\*x^n + c) + sqrt(a^2 - b^2)\*a)\*sin(d\*x^n + c))/(a^2\*cos(d\*x^n + c)^2 + 2\*a\*b\*cos(d\*x^n + c) + b^2)))/((a^3 - a\*b^2)\*d\*n), ((a^2 - b^2)\*d\*e^(n - 1)\*x^n - sqrt(-a^2 + b^2)\*b\*e^(n - 1)\*arctan(-(sqrt(-a^2 + b^2)\*b\*cos(d\*x^n + c) + sqrt(-a^2 + b^2)\*a)/((a^2 - b^2)\*sin(d\*x^n + c))))/((a^3 - a\*b^2)\*d\*n)]

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^{n-1}}{b \sec(dx^n + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^(-1+n)/(a+b\*sec(c+d\*x^n)), x, algorithm="giac")

[Out] integrate((e\*x)^(n - 1)/(b\*sec(d\*x^n + c) + a), x)

**maple** [C] time = 1.07, size = 314, normalized size = 3.61

$$\frac{x e^{\frac{(-1+n)(-i\pi \operatorname{csgn}(ie) \operatorname{csgn}(ix) \operatorname{csgn}(iex) + i\pi \operatorname{csgn}(ie) \operatorname{csgn}(iex)^2 + i\pi \operatorname{csgn}(ix) \operatorname{csgn}(iex)^2 - i\pi \operatorname{csgn}(iex)^3 + 2 \ln(x) + 2 \ln(e))}{2}}}{an} + \frac{2i \arctan\left(\frac{2ae^{i(dx^n+2c)} + 2e^{ic}b}{2\sqrt{a^2e^{2ic} - e^{2ic}b^2}}\right) e^n b e^{\frac{i(-\pi}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x)^(-1+n)/(a+b\*sec(c+d\*x^n)), x)

```
[Out] 1/a/n*x*exp(1/2*(-1+n)*(-I*Pi*csgn(I*e)*csgn(I*x)*csgn(I*e*x)+I*Pi*csgn(I*e)
)*csgn(I*e*x)^2+I*Pi*csgn(I*x)*csgn(I*e*x)^2-I*Pi*csgn(I*e*x)^3+2*ln(x)+2*ln
n(e))+2*I*arctan(1/2*(2*a*exp(I*(d*x^n+2*c))+2*exp(I*c)*b)/(a^2*exp(2*I*c)
-exp(2*I*c)*b^2)^(1/2))/(a^2*exp(2*I*c)-exp(2*I*c)*b^2)^(1/2)/d/e*e^n/n*b/a
*exp(1/2*I*(-Pi*n*csgn(I*e)*csgn(I*x)*csgn(I*e*x)+Pi*n*csgn(I*e)*csgn(I*e*x)
)^2+Pi*n*csgn(I*x)*csgn(I*e*x)^2-Pi*n*csgn(I*e*x)^3+Pi*csgn(I*e*x)*csgn(I*e
)*csgn(I*x)-Pi*csgn(I*e)*csgn(I*e*x)^2-Pi*csgn(I*x)*csgn(I*e*x)^2+Pi*csgn(I
*e*x)^3+2*c))
```

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+n)/(a+b*sec(c+d*x^n)),x, algorithm="maxima")
```

```
[Out] Timed out
```

**mupad** [B] time = 2.75, size = 223, normalized size = 2.56

$$\frac{x(e x)^{n-1}}{a n} + \frac{b x \ln \left( 2 b x e^{c 1 i} e^{d x^n 1 i} (e x)^{n-1} - \frac{b x (a+b e^{c 1 i} e^{d x^n 1 i}) (e x)^{n-1} 2 i}{\sqrt{a+b} \sqrt{a-b}} \right) (e x)^{n-1}}{a d n x^n \sqrt{a+b} \sqrt{a-b}} - \frac{b x \ln \left( 2 b x e^{c 1 i} e^{d x^n 1 i} (e x)^{n-1} + \frac{b x (a+b e^{c 1 i} e^{d x^n 1 i}) (e x)^{n-1} 2 i}{\sqrt{a+b} \sqrt{a-b}} \right) (e x)^{n-1}}{a d n x^n \sqrt{a+b} \sqrt{a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(n-1)/(a+b/cos(c+d*x^n)),x)
```

```
[Out] (x*(e*x)^(n-1))/(a*n) + (b*x*log(2*b*x*exp(c*1i)*exp(d*x^n*1i)*(e*x)^(n-1)
- (b*x*(a+b*exp(c*1i)*exp(d*x^n*1i))*(e*x)^(n-1)*2i)/((a+b)^(1/2)
*(a-b)^(1/2)))*(e*x)^(n-1))/(a*d*n*x^n*(a+b)^(1/2)*(a-b)^(1/2)) - (
b*x*log(2*b*x*exp(c*1i)*exp(d*x^n*1i)*(e*x)^(n-1) + (b*x*(a+b*exp(c*1i)
*exp(d*x^n*1i))*(e*x)^(n-1)*2i)/((a+b)^(1/2)*(a-b)^(1/2)))*(e*x)^(n-1)
)/(a*d*n*x^n*(a+b)^(1/2)*(a-b)^(1/2))
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^{n-1}}{a + b \sec(c + dx^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(-1+n)/(a+b*sec(c+d*x**n)),x)
```

```
[Out] Integral((e*x)**(n-1)/(a+b*sec(c+d*x**n)),x)
```

$$3.79 \quad \int \frac{(ex)^{-1+2n}}{a+b \sec(c+dx^n)} dx$$

**Optimal.** Leaf size=328

$$\frac{bx^{-2n}(ex)^{2n} \operatorname{Li}_2\left(-\frac{ae^{i(dx^n+c)}}{b-\sqrt{b^2-a^2}}\right)}{ad^2en\sqrt{b^2-a^2}} - \frac{bx^{-2n}(ex)^{2n} \operatorname{Li}_2\left(-\frac{ae^{i(dx^n+c)}}{b+\sqrt{b^2-a^2}}\right)}{ad^2en\sqrt{b^2-a^2}} + \frac{ibx^{-n}(ex)^{2n} \log\left(1 + \frac{ae^{i(c+dx^n)}}{b-\sqrt{b^2-a^2}}\right)}{aden\sqrt{b^2-a^2}} - \frac{ibx^{-n}(ex)^{2n} \log\left(1 + \frac{ae^{i(c+dx^n)}}{b+\sqrt{b^2-a^2}}\right)}{aden\sqrt{b^2-a^2}}$$

[Out]  $\frac{1}{2} * (e*x)^{(2*n)} / a / e / n + I * b * (e*x)^{(2*n)} * \ln(1 + a * \exp(I * (c + d*x^n))) / (b - (-a^2 + b^2)^{(1/2)}) / a / d / e / n / (x^n) / (-a^2 + b^2)^{(1/2)} - I * b * (e*x)^{(2*n)} * \ln(1 + a * \exp(I * (c + d*x^n))) / (b + (-a^2 + b^2)^{(1/2)}) / a / d / e / n / (x^n) / (-a^2 + b^2)^{(1/2)} + b * (e*x)^{(2*n)} * \operatorname{polylog}(2, -a * \exp(I * (c + d*x^n))) / (b - (-a^2 + b^2)^{(1/2)}) / a / d^2 / e / n / (x^{(2*n)}) / (-a^2 + b^2)^{(1/2)} - b * (e*x)^{(2*n)} * \operatorname{polylog}(2, -a * \exp(I * (c + d*x^n))) / (b + (-a^2 + b^2)^{(1/2)}) / a / d^2 / e / n / (x^{(2*n)}) / (-a^2 + b^2)^{(1/2)}$

**Rubi [A]** time = 0.60, antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {4208, 4204, 4191, 3321, 2264, 2190, 2279, 2391}

$$\frac{bx^{-2n}(ex)^{2n} \operatorname{PolyLog}\left(2, -\frac{ae^{i(c+dx^n)}}{b-\sqrt{b^2-a^2}}\right)}{ad^2en\sqrt{b^2-a^2}} - \frac{bx^{-2n}(ex)^{2n} \operatorname{PolyLog}\left(2, -\frac{ae^{i(c+dx^n)}}{\sqrt{b^2-a^2}+b}\right)}{ad^2en\sqrt{b^2-a^2}} + \frac{ibx^{-n}(ex)^{2n} \log\left(1 + \frac{ae^{i(c+dx^n)}}{b-\sqrt{b^2-a^2}}\right)}{aden\sqrt{b^2-a^2}} - \frac{ibx^{-n}(ex)^{2n} \log\left(1 + \frac{ae^{i(c+dx^n)}}{b+\sqrt{b^2-a^2}}\right)}{aden\sqrt{b^2-a^2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(e*x)^{-1+2*n}/(a+b*\operatorname{Sec}[c+d*x^n]),x]$

[Out]  $(e*x)^{(2*n)} / (2*a*e*n) + (I*b*(e*x)^{(2*n)} * \operatorname{Log}[1 + (a*E^{(I*(c+d*x^n))}) / (b - \operatorname{Sqrt}[-a^2 + b^2])]) / (a*\operatorname{Sqrt}[-a^2 + b^2]*d*e*n*x^n) - (I*b*(e*x)^{(2*n)} * \operatorname{Log}[1 + (a*E^{(I*(c+d*x^n))}) / (b + \operatorname{Sqrt}[-a^2 + b^2])]) / (a*\operatorname{Sqrt}[-a^2 + b^2]*d*e*n*x^n) + (b*(e*x)^{(2*n)} * \operatorname{PolyLog}[2, -((a*E^{(I*(c+d*x^n))}) / (b - \operatorname{Sqrt}[-a^2 + b^2]))]) / (a*\operatorname{Sqrt}[-a^2 + b^2]*d^2*e*n*x^{(2*n)}) - (b*(e*x)^{(2*n)} * \operatorname{PolyLog}[2, -((a*E^{(I*(c+d*x^n))}) / (b + \operatorname{Sqrt}[-a^2 + b^2]))]) / (a*\operatorname{Sqrt}[-a^2 + b^2]*d^2*e*n*x^{(2*n)})$

**Rule 2190**

$\operatorname{Int}[(((F_) ^ ((g_) * ((e_) + (f_) * (x_))) ^ (n_) * ((c_) + (d_) * (x_)) ^ (m_)) / ((a_) + (b_) * ((F_) ^ ((g_) * ((e_) + (f_) * (x_))) ^ (n_))), x\_Symbol] \rightarrow \operatorname{Simp} [((c + d*x)^m * \operatorname{Log}[1 + (b*(F^(g*(e + f*x)))^n)/a]] / (b*f*g*n*\operatorname{Log}[F]), x] - \operatorname{Dist} [(d*m) / (b*f*g*n*\operatorname{Log}[F]), \operatorname{Int} [(c + d*x)^(m-1) * \operatorname{Log}[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /;$  FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

**Rule 2264**

```
Int[((F_)^(u_)*((f_) + (g_)*(x_))^(m_))/((a_) + (b_)*(F_)^(u_) + (c_)
*(F_)^(v_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

### Rule 2279

```
Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2391

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 3321

```
Int[((c_) + (d_)*(x_))^(m_)/((a_) + (b_)*sin[(e_) + Pi*(k_) + (f_)*(
x_)]), x_Symbol] := Dist[2, Int[((c + d*x)^m*E^(I*Pi*(k - 1/2))*E^(I*(e + f
*x)))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(
e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

### Rule 4191

```
Int[(csc[(e_) + (f_)*(x_)]*(b_) + (a_))^(n_)*((c_) + (d_)*(x_))^(m_)
, x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

### Rule 4204

```
Int[(x_)^(m_)*((a_) + (b_)*Sec[(c_) + (d_)*(x_)^(n_)])^(p_), x_Symbol
] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

### Rule 4208

```
Int[((e_)*(x_))^(m_)*((a_) + (b_)*Sec[(c_) + (d_)*(x_)^(n_)])^(p_), x
_Symbol] := Dist[(e^IntPart[m]*(e*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a
+ b*Sec[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(ex)^{-1+2n}}{a + b \sec(c + dx^n)} dx &= \frac{(x^{-2n}(ex)^{2n}) \int \frac{x^{-1+2n}}{a+b \sec(c+dx^n)} dx}{e} \\
&= \frac{(x^{-2n}(ex)^{2n}) \text{Subst} \left( \int \frac{x}{a+b \sec(c+dx)} dx, x, x^n \right)}{en} \\
&= \frac{(x^{-2n}(ex)^{2n}) \text{Subst} \left( \int \left( \frac{x}{a} - \frac{bx}{a(b+a \cos(c+dx))} \right) dx, x, x^n \right)}{en} \\
&= \frac{(ex)^{2n}}{2aen} - \frac{(bx^{-2n}(ex)^{2n}) \text{Subst} \left( \int \frac{x}{b+a \cos(c+dx)} dx, x, x^n \right)}{aen} \\
&= \frac{(ex)^{2n}}{2aen} - \frac{(2bx^{-2n}(ex)^{2n}) \text{Subst} \left( \int \frac{e^{i(c+dx)}x}{a+2be^{i(c+dx)}+ae^{2i(c+dx)}} dx, x, x^n \right)}{aen} \\
&= \frac{(ex)^{2n}}{2aen} - \frac{(2bx^{-2n}(ex)^{2n}) \text{Subst} \left( \int \frac{e^{i(c+dx)}x}{2b-2\sqrt{-a^2+b^2}+2ae^{i(c+dx)}} dx, x, x^n \right)}{\sqrt{-a^2+b^2} en} + \frac{(2bx^{-2n}(ex)^{2n}) \text{Subst} \left( \int \frac{e^{i(c+dx)}x}{2b+2\sqrt{-a^2+b^2}+2ae^{i(c+dx)}} dx, x, x^n \right)}{\sqrt{-a^2+b^2} en} \\
&= \frac{(ex)^{2n}}{2aen} + \frac{ibx^{-n}(ex)^{2n} \log \left( 1 + \frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}} \right)}{a\sqrt{-a^2+b^2} den} - \frac{ibx^{-n}(ex)^{2n} \log \left( 1 + \frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}} \right)}{a\sqrt{-a^2+b^2} den} - \frac{(ibx^{-2n}(ex)^{2n}) \log \left( 1 + \frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}} \right)}{a\sqrt{-a^2+b^2} den} + \frac{(ibx^{-2n}(ex)^{2n}) \log \left( 1 + \frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}} \right)}{a\sqrt{-a^2+b^2} den} \\
&= \frac{(ex)^{2n}}{2aen} + \frac{ibx^{-n}(ex)^{2n} \log \left( 1 + \frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}} \right)}{a\sqrt{-a^2+b^2} den} - \frac{ibx^{-n}(ex)^{2n} \log \left( 1 + \frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}} \right)}{a\sqrt{-a^2+b^2} den} - \frac{(bx^{-2n}(ex)^{2n}) \log \left( 1 + \frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}} \right)}{a\sqrt{-a^2+b^2} den} + \frac{(bx^{-2n}(ex)^{2n}) \log \left( 1 + \frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}} \right)}{a\sqrt{-a^2+b^2} den}
\end{aligned}$$

**Mathematica [B]** time = 1.95, size = 861, normalized size = 2.62

$$\frac{(ex)^{2n} (b + a \cos(dx^n + c)) \left( 1 - \frac{2bx^{-2n} \left( 2(dx^n + c) \tanh^{-1} \left( \frac{(a+b) \cot \left( \frac{1}{2}(dx^n + c) \right)}{\sqrt{a^2 - b^2}} \right) \right) - 2 \left( c + \cos^{-1} \left( -\frac{b}{a} \right) \right) \tanh^{-1} \left( \frac{(a-b) \tan \left( \frac{1}{2}(dx^n + c) \right)}{\sqrt{a^2 - b^2}} \right) + \left( \cos^{-1} \left( -\frac{b}{a} \right) - 2 \right) \right)}{2a\sqrt{-a^2+b^2} den}$$

Antiderivative was successfully verified.



[In] Integrate[(e\*x)^(-1 + 2\*n)/(a + b\*Sec[c + d\*x^n]),x]

[Out]  $((e*x)^{(2*n)}*(b + a*\cos[c + d*x^n])*(1 - (2*b*(2*(c + d*x^n)*\operatorname{ArcTanh}[\frac{(a + b)*\cot[(c + d*x^n)/2]}{\sqrt{a^2 - b^2}}] - 2*(c + \operatorname{ArcCos}[-(b/a)])*\operatorname{ArcTanh}[\frac{(a - b)*\tan[(c + d*x^n)/2]}{\sqrt{a^2 - b^2}}] + (\operatorname{ArcCos}[-(b/a)] - (2*I)*\operatorname{ArcTanh}[\frac{(a + b)*\cot[(c + d*x^n)/2]}{\sqrt{a^2 - b^2}}] + (2*I)*\operatorname{ArcTanh}[\frac{(a - b)*\tan[(c + d*x^n)/2]}{\sqrt{a^2 - b^2}}])*\log[\frac{\sqrt{a^2 - b^2}}{\sqrt{2}*\sqrt{a}*e^{((I/2)*(c + d*x^n))*\sqrt{b + a*\cos[c + d*x^n]}}] + (\operatorname{ArcCos}[-(b/a)] + (2*I)*(\operatorname{ArcTanh}[\frac{(a + b)*\cot[(c + d*x^n)/2]}{\sqrt{a^2 - b^2}}] - \operatorname{ArcTanh}[\frac{(a - b)*\tan[(c + d*x^n)/2]}{\sqrt{a^2 - b^2}}])*\log[(\sqrt{a^2 - b^2})*e^{((I/2)*(c + d*x^n))}/(\sqrt{2}*\sqrt{a}*\sqrt{b + a*\cos[c + d*x^n]})] - (\operatorname{ArcCos}[-(b/a)] - (2*I)*\operatorname{ArcTanh}[\frac{(a - b)*\tan[(c + d*x^n)/2]}{\sqrt{a^2 - b^2}}])*\log[(a + b)*(a - b - I*\sqrt{a^2 - b^2})*(1 + I*\tan[(c + d*x^n)/2])]/(a*(a + b + \sqrt{a^2 - b^2})*\tan[(c + d*x^n)/2])) - (\operatorname{ArcCos}[-(b/a)] + (2*I)*\operatorname{ArcTanh}[\frac{(a - b)*\tan[(c + d*x^n)/2]}{\sqrt{a^2 - b^2}}])*\log[(a + b)*((-I)*a + I*b + \sqrt{a^2 - b^2})*(I + \tan[(c + d*x^n)/2])]/(a*(a + b + \sqrt{a^2 - b^2})*\tan[(c + d*x^n)/2])) + I*(\operatorname{PolyLog}[2, ((b - I*\sqrt{a^2 - b^2})*(a + b - \sqrt{a^2 - b^2})*\tan[(c + d*x^n)/2])]/(a*(a + b + \sqrt{a^2 - b^2})*\tan[(c + d*x^n)/2])) - \operatorname{PolyLog}[2, ((b + I*\sqrt{a^2 - b^2})*(a + b - \sqrt{a^2 - b^2})*\tan[(c + d*x^n)/2])]/(a*(a + b + \sqrt{a^2 - b^2})*\tan[(c + d*x^n)/2])))/(\sqrt{a^2 - b^2}*d^{2*x^{2*n}})*\operatorname{Sec}[c + d*x^n]/(2*a*e^n*(a + b*\operatorname{Sec}[c + d*x^n]))$

**fricas [B]** time = 0.71, size = 1291, normalized size = 3.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^(-1+2\*n)/(a+b\*sec(c+d\*x^n)),x, algorithm="fricas")

[Out]  $-1/4*(2*I*a*b*c*e^{(2*n - 1)*\sqrt{-(a^2 - b^2)/a^2}}*\log(2*a*\cos(d*x^n + c) + 2*I*a*\sin(d*x^n + c) + 2*a*\sqrt{-(a^2 - b^2)/a^2} + 2*b) - 2*I*a*b*c*e^{(2*n - 1)*\sqrt{-(a^2 - b^2)/a^2}}*\log(2*a*\cos(d*x^n + c) - 2*I*a*\sin(d*x^n + c) + 2*a*\sqrt{-(a^2 - b^2)/a^2} + 2*b) + 2*I*a*b*c*e^{(2*n - 1)*\sqrt{-(a^2 - b^2)/a^2}}*\log(-2*a*\cos(d*x^n + c) + 2*I*a*\sin(d*x^n + c) + 2*a*\sqrt{-(a^2 - b^2)/a^2} - 2*b) - 2*I*a*b*c*e^{(2*n - 1)*\sqrt{-(a^2 - b^2)/a^2}}*\log(-2*a*\cos(d*x^n + c) - 2*I*a*\sin(d*x^n + c) + 2*a*\sqrt{-(a^2 - b^2)/a^2} - 2*b) - 2*(a^2 - b^2)*d^2*e^{(2*n - 1)*x^{2*n}} + 2*a*b*e^{(2*n - 1)*\sqrt{-(a^2 - b^2)/a^2}}*\operatorname{dilog}(-1/2*(2*(a*\sqrt{-(a^2 - b^2)/a^2} + b)*\cos(d*x^n + c) + (2*I*a*\sqrt{-(a^2 - b^2)/a^2} + 2*I*b)*\sin(d*x^n + c) + 2*a)/a + 1) + 2*a*b*e^{(2*n - 1)*\sqrt{-(a^2 - b^2)/a^2}}*\operatorname{dilog}(-1/2*(2*(a*\sqrt{-(a^2 - b^2)/a^2} + b)*\cos(d*x^n + c) + (-2*I*a*\sqrt{-(a^2 - b^2)/a^2} - 2*I*b)*\sin(d*x^n + c) + 2*a)/a + 1) - 2*a*b*e^{(2*n - 1)*\sqrt{-(a^2 - b^2)/a^2}}*\operatorname{dilog}(1/2*(2*(a*\sqrt{-(a^2 - b^2)/a^2} - b)*\cos(d*x^n + c) - (2*I*a*\sqrt{-(a^2 - b^2)/a^2} - 2*I*b)*\sin(d*x^n + c) - 2*a)/a + 1) - 2*a*b*e^{(2*n - 1)*\sqrt{-(a^2 - b^2)/a^2}}*\operatorname{dilog}(1/2*(2*(a*\sqrt{-(a^2 - b^2)/a^2} - b)*\cos(d*x^n + c) - (-2*I*a*\sqrt{-(a^2 - b^2)/a^2} - 2*I*b)*\sin(d*x^n + c) - 2*a)/a + 1)$



$$\begin{aligned}
& n(I*x)*csgn(I*e*x))*exp(I*Pi*n*csgn(I*e)*csgn(I*e*x)^2)*exp(I*Pi*n*csgn(I*x) \\
& )*csgn(I*e*x)^2)*exp(-I*Pi*n*csgn(I*e*x)^3)*exp(1/2*I*Pi*csgn(I*e*x)^3)*exp \\
& (I*c)+1/d^2/n/e*(e^n)^2/a*b*dilog(a/(exp(I*c)*b-(exp(2*I*c)*b^2-a^2*exp(2*I \\
& *c))^(1/2))*exp(I*(d*x^n+2*c))+1/(exp(I*c)*b-(exp(2*I*c)*b^2-a^2*exp(2*I*c) \\
& )^(1/2))*exp(I*c)*b-1/(exp(I*c)*b-(exp(2*I*c)*b^2-a^2*exp(2*I*c))^(1/2))*(e \\
& xp(2*I*c)*b^2-a^2*exp(2*I*c))^(1/2))/(exp(2*I*c)*b^2-a^2*exp(2*I*c))^(1/2)* \\
& (-1)^(1/2*csgn(I*e)*csgn(I*x)*csgn(I*e*x))*(-1)^(-1/2*csgn(I*x)*csgn(I*e*x) \\
& ^2)*(-1)^(-1/2*csgn(I*e)*csgn(I*e*x)^2)*exp(-I*Pi*n*csgn(I*e)*csgn(I*x)*csg \\
& n(I*e*x))*exp(I*Pi*n*csgn(I*e)*csgn(I*e*x)^2)*exp(I*Pi*n*csgn(I*x)*csgn(I*e \\
& *x)^2)*exp(-I*Pi*n*csgn(I*e*x)^3)*exp(1/2*I*Pi*csgn(I*e*x)^3)*exp(I*c)-1/d^ \\
& 2/n/e*(e^n)^2/a*b*dilog(a/(exp(I*c)*b+(exp(2*I*c)*b^2-a^2*exp(2*I*c))^(1/2) \\
& )*exp(I*(d*x^n+2*c))+1/(exp(I*c)*b+(exp(2*I*c)*b^2-a^2*exp(2*I*c))^(1/2))*e \\
& xp(I*c)*b+1/(exp(I*c)*b+(exp(2*I*c)*b^2-a^2*exp(2*I*c))^(1/2))*(exp(2*I*c)* \\
& b^2-a^2*exp(2*I*c))^(1/2))/(exp(2*I*c)*b^2-a^2*exp(2*I*c))^(1/2)*(-1)^(1/2* \\
& csgn(I*e)*csgn(I*x)*csgn(I*e*x))*(-1)^(-1/2*csgn(I*x)*csgn(I*e*x)^2)*(-1)^( \\
& -1/2*csgn(I*e)*csgn(I*e*x)^2)*exp(-I*Pi*n*csgn(I*e)*csgn(I*x)*csgn(I*e*x))* \\
& exp(I*Pi*n*csgn(I*e)*csgn(I*e*x)^2)*exp(I*Pi*n*csgn(I*x)*csgn(I*e*x)^2)*exp \\
& (-I*Pi*n*csgn(I*e*x)^3)*exp(1/2*I*Pi*csgn(I*e*x)^3)*exp(I*c)
\end{aligned}$$

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^(-1+2\*n)/(a+b\*sec(c+d\*x^n)),x, algorithm="maxima")

[Out] Timed out

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(ex)^{2n-1}}{a + \frac{b}{\cos(c+dx^n)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x)^(2\*n - 1)/(a + b/cos(c + d\*x^n)),x)

[Out] int((e\*x)^(2\*n - 1)/(a + b/cos(c + d\*x^n)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^{2n-1}}{a + b \sec(c + dx^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(-1+2*n)/(a+b*sec(c+d*x**n)),x)
```

```
[Out] Integral((e*x)**(2*n - 1)/(a + b*sec(c + d*x**n)), x)
```

$$3.80 \quad \int \frac{(ex)^{-1+3n}}{a+b \sec(c+dx^n)} dx$$

**Optimal.** Leaf size=485

$$\frac{2ibx^{-3n}(ex)^{3n}\text{Li}_3\left(-\frac{ae^{i(dx^n+c)}}{b-\sqrt{b^2-a^2}}\right)}{ad^3en\sqrt{b^2-a^2}} - \frac{2ibx^{-3n}(ex)^{3n}\text{Li}_3\left(-\frac{ae^{i(dx^n+c)}}{b+\sqrt{b^2-a^2}}\right)}{ad^3en\sqrt{b^2-a^2}} + \frac{2bx^{-2n}(ex)^{3n}\text{Li}_2\left(-\frac{ae^{i(dx^n+c)}}{b-\sqrt{b^2-a^2}}\right)}{ad^2en\sqrt{b^2-a^2}} - \frac{2bx^{-2n}(ex)^{3n}\text{Li}_2\left(-\frac{ae^{i(dx^n+c)}}{b+\sqrt{b^2-a^2}}\right)}{ad^2en\sqrt{b^2-a^2}}$$

[Out]  $1/3*(e*x)^{(3*n)}/a/e/n/I*b*(e*x)^{(3*n)}*\ln(1+a*\exp(I*(c+d*x^n)))/(b-(-a^2+b^2)^{(1/2)))/a/d/e/n/(x^n)/(-a^2+b^2)^{(1/2)}-I*b*(e*x)^{(3*n)}*\ln(1+a*\exp(I*(c+d*x^n)))/(b+(-a^2+b^2)^{(1/2)))/a/d/e/n/(x^n)/(-a^2+b^2)^{(1/2)}+2*b*(e*x)^{(3*n)}*polylog(2,-a*\exp(I*(c+d*x^n)))/(b-(-a^2+b^2)^{(1/2)))/a/d^2/e/n/(x^{(2*n)})/(-a^2+b^2)^{(1/2)}-2*b*(e*x)^{(3*n)}*polylog(2,-a*\exp(I*(c+d*x^n)))/(b+(-a^2+b^2)^{(1/2)))/a/d^2/e/n/(x^{(2*n)})/(-a^2+b^2)^{(1/2)}+2*I*b*(e*x)^{(3*n)}*polylog(3,-a*\exp(I*(c+d*x^n)))/(b-(-a^2+b^2)^{(1/2)))/a/d^3/e/n/(x^{(3*n)})/(-a^2+b^2)^{(1/2)}-2*I*b*(e*x)^{(3*n)}*polylog(3,-a*\exp(I*(c+d*x^n)))/(b+(-a^2+b^2)^{(1/2)))/a/d^3/e/n/(x^{(3*n)})/(-a^2+b^2)^{(1/2)}$

**Rubi [A]** time = 0.92, antiderivative size = 485, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {4208, 4204, 4191, 3321, 2264, 2190, 2531, 2282, 6589}

$$\frac{2ibx^{-3n}(ex)^{3n}\text{PolyLog}\left(3, -\frac{ae^{i(c+dx^n)}}{b-\sqrt{b^2-a^2}}\right)}{ad^3en\sqrt{b^2-a^2}} - \frac{2ibx^{-3n}(ex)^{3n}\text{PolyLog}\left(3, -\frac{ae^{i(c+dx^n)}}{\sqrt{b^2-a^2}+b}\right)}{ad^3en\sqrt{b^2-a^2}} + \frac{2bx^{-2n}(ex)^{3n}\text{PolyLog}\left(2, -\frac{ae^{i(c+dx^n)}}{b-\sqrt{b^2-a^2}}\right)}{ad^2en\sqrt{b^2-a^2}} - \frac{2bx^{-2n}(ex)^{3n}\text{PolyLog}\left(2, -\frac{ae^{i(c+dx^n)}}{\sqrt{b^2-a^2}+b}\right)}{ad^2en\sqrt{b^2-a^2}}$$

Antiderivative was successfully verified.

[In] Int[(e\*x)^(-1 + 3\*n)/(a + b\*Sec[c + d\*x^n]), x]

[Out]  $(e*x)^{(3*n)}/(3*a*e*n) + (I*b*(e*x)^{(3*n)}*\text{Log}[1 + (a*E^{I*(c + d*x^n)})]/(b - \text{Sqrt}[-a^2 + b^2]))/(a*\text{Sqrt}[-a^2 + b^2]*d*e*n*x^n) - (I*b*(e*x)^{(3*n)}*\text{Log}[1 + (a*E^{I*(c + d*x^n)})]/(b + \text{Sqrt}[-a^2 + b^2]))/(a*\text{Sqrt}[-a^2 + b^2]*d*e*n*x^n) + (2*b*(e*x)^{(3*n)}*\text{PolyLog}[2, -((a*E^{I*(c + d*x^n)})/(b - \text{Sqrt}[-a^2 + b^2]))]/(a*\text{Sqrt}[-a^2 + b^2]*d^2*e*n*x^{(2*n)}) - (2*b*(e*x)^{(3*n)}*\text{PolyLog}[2, -((a*E^{I*(c + d*x^n)})/(b + \text{Sqrt}[-a^2 + b^2]))]/(a*\text{Sqrt}[-a^2 + b^2]*d^2*e*n*x^{(2*n)}) + ((2*I)*b*(e*x)^{(3*n)}*\text{PolyLog}[3, -((a*E^{I*(c + d*x^n)})/(b - \text{Sqrt}[-a^2 + b^2]))]/(a*\text{Sqrt}[-a^2 + b^2]*d^3*e*n*x^{(3*n)}) - ((2*I)*b*(e*x)^{(3*n)}*\text{PolyLog}[3, -((a*E^{I*(c + d*x^n)})/(b + \text{Sqrt}[-a^2 + b^2]))]/(a*\text{Sqrt}[-a^2 + b^2]*d^3*e*n*x^{(3*n)})$

**Rule 2190**

Int[(((F\_)^(g\_)\*((e\_) + (f\_)\*(x\_)))^(n\_)\*((c\_) + (d\_)\*(x\_))^(m\_))/((a\_) + (b\_)\*((F\_)^(g\_)\*((e\_) + (f\_)\*(x\_)))^(n\_)), x\_Symbol] := Simp

```

[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

```

### Rule 2264

```

Int[((F_)^(u_)*((f_) + (g_)*(x_))^(m_))/((a_) + (b_)*(F_)^(u_) + (c_)
*(F_)^(v_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]

```

### Rule 2282

```

Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

```

### Rule 2531

```

Int[Log[1 + (e_)*((F_)^(c_)*((a_) + (b_)*(x_)))^(n_)]*((f_) + (g_)
*(x_))^(m_), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
))^n]])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]

```

### Rule 3321

```

Int[((c_) + (d_)*(x_))^(m_)/((a_) + (b_)*sin[(e_) + Pi*(k_) + (f_)*(
x_)]), x_Symbol] := Dist[2, Int[((c + d*x)^m*E^(I*Pi*(k - 1/2))*E^(I*(e + f
*x)))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(
e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]

```

### Rule 4191

```

Int[(csc[(e_) + (f_)*(x_)]*(b_) + (a_))^(n_)*((c_) + (d_)*(x_))^(m_)
, x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x]^n)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]

```

### Rule 4204

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol]
  := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]
```

### Rule 4208

```
Int[((e_)*(x_))^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol]
  := Dist[(e^IntPart[m]*(e*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a + b*Sec[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol]
  := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(ex)^{-1+3n}}{a + b \sec(c + dx^n)} dx &= \frac{(x^{-3n}(ex)^{3n}) \int \frac{x^{-1+3n}}{a+b \sec(c+dx^n)} dx}{e} \\
&= \frac{(x^{-3n}(ex)^{3n}) \text{Subst} \left( \int \frac{x^2}{a+b \sec(c+dx)} dx, x, x^n \right)}{en} \\
&= \frac{(x^{-3n}(ex)^{3n}) \text{Subst} \left( \int \left( \frac{x^2}{a} - \frac{bx^2}{a(b+a \cos(c+dx))} \right) dx, x, x^n \right)}{en} \\
&= \frac{(ex)^{3n}}{3aen} - \frac{(bx^{-3n}(ex)^{3n}) \text{Subst} \left( \int \frac{x^2}{b+a \cos(c+dx)} dx, x, x^n \right)}{aen} \\
&= \frac{(ex)^{3n}}{3aen} - \frac{(2bx^{-3n}(ex)^{3n}) \text{Subst} \left( \int \frac{e^{i(c+dx)} x^2}{a+2be^{i(c+dx)}+ae^{2i(c+dx)}} dx, x, x^n \right)}{aen} \\
&= \frac{(ex)^{3n}}{3aen} - \frac{(2bx^{-3n}(ex)^{3n}) \text{Subst} \left( \int \frac{e^{i(c+dx)} x^2}{2b-2\sqrt{-a^2+b^2}+2ae^{i(c+dx)}} dx, x, x^n \right)}{\sqrt{-a^2+b^2} en} + \frac{(2bx^{-3n}(ex)^{3n}) \text{Subst} \left( \int \frac{e^{i(c+dx)} x^2}{2b+2\sqrt{-a^2+b^2}+2ae^{i(c+dx)}} dx, x, x^n \right)}{\sqrt{-a^2+b^2} en} \\
&= \frac{(ex)^{3n}}{3aen} + \frac{ibx^{-n}(ex)^{3n} \log \left( 1 + \frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}} \right)}{a\sqrt{-a^2+b^2} den} - \frac{ibx^{-n}(ex)^{3n} \log \left( 1 + \frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}} \right)}{a\sqrt{-a^2+b^2} den} - \frac{(2ibx^{-2n}(ex)^{3n}) \text{Subst} \left( \int \frac{e^{i(c+dx)} x^2}{2b-2\sqrt{-a^2+b^2}+2ae^{i(c+dx)}} dx, x, x^n \right)}{\sqrt{-a^2+b^2} en} + \frac{(2ibx^{-2n}(ex)^{3n}) \text{Subst} \left( \int \frac{e^{i(c+dx)} x^2}{2b+2\sqrt{-a^2+b^2}+2ae^{i(c+dx)}} dx, x, x^n \right)}{\sqrt{-a^2+b^2} en} \\
&= \frac{(ex)^{3n}}{3aen} + \frac{ibx^{-n}(ex)^{3n} \log \left( 1 + \frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}} \right)}{a\sqrt{-a^2+b^2} den} - \frac{ibx^{-n}(ex)^{3n} \log \left( 1 + \frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}} \right)}{a\sqrt{-a^2+b^2} den} + \frac{2bx^{-2n}(ex)^{3n}}{2en} \\
&= \frac{(ex)^{3n}}{3aen} + \frac{ibx^{-n}(ex)^{3n} \log \left( 1 + \frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}} \right)}{a\sqrt{-a^2+b^2} den} - \frac{ibx^{-n}(ex)^{3n} \log \left( 1 + \frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}} \right)}{a\sqrt{-a^2+b^2} den} + \frac{2bx^{-2n}(ex)^{3n}}{2en} \\
&= \frac{(ex)^{3n}}{3aen} + \frac{ibx^{-n}(ex)^{3n} \log \left( 1 + \frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}} \right)}{a\sqrt{-a^2+b^2} den} - \frac{ibx^{-n}(ex)^{3n} \log \left( 1 + \frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}} \right)}{a\sqrt{-a^2+b^2} den} + \frac{2bx^{-2n}(ex)^{3n}}{2en}
\end{aligned}$$

**Mathematica** [F] time = 1.84, size = 0, normalized size = 0.00

$$\int \frac{(ex)^{-1+3n}}{a + b \sec(c + dx^n)} dx$$

Verification is Not applicable to the result.

[In] Integrate[(e\*x)^(-1 + 3\*n)/(a + b\*Sec[c + d\*x^n]), x]





$$\begin{aligned} & \sqrt{2n} \sqrt{-(a^2 - b^2)/a^2} + 6Ia*b*c^2 * e^{(3n-1)} \sqrt{-(a^2 - b^2)/a^2} \\ & \log(-1/2 * (2 * (a \sqrt{-(a^2 - b^2)/a^2} - b) \cos(dx^n + c) - (-2Ia \sqrt{-(a^2 - b^2)/a^2} + 2Ib) \sin(dx^n + c) - 2a/a)) / ((a^3 - a*b^2) * d^{3n}) \end{aligned}$$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^{3n-1}}{b \sec(dx^n + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^(-1+3\*n)/(a+b\*sec(c+d\*x^n)),x, algorithm="giac")

[Out] integrate((e\*x)^(3\*n - 1)/(b\*sec(d\*x^n + c) + a), x)

**maple** [F] time = 1.95, size = 0, normalized size = 0.00

$$\int \frac{(ex)^{-1+3n}}{a + b \sec(c + dx^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x)^(-1+3\*n)/(a+b\*sec(c+d\*x^n)),x)

[Out] int((e\*x)^(-1+3\*n)/(a+b\*sec(c+d\*x^n)),x)

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^(-1+3\*n)/(a+b\*sec(c+d\*x^n)),x, algorithm="maxima")

[Out] Timed out

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(ex)^{3n-1}}{a + \frac{b}{\cos(c+dx^n)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x)^(3\*n - 1)/(a + b/cos(c + d\*x^n)),x)

[Out] int((e\*x)^(3\*n - 1)/(a + b/cos(c + d\*x^n)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^{3n-1}}{a + b \sec(c + dx^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)\*\*(-1+3\*n)/(a+b\*sec(c+d\*x\*\*n)), x)

[Out] Integral((e\*x)\*\*(3\*n - 1)/(a + b\*sec(c + d\*x\*\*n)), x)

$$3.81 \quad \int \frac{(ex)^{-1+n}}{(a+b \sec(c+dx^n))^2} dx$$

**Optimal.** Leaf size=157

$$\frac{2b(2a^2 - b^2)x^{-n}(ex)^n \tanh^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx^n)\right)}{\sqrt{a+b}}\right)}{a^2den(a-b)^{3/2}(a+b)^{3/2}} + \frac{b^2x^{-n}(ex)^n \tan(c+dx^n)}{aden(a^2 - b^2)(a+b \sec(c+dx^n))} + \frac{(ex)^n}{a^2en}$$

[Out] (e\*x)^n/a^2/e/n-2\*b\*(2\*a^2-b^2)\*(e\*x)^n\*arctanh((a-b)^(1/2)\*tan(1/2\*c+1/2\*d\*x^n)/(a+b)^(1/2))/a^2/(a-b)^(3/2)/(a+b)^(3/2)/d/e/n/(x^n)+b^2\*(e\*x)^n\*tan(c+d\*x^n)/a/(a^2-b^2)/d/e/n/(x^n)/(a+b\*sec(c+d\*x^n))

**Rubi [A]** time = 0.29, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$ , Rules used = {4208, 4204, 3785, 3919, 3831, 2659, 208}

$$\frac{2b(2a^2 - b^2)x^{-n}(ex)^n \tanh^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx^n)\right)}{\sqrt{a+b}}\right)}{a^2den(a-b)^{3/2}(a+b)^{3/2}} + \frac{b^2x^{-n}(ex)^n \tan(c+dx^n)}{aden(a^2 - b^2)(a+b \sec(c+dx^n))} + \frac{(ex)^n}{a^2en}$$

Antiderivative was successfully verified.

[In] Int[(e\*x)^(-1 + n)/(a + b\*Sec[c + d\*x^n])^2, x]

[Out] (e\*x)^n/(a^2\*e\*n) - (2\*b\*(2\*a^2 - b^2)\*(e\*x)^n\*ArcTanh[(Sqrt[a - b]\*Tan[(c + d\*x^n)/2])/Sqrt[a + b]])/(a^2\*(a - b)^(3/2)\*(a + b)^(3/2)\*d\*e\*n\*x^n) + (b^2\*(e\*x)^n\*Tan[c + d\*x^n])/(a\*(a^2 - b^2)\*d\*e\*n\*x^n\*(a + b\*Sec[c + d\*x^n]))

Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 2659

Int[((a\_) + (b\_.)\*sin[Pi/2 + (c\_.) + (d\_.)\*(x\_)])^(-1), x\_Symbol] := With[{e = FreeFactors[Tan[(c + d\*x)/2], x]}, Dist[(2\*e)/d, Subst[Int[1/(a + b + (a - b)\*e^2\*x^2), x], x, Tan[(c + d\*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 3785

```
Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.) + (a_.))^(n_), x_Symbol] := Simp[(b^2*Cot
[c + d*x]*(a + b*Csc[c + d*x])^(n + 1))/(a*d*(n + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

### Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_.)]/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol]
:= Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

### Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))/(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_.)), x_Symbol] := Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x]
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
a*d, 0]
```

### Rule 4204

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol]
:= Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

### Rule 4208

```
Int[((e_.)*(x_.))^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x
_Symbol] := Dist[(e^IntPart[m]*(e*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a
+ b*Sec[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(ex)^{-1+n}}{(a + b \sec(c + dx^n))^2} dx &= \frac{(x^{-n}(ex)^n) \int \frac{x^{-1+n}}{(a+b \sec(c+dx^n))^2} dx}{e} \\
&= \frac{(x^{-n}(ex)^n) \text{Subst} \left( \int \frac{1}{(a+b \sec(c+dx))^2} dx, x, x^n \right)}{en} \\
&= \frac{b^2 x^{-n}(ex)^n \tan(c + dx^n)}{a(a^2 - b^2) \text{den}(a + b \sec(c + dx^n))} - \frac{(x^{-n}(ex)^n) \text{Subst} \left( \int \frac{-a^2 + b^2 + ab \sec(c+dx)}{a+b \sec(c+dx)} dx, x, x^n \right)}{a(a^2 - b^2) en} \\
&= \frac{(ex)^n}{a^2 en} + \frac{b^2 x^{-n}(ex)^n \tan(c + dx^n)}{a(a^2 - b^2) \text{den}(a + b \sec(c + dx^n))} + \frac{\left( (-a^2 b + b(-a^2 + b^2)) x^{-n}(ex)^n \right) \text{Subst} \left( \int \frac{1}{a+b \sec(c+dx)} dx, x, x^n \right)}{a^2(a^2 - b^2) en} \\
&= \frac{(ex)^n}{a^2 en} + \frac{b^2 x^{-n}(ex)^n \tan(c + dx^n)}{a(a^2 - b^2) \text{den}(a + b \sec(c + dx^n))} + \frac{\left( (-a^2 b + b(-a^2 + b^2)) x^{-n}(ex)^n \right) \text{Subst} \left( \int \frac{1}{a+b \sec(c+dx)} dx, x, x^n \right)}{a^2 b(a^2 - b^2) en} \\
&= \frac{(ex)^n}{a^2 en} + \frac{b^2 x^{-n}(ex)^n \tan(c + dx^n)}{a(a^2 - b^2) \text{den}(a + b \sec(c + dx^n))} + \frac{\left( 2(-a^2 b + b(-a^2 + b^2)) x^{-n}(ex)^n \right) \text{Subst} \left( \int \frac{1}{a+b \sec(c+dx)} dx, x, x^n \right)}{a^2 b(a^2 - b^2) en} \\
&= \frac{(ex)^n}{a^2 en} - \frac{2b(2a^2 - b^2) x^{-n}(ex)^n \tanh^{-1} \left( \frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx^n)\right)}{\sqrt{a+b}} \right)}{a^2(a-b)^{3/2}(a+b)^{3/2} \text{den}} + \frac{b^2 x^{-n}(ex)^n \tan(c + dx^n)}{a(a^2 - b^2) \text{den}(a + b \sec(c + dx^n))}
\end{aligned}$$

**Mathematica [A]** time = 0.93, size = 191, normalized size = 1.22

$$\frac{x^{-n}(ex)^n \left( \sqrt{a^2 - b^2} \left( b \left( (a^2 - b^2)(c + dx^n) + ab \sin(c + dx^n) \right) + a(a^2 - b^2)(c + dx^n) \cos(c + dx^n) \right) - 2b(b^2 - 2a^2) \right)}{a^2 \text{den}(a - b)(a + b) \sqrt{a^2 - b^2} (a \cos(c + dx^n) + b)}$$

Antiderivative was successfully verified.

[In] Integrate[(e\*x)^(-1 + n)/(a + b\*Sec[c + d\*x^n])^2, x]

[Out] ((e\*x)^n\*(-2\*b\*(-2\*a^2 + b^2)\*ArcTanh[((-a + b)\*Tan[(c + d\*x^n)/2])/Sqrt[a^2 - b^2]])\*(b + a\*Cos[c + d\*x^n]) + Sqrt[a^2 - b^2]\*(a\*(a^2 - b^2)\*(c + d\*x^n)\*Cos[c + d\*x^n] + b\*((a^2 - b^2)\*(c + d\*x^n) + a\*b\*Sin[c + d\*x^n])))/(a^2\*(a - b)\*(a + b)\*Sqrt[a^2 - b^2]\*d\*e\*n\*x^n\*(b + a\*Cos[c + d\*x^n]))

**fricas** [A] time = 0.66, size = 628, normalized size = 4.00

$$\frac{2(a^5 - 2a^3b^2 + ab^4)de^{n-1}x^n \cos(dx^n + c) + 2(a^4b - 2a^2b^3 + b^5)de^{n-1}x^n + 2(a^3b^2 - ab^4)e^{n-1} \sin(dx^n + c) + \dots}{2((a^7 - 2a^5b^2 + a^3b^4)d^n \cos(dx^n + c) + (a^6b - 2a^4b^3 + a^2b^5)d^n)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^(-1+n)/(a+b\*sec(c+d\*x^n))^2,x, algorithm="fricas")

[Out] [1/2\*(2\*(a^5 - 2\*a^3\*b^2 + a\*b^4)\*d\*e^(n - 1)\*x^n\*cos(d\*x^n + c) + 2\*(a^4\*b - 2\*a^2\*b^3 + b^5)\*d\*e^(n - 1)\*x^n + 2\*(a^3\*b^2 - a\*b^4)\*e^(n - 1)\*sin(d\*x^n + c) + ((2\*a^3\*b - a\*b^3)\*sqrt(a^2 - b^2)\*e^(n - 1)\*cos(d\*x^n + c) + (2\*a^2\*b^2 - b^4)\*sqrt(a^2 - b^2)\*e^(n - 1))\*log((2\*a\*b\*cos(d\*x^n + c) - (a^2 - 2\*b^2)\*cos(d\*x^n + c)^2 + 2\*a^2 - b^2 - 2\*(sqrt(a^2 - b^2)\*b\*cos(d\*x^n + c) + sqrt(a^2 - b^2)\*a)\*sin(d\*x^n + c))/(a^2\*cos(d\*x^n + c)^2 + 2\*a\*b\*cos(d\*x^n + c) + b^2)))/((a^7 - 2\*a^5\*b^2 + a^3\*b^4)\*d^n\*cos(d\*x^n + c) + (a^6\*b - 2\*a^4\*b^3 + a^2\*b^5)\*d^n), ((a^5 - 2\*a^3\*b^2 + a\*b^4)\*d\*e^(n - 1)\*x^n\*cos(d\*x^n + c) + (a^4\*b - 2\*a^2\*b^3 + b^5)\*d\*e^(n - 1)\*x^n + (a^3\*b^2 - a\*b^4)\*e^(n - 1)\*sin(d\*x^n + c) - ((2\*a^3\*b - a\*b^3)\*sqrt(-a^2 + b^2)\*e^(n - 1)\*cos(d\*x^n + c) + (2\*a^2\*b^2 - b^4)\*sqrt(-a^2 + b^2)\*e^(n - 1))\*arctan(-(sqrt(-a^2 + b^2)\*b\*cos(d\*x^n + c) + sqrt(-a^2 + b^2)\*a)/((a^2 - b^2)\*sin(d\*x^n + c)))/((a^7 - 2\*a^5\*b^2 + a^3\*b^4)\*d^n\*cos(d\*x^n + c) + (a^6\*b - 2\*a^4\*b^3 + a^2\*b^5)\*d^n)]

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^{n-1}}{(b \sec(dx^n + c) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^(-1+n)/(a+b\*sec(c+d\*x^n))^2,x, algorithm="giac")

[Out] integrate((e\*x)^(n - 1)/(b\*sec(d\*x^n + c) + a)^2, x)

**maple** [C] time = 1.33, size = 704, normalized size = 4.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x)^(-1+n)/(a+b\*sec(c+d\*x^n))^2,x)

```
[Out] 1/a^2/n*x*exp(1/2*(-1+n)*(-I*Pi*csgn(I*e)*csgn(I*x)*csgn(I*e*x)+I*Pi*csgn(I
*e)*csgn(I*e*x)^2+I*Pi*csgn(I*x)*csgn(I*e*x)^2-I*Pi*csgn(I*e*x)^3+2*ln(x)+2
*ln(e))+2*I*b^2*x/a^2/(a^2-b^2)/d/n/(x^n)/(exp(2*I*(c+d*x^n))*a+2*b*exp(I*
(c+d*x^n))+a)*(b*e^n*x^n/e/x*(-1)^(-1/2*csgn(I*x)*csgn(I*e*x)^2)*(-1)^(-1/2
*csgn(I*e)*csgn(I*e*x)^2)*(-1)^(1/2*csgn(I*e)*csgn(I*x)*csgn(I*e*x))*exp(1/
2*I*Pi*csgn(I*e*x)^3)*exp(-1/2*I*Pi*n*csgn(I*e)*csgn(I*x)*csgn(I*e*x))*exp(
1/2*I*Pi*n*csgn(I*e)*csgn(I*e*x)^2)*exp(1/2*I*Pi*n*csgn(I*x)*csgn(I*e*x)^2)
*exp(-1/2*I*Pi*n*csgn(I*e*x)^3)*exp(I*x^n*d)*exp(I*c)+a*e^n*x^n/e/x*(-1)^(-
1/2*csgn(I*x)*csgn(I*e*x)^2)*(-1)^(-1/2*csgn(I*e)*csgn(I*e*x)^2)*(-1)^(1/2*
csgn(I*e)*csgn(I*x)*csgn(I*e*x))*exp(1/2*I*Pi*csgn(I*e*x)*(-n*csgn(I*e)*csg
n(I*x)+n*csgn(I*e)*csgn(I*e*x)+n*csgn(I*x)*csgn(I*e*x)-n*csgn(I*e*x)^2+csgn
(I*e*x)^2))+2*I*arctan(1/2*(2*a*exp(I*(d*x^n+2*c))+2*exp(I*c)*b)/(a^2*exp(
2*I*c)-exp(2*I*c)*b^2)^(1/2))/(a^2*exp(2*I*c)-exp(2*I*c)*b^2)^(1/2)/d/e*e^n
/n/(-a^2+b^2)*(-2*a^2+b^2)/a^2*b*exp(1/2*I*(-Pi*n*csgn(I*e)*csgn(I*x)*csgn(
I*e*x)+Pi*n*csgn(I*e)*csgn(I*e*x)^2+Pi*n*csgn(I*x)*csgn(I*e*x)^2-Pi*n*csgn(
I*e*x)^3+Pi*csgn(I*e*x)*csgn(I*e)*csgn(I*x)-Pi*csgn(I*e)*csgn(I*e*x)^2-Pi*c
sgn(I*x)*csgn(I*e*x)^2+Pi*csgn(I*e*x)^3+2*c))
```

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+n)/(a+b*sec(c+d*x^n))^2,x, algorithm="maxima")
```

[Out] Timed out

**mupad** [B] time = 5.33, size = 461, normalized size = 2.94

$$\frac{\frac{b^2 x (e x)^{n-1} 2i}{a d n x^n (a^2 - b^2)} + \frac{b^3 x e^{c 1i + d x^n 1i} (e x)^{n-1} 2i}{a^2 d n x^n (a^2 - b^2)}}{a + a e^{c 2i + d x^n 2i} + 2 b e^{c 1i + d x^n 1i}} + \frac{x (e x)^{n-1}}{a^2 n} + \frac{b x \ln\left(-2 e^{c 1i} e^{d x^n 1i} \left(b^3 x (e x)^{n-1} - 2 a^2 b x (e x)^{n-1}\right) - \frac{b x (a^4 - a^2 b^2)}{a^2 d n x^n (a + b)^{3/2}}\right)}{a^2 d n x^n (a + b)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(n - 1)/(a + b/cos(c + d*x^n))^2,x)
```

```
[Out] ((b^2*x*(e*x)^(n - 1)*2i)/(a*d*n*x^n*(a^2 - b^2)) + (b^3*x*exp(c*1i + d*x^n
*1i)*(e*x)^(n - 1)*2i)/(a^2*d*n*x^n*(a^2 - b^2)))/(a + a*exp(c*2i + d*x^n*2
i) + 2*b*exp(c*1i + d*x^n*1i)) + (x*(e*x)^(n - 1))/(a^2*n) + (b*x*log(- 2*e
xp(c*1i)*exp(d*x^n*1i)*(b^3*x*(e*x)^(n - 1) - 2*a^2*b*x*(e*x)^(n - 1)) - (b
*x*(a^4 - a^2*b^2)*(a + b*exp(c*1i)*exp(d*x^n*1i))*(e*x)^(n - 1)*(2*a^2 - b
^2)*2i)/(a^2*(a + b)^(3/2)*(a - b)^(3/2)))*(e*x)^(n - 1)*(2*a^2 - b^2))/(a^
2*d*n*x^n*(a + b)^(3/2)*(a - b)^(3/2)) - (b*x*log((b*x*(a^4 - a^2*b^2)*(a +
b*exp(c*1i)*exp(d*x^n*1i))*(e*x)^(n - 1)*(2*a^2 - b^2)*2i)/(a^2*(a + b)^(3
```



$$\frac{1}{2}(a - b)^{3/2} - 2\exp(c) \exp(dx^n) (b^3 x (e^x)^{n-1} - 2a^2 b x (e^x)^{n-1}) (e^x)^{n-1} (2a^2 - b^2) / (a^2 d n x^n (a + b)^{3/2}) (a - b)^{3/2}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)\*\*(-1+n)/(a+b\*sec(c+d\*x\*\*n))\*\*2,x)

[Out] Timed out

$$3.82 \quad \int \frac{(ex)^{-1+2n}}{(a+b \sec(c+dx^n))^2} dx$$

**Optimal.** Leaf size=757

$$\frac{2bx^{-2n}(ex)^{2n}\text{Li}_2\left(-\frac{ae^{i(dx^n+c)}}{b-\sqrt{b^2-a^2}}\right)}{a^2d^2en\sqrt{b^2-a^2}} - \frac{2bx^{-2n}(ex)^{2n}\text{Li}_2\left(-\frac{ae^{i(dx^n+c)}}{b+\sqrt{b^2-a^2}}\right)}{a^2d^2en\sqrt{b^2-a^2}} + \frac{b^2x^{-2n}(ex)^{2n}\log(a\cos(c+dx^n)+b)}{a^2d^2en(a^2-b^2)} + \frac{2ibx^{-n}(ex)^{2n}}{a^2d}$$

[Out]  $\frac{1}{2}*(e*x)^{(2*n)}/a^2/e/n+b^2*(e*x)^{(2*n)}*\ln(b+a*\cos(c+d*x^n))/a^2/(a^2-b^2)/d^2/e/n/(x^{(2*n)})-I*b^3*(e*x)^{(2*n)}*\ln(1+a*\exp(I*(c+d*x^n)))/(b-(-a^2+b^2)^{(1/2)))/a^2/(-a^2+b^2)^{(3/2)}/d/e/n/(x^n)+I*b^3*(e*x)^{(2*n)}*\ln(1+a*\exp(I*(c+d*x^n)))/(b+(-a^2+b^2)^{(1/2)))/a^2/(-a^2+b^2)^{(3/2)}/d/e/n/(x^n)-b^3*(e*x)^{(2*n)}*\text{polylog}(2,-a*\exp(I*(c+d*x^n)))/(b-(-a^2+b^2)^{(1/2)))/a^2/(-a^2+b^2)^{(3/2)}/d^2/e/n/(x^{(2*n)})+b^3*(e*x)^{(2*n)}*\text{polylog}(2,-a*\exp(I*(c+d*x^n)))/(b+(-a^2+b^2)^{(1/2)))/a^2/(-a^2+b^2)^{(3/2)}/d^2/e/n/(x^{(2*n)})+b^2*(e*x)^{(2*n)}*\sin(c+d*x^n)/a/(a^2-b^2)/d/e/n/(x^n)/(b+a*\cos(c+d*x^n))+2*I*b*(e*x)^{(2*n)}*\ln(1+a*\exp(I*(c+d*x^n)))/(b-(-a^2+b^2)^{(1/2)))/a^2/d/e/n/(x^n)/(-a^2+b^2)^{(1/2)}-2*I*b*(e*x)^{(2*n)}*\ln(1+a*\exp(I*(c+d*x^n)))/(b+(-a^2+b^2)^{(1/2)))/a^2/d/e/n/(x^n)/(-a^2+b^2)^{(1/2)}+2*b*(e*x)^{(2*n)}*\text{polylog}(2,-a*\exp(I*(c+d*x^n)))/(b-(-a^2+b^2)^{(1/2)))/a^2/d^2/e/n/(x^{(2*n)})/(-a^2+b^2)^{(1/2)}-2*b*(e*x)^{(2*n)}*\text{polylog}(2,-a*\exp(I*(c+d*x^n)))/(b+(-a^2+b^2)^{(1/2)))/a^2/d^2/e/n/(x^{(2*n)})/(-a^2+b^2)^{(1/2)}$

**Rubi [A]** time = 1.27, antiderivative size = 757, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 11, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$ , Rules used = {4208, 4204, 4191, 3324, 3321, 2264, 2190, 2279, 2391, 2668, 31}

$$\frac{2bx^{-2n}(ex)^{2n}\text{PolyLog}\left(2,-\frac{ae^{i(c+dx^n)}}{b-\sqrt{b^2-a^2}}\right)}{a^2d^2en\sqrt{b^2-a^2}} - \frac{b^3x^{-2n}(ex)^{2n}\text{PolyLog}\left(2,-\frac{ae^{i(c+dx^n)}}{b+\sqrt{b^2-a^2}}\right)}{a^2d^2en(b^2-a^2)^{3/2}} - \frac{2bx^{-2n}(ex)^{2n}\text{PolyLog}\left(2,-\frac{ae^{i(c+dx^n)}}{\sqrt{b^2-a^2}}\right)}{a^2d^2en\sqrt{b^2-a^2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(e*x)^{-1+2n}/(a+b*\text{Sec}[c+d*x^n])^2,x]$

[Out]  $(e*x)^{(2*n)}/(2*a^2*e^n) - (I*b^3*(e*x)^{(2*n)}*\text{Log}[1+(a*E^{(I*(c+d*x^n))})]/(b-\text{Sqrt}[-a^2+b^2]))/(a^2*(-a^2+b^2)^{(3/2)}*d*e^n*x^n) + ((2*I)*b*(e*x)^{(2*n)}*\text{Log}[1+(a*E^{(I*(c+d*x^n))})]/(b-\text{Sqrt}[-a^2+b^2]))/(a^2*\text{Sqrt}[-a^2+b^2]*d*e^n*x^n) + (I*b^3*(e*x)^{(2*n)}*\text{Log}[1+(a*E^{(I*(c+d*x^n))})]/(b+\text{Sqrt}[-a^2+b^2]))/(a^2*(-a^2+b^2)^{(3/2)}*d*e^n*x^n) - ((2*I)*b*(e*x)^{(2*n)}*\text{Log}[1+(a*E^{(I*(c+d*x^n))})]/(b+\text{Sqrt}[-a^2+b^2]))/(a^2*\text{Sqrt}[-a^2$

$$+ b^2] * d * e * n * x^n) + (b^2 * (e * x)^{(2 * n)} * \text{Log}[b + a * \text{Cos}[c + d * x^n]]) / (a^2 * (a^2 - b^2) * d^2 * e * n * x^{(2 * n)}) - (b^3 * (e * x)^{(2 * n)} * \text{PolyLog}[2, -((a * E^{(I * (c + d * x^n))}) / (b - \text{Sqrt}[-a^2 + b^2]))]) / (a^2 * (-a^2 + b^2)^{(3/2)} * d^2 * e * n * x^{(2 * n)}) + (2 * b * (e * x)^{(2 * n)} * \text{PolyLog}[2, -((a * E^{(I * (c + d * x^n))}) / (b - \text{Sqrt}[-a^2 + b^2]))]) / (a^2 * \text{Sqrt}[-a^2 + b^2] * d^2 * e * n * x^{(2 * n)}) + (b^3 * (e * x)^{(2 * n)} * \text{PolyLog}[2, -((a * E^{(I * (c + d * x^n))}) / (b + \text{Sqrt}[-a^2 + b^2]))]) / (a^2 * (-a^2 + b^2)^{(3/2)} * d^2 * e * n * x^{(2 * n)}) - (2 * b * (e * x)^{(2 * n)} * \text{PolyLog}[2, -((a * E^{(I * (c + d * x^n))}) / (b + \text{Sqrt}[-a^2 + b^2]))]) / (a^2 * \text{Sqrt}[-a^2 + b^2] * d^2 * e * n * x^{(2 * n)}) + (b^2 * (e * x)^{(2 * n)} * \text{Sin}[c + d * x^n]) / (a * (a^2 - b^2) * d * e * n * x^n * (b + a * \text{Cos}[c + d * x^n]))$$

### Rule 31

```
Int[((a_) + (b_)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

### Rule 2190

```
Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_))/((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp[((c + d*x)^m * Log[1 + (b*(F^(g*(e + f*x))))^n]/a]/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1) * Log[1 + (b*(F^(g*(e + f*x))))^n]/a], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2264

```
Int[((F_)^(u_)*((f_) + (g_)*(x_))^(m_))/((a_) + (b_)*(F_)^(u_) + (c_) * (F_)^(v_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[(f + g*x)^m * F^u]/(b - q + 2*c * F^u), x], x] - Dist[(2*c)/q, Int[(f + g*x)^m * F^u]/(b + q + 2*c * F^u), x], x] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

### Rule 2279

```
Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2391

```
Int[Log[(c_)*((d_) + (e_)*(x_))^(n_) ]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 2668

```
Int[cos[(e_) + (f_)*(x_)]^(p_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_), x_Symbol] := Dist[1/(b^p*f), Subst[Int[(a + x)^m * (b^2 - x^2)^((p - 1)/
```

2), x], x, b\*Sin[e + f\*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]

### Rule 3321

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)/((a\_.) + (b\_.)\*sin[(e\_.) + Pi\*(k\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Dist[2, Int[((c + d\*x)^m\*E^(I\*Pi\*(k - 1/2))\*E^(I\*(e + f\*x)))/(b + 2\*a\*E^(I\*Pi\*(k - 1/2))\*E^(I\*(e + f\*x)) - b\*E^(2\*I\*k\*Pi)\*E^(2\*I\*(e + f\*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2\*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]

### Rule 3324

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)/((a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)]^2, x\_Symbol] := Simp[(b\*(c + d\*x)^m\*Cos[e + f\*x])/(f\*(a^2 - b^2)\*(a + b\*Sin[e + f\*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d\*x)^m/(a + b\*Sin[e + f\*x]), x], x] - Dist[(b\*d\*m)/(f\*(a^2 - b^2)), Int[((c + d\*x)^(m - 1)\*Cos[e + f\*x])/(a + b\*Sin[e + f\*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]

### Rule 4191

Int[(csc[(e\_.) + (f\_.)\*(x\_)]\*(b\_.) + (a\_.))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Int[ExpandIntegrand[(c + d\*x)^m, 1/(Sin[e + f\*x]^n/(b + a\*Sin[e + f\*x]^n)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGtQ[m, 0]

### Rule 4204

Int[(x\_)^(m\_.)\*((a\_.) + (b\_.)\*Sec[(c\_.) + (d\_.)\*(x\_)^(n\_)])^(p\_.), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*Sec[c + d\*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]

### Rule 4208

Int[((e\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*Sec[(c\_.) + (d\_.)\*(x\_)^(n\_)])^(p\_.), x\_Symbol] := Dist[(e^IntPart[m]\*(e\*x)^FracPart[m])/x^FracPart[m], Int[x^m\*(a + b\*Sec[c + d\*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]

### Rubi steps

$$\begin{aligned}
\int \frac{(ex)^{-1+2n}}{(a+b \sec(c+dx^n))^2} dx &= \frac{(x^{-2n}(ex)^{2n}) \int \frac{x^{-1+2n}}{(a+b \sec(c+dx))^2} dx}{e} \\
&= \frac{(x^{-2n}(ex)^{2n}) \text{Subst} \left( \int \frac{x}{(a+b \sec(c+dx))^2} dx, x, x^n \right)}{e} \\
&= \frac{(x^{-2n}(ex)^{2n}) \text{Subst} \left( \int \left( \frac{x}{a^2} + \frac{b^2 x}{a^2(b+a \cos(c+dx))^2} - \frac{2bx}{a^2(b+a \cos(c+dx))} \right) dx, x, x^n \right)}{e} \\
&= \frac{(ex)^{2n}}{2a^2 en} - \frac{(2bx^{-2n}(ex)^{2n}) \text{Subst} \left( \int \frac{x}{b+a \cos(c+dx)} dx, x, x^n \right)}{a^2 en} + \frac{(b^2 x^{-2n}(ex)^{2n}) \text{Subst} \left( \int \frac{x}{(b+a \cos(c+dx))^2} dx, x, x^n \right)}{a^2 en} \\
&= \frac{(ex)^{2n}}{2a^2 en} + \frac{b^2 x^{-n}(ex)^{2n} \sin(c+dx^n)}{a(a^2-b^2) \text{den}(b+a \cos(c+dx^n))} - \frac{(4bx^{-2n}(ex)^{2n}) \text{Subst} \left( \int \frac{e^{i(c+dx^n)}}{a+2be^{i(c+dx^n)}} dx, x, x^n \right)}{a^2 en} \\
&= \frac{(ex)^{2n}}{2a^2 en} + \frac{b^2 x^{-n}(ex)^{2n} \sin(c+dx^n)}{a(a^2-b^2) \text{den}(b+a \cos(c+dx^n))} - \frac{(2b^3 x^{-2n}(ex)^{2n}) \text{Subst} \left( \int \frac{e^{i(c+dx^n)}}{a+2be^{i(c+dx^n)}} dx, x, x^n \right)}{a^2(a^2-b^2) en} \\
&= \frac{(ex)^{2n}}{2a^2 en} + \frac{2ibx^{-n}(ex)^{2n} \log \left( 1 + \frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}} \right)}{a^2 \sqrt{-a^2+b^2} \text{den}} - \frac{2ibx^{-n}(ex)^{2n} \log \left( 1 + \frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}} \right)}{a^2 \sqrt{-a^2+b^2} \text{den}} + \\
&= \frac{(ex)^{2n}}{2a^2 en} - \frac{ib^3 x^{-n}(ex)^{2n} \log \left( 1 + \frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}} \right)}{a^2 (-a^2+b^2)^{3/2} \text{den}} + \frac{2ibx^{-n}(ex)^{2n} \log \left( 1 + \frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}} \right)}{a^2 \sqrt{-a^2+b^2} \text{den}} + \\
&= \frac{(ex)^{2n}}{2a^2 en} - \frac{ib^3 x^{-n}(ex)^{2n} \log \left( 1 + \frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}} \right)}{a^2 (-a^2+b^2)^{3/2} \text{den}} + \frac{2ibx^{-n}(ex)^{2n} \log \left( 1 + \frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}} \right)}{a^2 \sqrt{-a^2+b^2} \text{den}} + \\
&= \frac{(ex)^{2n}}{2a^2 en} - \frac{ib^3 x^{-n}(ex)^{2n} \log \left( 1 + \frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}} \right)}{a^2 (-a^2+b^2)^{3/2} \text{den}} + \frac{2ibx^{-n}(ex)^{2n} \log \left( 1 + \frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}} \right)}{a^2 \sqrt{-a^2+b^2} \text{den}} +
\end{aligned}$$

**Mathematica [B]** time = 10.47, size = 2450, normalized size = 3.24

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(e\*x)^(-1 + 2\*n)/(a + b\*Sec[c + d\*x^n])^2,x]

[Out] 
$$\begin{aligned} & (-2*b*x^{(1-2*n)}*(e*x)^{-1+2*n}*(b+a*\cos[c+d*x^n])^2*(2*(c+d*x^n)* \\ & \operatorname{ArcTanh}[\frac{(a+b)*\cot[(c+d*x^n)/2]}{\sqrt{a^2-b^2}}] - 2*(c+\operatorname{ArcCos}[-(b/a)]) \\ & )*\operatorname{ArcTanh}[\frac{(a-b)*\tan[(c+d*x^n)/2]}{\sqrt{a^2-b^2}}] + (\operatorname{ArcCos}[-(b/a)] \\ & - (2*I)*(\operatorname{ArcTanh}[\frac{(a+b)*\cot[(c+d*x^n)/2]}{\sqrt{a^2-b^2}}] - \operatorname{ArcTanh}[\frac{(a-b)*\tan[(c+d*x^n)/2]}{\sqrt{a^2-b^2}}])) \\ & )*\log[\frac{\sqrt{a^2-b^2}}{\sqrt{2}*\sqrt{a}*e^{(I/2)*(c+d*x^n)}*\sqrt{b+a*\cos[c+d*x^n]}}] + (\operatorname{ArcCos}[-(b/a)] \\ & + (2*I)*(\operatorname{ArcTanh}[\frac{(a+b)*\cot[(c+d*x^n)/2]}{\sqrt{a^2-b^2}}] - \operatorname{ArcTanh}[\frac{(a-b)*\tan[(c+d*x^n)/2]}{\sqrt{a^2-b^2}}])) \\ & )*\log[(\sqrt{a^2-b^2}*e^{(I/2)*(c+d*x^n)})/(\sqrt{2}*\sqrt{a}*sqrt{b+a*\cos[c+d*x^n]})] - (\operatorname{ArcCos}[-(b/a)] \\ & + (2*I)*\operatorname{ArcTanh}[\frac{(a-b)*\tan[(c+d*x^n)/2]}{\sqrt{a^2-b^2}}])*\log[1 - ((b - I*\sqrt{a^2-b^2})*(a+b - \sqrt{a^2-b^2})*\tan[(c+d*x^n)/2])]/( \\ & a*(a+b + \sqrt{a^2-b^2}*\tan[(c+d*x^n)/2])) + (-\operatorname{ArcCos}[-(b/a)] + (2*I) \\ & )*\operatorname{ArcTanh}[\frac{(a-b)*\tan[(c+d*x^n)/2]}{\sqrt{a^2-b^2}}])*\log[1 - ((b + I*\sqrt{a^2-b^2})*(a+b - \sqrt{a^2-b^2})*\tan[(c+d*x^n)/2])]/( \\ & a*(a+b + \sqrt{a^2-b^2}*\tan[(c+d*x^n)/2])) + I*(\operatorname{PolyLog}[2, ((b - I*\sqrt{a^2-b^2})*(a+b - \sqrt{a^2-b^2})*\tan[(c+d*x^n)/2])]/( \\ & a*(a+b + \sqrt{a^2-b^2}*\tan[(c+d*x^n)/2])) - \operatorname{PolyLog}[2, ((b + I*\sqrt{a^2-b^2})*(a+b - \sqrt{a^2-b^2})*\tan[(c+d*x^n)/2])]/( \\ & a*(a+b + \sqrt{a^2-b^2}*\tan[(c+d*x^n)/2])))]* \operatorname{Sec}[c+d*x^n]^2/((a^2-b^2)^{(3/2)}*d^{2*n}*(a+b*\operatorname{Sec}[c+d*x^n])^2) \\ & + (b^3*x^{(1-2*n)}*(e*x)^{-1+2*n}*(b+a*\cos[c+d*x^n])^2*(2*(c+d*x^n)* \\ & \operatorname{ArcTanh}[\frac{(a+b)*\cot[(c+d*x^n)/2]}{\sqrt{a^2-b^2}}] - 2*(c+\operatorname{ArcCos}[-(b/a)]) \\ & )*\operatorname{ArcTanh}[\frac{(a-b)*\tan[(c+d*x^n)/2]}{\sqrt{a^2-b^2}}] + (\operatorname{ArcCos}[-(b/a)] \\ & - (2*I)*(\operatorname{ArcTanh}[\frac{(a+b)*\cot[(c+d*x^n)/2]}{\sqrt{a^2-b^2}}] - \operatorname{ArcTanh}[\frac{(a-b)*\tan[(c+d*x^n)/2]}{\sqrt{a^2-b^2}}])) \\ & )*\log[\frac{\sqrt{a^2-b^2}}{\sqrt{2}*\sqrt{a}*e^{(I/2)*(c+d*x^n)}*\sqrt{b+a*\cos[c+d*x^n]}}] + (\operatorname{ArcCos}[-(b/a)] \\ & + (2*I)*(\operatorname{ArcTanh}[\frac{(a+b)*\cot[(c+d*x^n)/2]}{\sqrt{a^2-b^2}}] - \operatorname{ArcTanh}[\frac{(a-b)*\tan[(c+d*x^n)/2]}{\sqrt{a^2-b^2}}])) \\ & )*\log[(\sqrt{a^2-b^2}*e^{(I/2)*(c+d*x^n)})/(\sqrt{2}*\sqrt{a}*sqrt{b+a*\cos[c+d*x^n]})] - (\operatorname{ArcCos}[-(b/a)] \\ & + (2*I)*\operatorname{ArcTanh}[\frac{(a-b)*\tan[(c+d*x^n)/2]}{\sqrt{a^2-b^2}}])*\log[1 - ((b - I*\sqrt{a^2-b^2})*(a+b - \sqrt{a^2-b^2})*\tan[(c+d*x^n)/2])]/( \\ & a*(a+b + \sqrt{a^2-b^2}*\tan[(c+d*x^n)/2])) + (-\operatorname{ArcCos}[-(b/a)] + (2 \\ & *I)*\operatorname{ArcTanh}[\frac{(a-b)*\tan[(c+d*x^n)/2]}{\sqrt{a^2-b^2}}])*\log[1 - ((b + I*\sqrt{a^2-b^2})*(a+b - \sqrt{a^2-b^2})*\tan[(c+d*x^n)/2])]/( \\ & a*(a+b + \sqrt{a^2-b^2}*\tan[(c+d*x^n)/2])) + I*(\operatorname{PolyLog}[2, ((b - I*\sqrt{a^2-b^2})*(a+b - \sqrt{a^2-b^2})*\tan[(c+d*x^n)/2])]/( \\ & a*(a+b + \sqrt{a^2-b^2}*\tan[(c+d*x^n)/2])) - \operatorname{PolyLog}[2, ((b + I*\sqrt{a^2-b^2})*(a+b - \sqrt{a^2-b^2})*\tan[(c+d*x^n)/2])]/( \\ & a*(a+b + \sqrt{a^2-b^2}*\tan[(c+d*x^n)/2])))]* \operatorname{Sec}[c+d*x^n]^2/(a^2*(a^2-b^2)^{(3/2)}*d^{2*n}*(a+b*\operatorname{Sec}[c+d*x^n])^2) \\ & + (x^{(1-n)}*(e*x)^{-1+2*n}*(b+a*\cos[c+d*x^n])^2*\operatorname{Sec}[c+d*x^n]^2*(a^2*d*x^n*\cos[c] - b^2*d*x^n*\cos[c] + 2*b^2*\sin[c]))/(2*a^2*(a-b)* \\ & (a+b)*d*n*(a+b*\operatorname{Sec}[c+d*x^n])^2*(\cos[c/2] - \sin[c/2])*(\cos[c/2] + \sin[c/2])) + (b^2*x^{(1-2*n)}*(e*x)^{-1+2*n}*(b+a*\cos[c+d*x^n])^2*\operatorname{Sec}[c]* \\ & \operatorname{Sec}[c+d*x^n]^2*(a*\cos[c]*\log[b+a*\cos[c]*\cos[d*x^n] - a*\sin[c]*\sin[d*x^n] \end{aligned}$$

$$\begin{aligned} & ] + a*d*x^n*\sin[c] - ((2*I)*a*b*\text{ArcTan}[((-I)*a*\sin[c] - I*(-b + a*\cos[c]))* \\ & \text{Tan}[(d*x^n)/2]]/\text{Sqrt}[-b^2 + a^2*\cos[c]^2 + a^2*\sin[c]^2]*\sin[c])/\text{Sqrt}[-b^2 \\ & + a^2*\cos[c]^2 + a^2*\sin[c]^2))/ (a*(a^2 - b^2)*d^2*n*(a + b*\sec[c + d*x^n \\ & ])^2*(a^2*\cos[c]^2 + a^2*\sin[c]^2)) + (b^2*x^(1 - n)*(e*x)^(-1 + 2*n)*(b + \\ & a*\cos[c + d*x^n])*Sec[c + d*x^n]^2*(b*\sin[c] - a*\sin[d*x^n]))/(a^2*(-a + b) \\ & *(a + b)*d*n*(a + b*\sec[c + d*x^n])^2*(\cos[c/2] - \sin[c/2])*(\cos[c/2] + \sin \\ & [c/2])) + (b^2*x^(1 - n)*(e*x)^(-1 + 2*n)*(b + a*\cos[c + d*x^n])^2*Sec[c + \\ & d*x^n]^2*\text{Tan}[c])/ (a^2*(-a^2 + b^2)*d*n*(a + b*\sec[c + d*x^n])^2) - ((2*I)*b \\ & ^3*x^(1 - 2*n)*(e*x)^(-1 + 2*n)*\text{ArcTan}[(b + a*\cos[c + d*x^n] + I*a*\sin[c + \\ & d*x^n)]/\text{Sqrt}[a^2 - b^2]]*(b + a*\cos[c + d*x^n])^2*Sec[c + d*x^n]^2*\text{Tan}[c])/ \\ & (a^2*(a^2 - b^2)^(3/2)*d^2*n*(a + b*\sec[c + d*x^n])^2) \end{aligned}$$

**fricas [B]** time = 1.10, size = 2531, normalized size = 3.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^(-1+2\*n)/(a+b\*sec(c+d\*x^n))^2,x, algorithm="fricas")

[Out] 
$$\begin{aligned} & 1/4*(2*(a^5 - 2*a^3*b^2 + a*b^4)*d^2*e^(2*n - 1)*x^(2*n)*\cos(d*x^n + c) + 2 \\ & *(a^4*b - 2*a^2*b^3 + b^5)*d^2*e^(2*n - 1)*x^(2*n) + 4*(a^3*b^2 - a*b^4)*d* \\ & e^(2*n - 1)*x^n*\sin(d*x^n + c) - 2*((2*a^4*b - a^2*b^3)*e^(2*n - 1)*\text{sqrt}(-( \\ & a^2 - b^2)/a^2)*\cos(d*x^n + c) + (2*a^3*b^2 - a*b^4)*e^(2*n - 1)*\text{sqrt}(-(a^2 \\ & - b^2)/a^2))*\text{dilog}(-1/2*(2*(a*\text{sqrt}(-(a^2 - b^2)/a^2) + b)*\cos(d*x^n + c) + \\ & (2*I*a*\text{sqrt}(-(a^2 - b^2)/a^2) + 2*I*b)*\sin(d*x^n + c) + 2*a)/a + 1) - 2*(( \\ & 2*a^4*b - a^2*b^3)*e^(2*n - 1)*\text{sqrt}(-(a^2 - b^2)/a^2)*\cos(d*x^n + c) + (2*a \\ & ^3*b^2 - a*b^4)*e^(2*n - 1)*\text{sqrt}(-(a^2 - b^2)/a^2))*\text{dilog}(-1/2*(2*(a*\text{sqrt}(- \\ & (a^2 - b^2)/a^2) + b)*\cos(d*x^n + c) + (-2*I*a*\text{sqrt}(-(a^2 - b^2)/a^2) - 2*I \\ & *b)*\sin(d*x^n + c) + 2*a)/a + 1) + 2*((2*a^4*b - a^2*b^3)*e^(2*n - 1)*\text{sqrt}(- \\ & (a^2 - b^2)/a^2)*\cos(d*x^n + c) + (2*a^3*b^2 - a*b^4)*e^(2*n - 1)*\text{sqrt}(-(a \\ & ^2 - b^2)/a^2))*\text{dilog}(1/2*(2*(a*\text{sqrt}(-(a^2 - b^2)/a^2) - b)*\cos(d*x^n + c) \\ & - (2*I*a*\text{sqrt}(-(a^2 - b^2)/a^2) - 2*I*b)*\sin(d*x^n + c) - 2*a)/a + 1) + 2*( \\ & (2*a^4*b - a^2*b^3)*e^(2*n - 1)*\text{sqrt}(-(a^2 - b^2)/a^2)*\cos(d*x^n + c) + (2* \\ & a^3*b^2 - a*b^4)*e^(2*n - 1)*\text{sqrt}(-(a^2 - b^2)/a^2))*\text{dilog}(1/2*(2*(a*\text{sqrt}(- \\ & (a^2 - b^2)/a^2) - b)*\cos(d*x^n + c) - (-2*I*a*\text{sqrt}(-(a^2 - b^2)/a^2) + 2*I \\ & *b)*\sin(d*x^n + c) - 2*a)/a + 1) + (2*(a^3*b^2 - a*b^4 - I*(2*a^4*b - a^2*b \\ & ^3)*c*\text{sqrt}(-(a^2 - b^2)/a^2))*e^(2*n - 1)*\cos(d*x^n + c) + (2*a^2*b^3 - 2*b \\ & ^5 - 2*I*(2*a^3*b^2 - a*b^4)*c*\text{sqrt}(-(a^2 - b^2)/a^2))*e^(2*n - 1))*\log(2*a \\ & *\cos(d*x^n + c) + 2*I*a*\sin(d*x^n + c) + 2*a*\text{sqrt}(-(a^2 - b^2)/a^2) + 2*b) \\ & + (2*(a^3*b^2 - a*b^4 + I*(2*a^4*b - a^2*b^3)*c*\text{sqrt}(-(a^2 - b^2)/a^2))*e^( \\ & 2*n - 1)*\cos(d*x^n + c) + (2*a^2*b^3 - 2*b^5 + 2*I*(2*a^3*b^2 - a*b^4)*c*\text{sq} \\ & \text{rt}(-(a^2 - b^2)/a^2))*e^(2*n - 1))*\log(2*a*\cos(d*x^n + c) - 2*I*a*\sin(d*x^n \\ & + c) + 2*a*\text{sqrt}(-(a^2 - b^2)/a^2) + 2*b) + (2*(a^3*b^2 - a*b^4 - I*(2*a^4*b \\ & - a^2*b^3)*c*\text{sqrt}(-(a^2 - b^2)/a^2))*e^(2*n - 1)*\cos(d*x^n + c) + (2*a^2*b \\ & ^3 - 2*b^5 - 2*I*(2*a^3*b^2 - a*b^4)*c*\text{sqrt}(-(a^2 - b^2)/a^2))*e^(2*n - 1) \end{aligned}$$

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)*log(-2*a*cos(d*x^n + c) + 2*I*a*sin(d*x^n + c) + 2*a*sqrt(-(a^2 - b^2)/a^2) - 2*b) + (2*(a^3*b^2 - a*b^4 + I*(2*a^4*b - a^2*b^3))*c*sqrt(-(a^2 - b^2)/a^2))*e^(2*n - 1)*cos(d*x^n + c) + (2*a^2*b^3 - 2*b^5 + 2*I*(2*a^3*b^2 - a*b^4))*c*sqrt(-(a^2 - b^2)/a^2))*e^(2*n - 1))*log(-2*a*cos(d*x^n + c) - 2*I*a*sin(d*x^n + c) + 2*a*sqrt(-(a^2 - b^2)/a^2) - 2*b) + (-2*I*(2*a^3*b^2 - a*b^4)*d*e^(2*n - 1)*x^n*sqrt(-(a^2 - b^2)/a^2) - 2*I*(2*a^3*b^2 - a*b^4))*c*e^(2*n - 1)*sqrt(-(a^2 - b^2)/a^2) + (-2*I*(2*a^4*b - a^2*b^3)*d*e^(2*n - 1)*x^n*sqrt(-(a^2 - b^2)/a^2) - 2*I*(2*a^4*b - a^2*b^3))*c*e^(2*n - 1)*sqrt(-(a^2 - b^2)/a^2))*cos(d*x^n + c))*log(1/2*(2*(a*sqrt(-(a^2 - b^2)/a^2) + b)*cos(d*x^n + c) + (2*I*a*sqrt(-(a^2 - b^2)/a^2) + 2*I*b)*sin(d*x^n + c) + 2*a)/a) + (2*I*(2*a^3*b^2 - a*b^4)*d*e^(2*n - 1)*x^n*sqrt(-(a^2 - b^2)/a^2) + 2*I*(2*a^3*b^2 - a*b^4))*c*e^(2*n - 1)*sqrt(-(a^2 - b^2)/a^2) + (2*I*(2*a^4*b - a^2*b^3)*d*e^(2*n - 1)*x^n*sqrt(-(a^2 - b^2)/a^2) + 2*I*(2*a^4*b - a^2*b^3))*c*e^(2*n - 1)*sqrt(-(a^2 - b^2)/a^2))*cos(d*x^n + c))*log(1/2*(2*(a*sqrt(-(a^2 - b^2)/a^2) + b)*cos(d*x^n + c) + (-2*I*a*sqrt(-(a^2 - b^2)/a^2) - 2*I*b)*sin(d*x^n + c) + 2*a)/a) + (-2*I*(2*a^3*b^2 - a*b^4)*d*e^(2*n - 1)*x^n*sqrt(-(a^2 - b^2)/a^2) - 2*I*(2*a^3*b^2 - a*b^4))*c*e^(2*n - 1)*sqrt(-(a^2 - b^2)/a^2) + (-2*I*(2*a^4*b - a^2*b^3)*d*e^(2*n - 1)*x^n*sqrt(-(a^2 - b^2)/a^2) - 2*I*(2*a^4*b - a^2*b^3))*c*e^(2*n - 1)*sqrt(-(a^2 - b^2)/a^2))*cos(d*x^n + c))*log(-1/2*(2*(a*sqrt(-(a^2 - b^2)/a^2) - b)*cos(d*x^n + c) - (2*I*a*sqrt(-(a^2 - b^2)/a^2) - 2*I*b)*sin(d*x^n + c) - 2*a)/a) + (2*I*(2*a^3*b^2 - a*b^4)*d*e^(2*n - 1)*x^n*sqrt(-(a^2 - b^2)/a^2) + 2*I*(2*a^3*b^2 - a*b^4))*c*e^(2*n - 1)*sqrt(-(a^2 - b^2)/a^2) + (2*I*(2*a^4*b - a^2*b^3)*d*e^(2*n - 1)*x^n*sqrt(-(a^2 - b^2)/a^2) + 2*I*(2*a^4*b - a^2*b^3))*c*e^(2*n - 1)*sqrt(-(a^2 - b^2)/a^2))*cos(d*x^n + c))*log(-1/2*(2*(a*sqrt(-(a^2 - b^2)/a^2) - b)*cos(d*x^n + c) - (-2*I*a*sqrt(-(a^2 - b^2)/a^2) + 2*I*b)*sin(d*x^n + c) - 2*a)/a))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d^2*n*cos(d*x^n + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d^2*n)

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**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^{2n-1}}{(b \sec(dx^n + c) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^(-1+2\*n)/(a+b\*sec(c+d\*x^n))^2,x, algorithm="giac")

[Out] integrate((e\*x)^(2\*n - 1)/(b\*sec(d\*x^n + c) + a)^2, x)

**maple** [C] time = 1.14, size = 2817, normalized size = 3.72

output too large to display

Verification of antiderivative is not currently implemented for this CAS.



[In]  $\text{int}((e*x)^{-1+2*n}/(a+b*\text{sec}(c+d*x^n))^2, x)$

[Out]  $\frac{1}{2} \frac{1}{a^2} \frac{1}{n} x \exp\left(\frac{1}{2}(-1+2n)(-i\pi \text{csgn}(Ie) \text{csgn}(Ix) \text{csgn}(Ie*x) + i\pi \text{csgn}(Ie) \text{csgn}(Ie*x)^2 + i\pi \text{csgn}(Ix) \text{csgn}(Ie*x)^2 - i\pi \text{csgn}(Ie*x)^3 + 2 \ln(x) + 2 \ln(e))\right) + 2 \frac{b^2}{a^2} \frac{1}{(a^2-b^2)} \frac{1}{d} \frac{1}{n} \frac{1}{(x^n)} \frac{1}{(\exp(2I(c+d*x^n)) + a + 2b \exp(I(c+d*x^n)))} + a \frac{1}{(b(e^n)^2(x^n)^2/e/x(-1)^{-1/2 \text{csgn}(Ix) \text{csgn}(Ie*x)^2})} (-1)^{-1/2 \text{csgn}(Ie) \text{csgn}(Ie*x)^2} (-1)^{1/2 \text{csgn}(Ie) \text{csgn}(Ix) \text{csgn}(Ie*x)} \exp\left(\frac{1}{2} i\pi \text{csgn}(Ie*x)^3\right) \exp(-i\pi n \text{csgn}(Ie) \text{csgn}(Ix) \text{csgn}(Ie*x)) \exp(i\pi n \text{csgn}(Ie) \text{csgn}(Ie*x)^2) \exp(i\pi n \text{csgn}(Ix) \text{csgn}(Ie*x)^2) \exp(-i\pi n \text{csgn}(Ie*x)^3) \exp(Ix^n d) \exp(Ic) + a (e^n)^2 (x^n)^2 / e / x (-1)^{-1/2 \text{csgn}(Ix) \text{csgn}(Ie*x)^2} (-1)^{-1/2 \text{csgn}(Ie) \text{csgn}(Ie*x)^2} (-1)^{1/2 \text{csgn}(Ie) \text{csgn}(Ix) \text{csgn}(Ie*x)} \exp\left(\frac{1}{2} i\pi \text{csgn}(Ie*x) (-2n \text{csgn}(Ie) \text{csgn}(Ix) + 2n \text{csgn}(Ie) \text{csgn}(Ie*x) + 2n \text{csgn}(Ix) \text{csgn}(Ie*x) - 2n \text{csgn}(Ie*x)^2 + \text{csgn}(Ie*x)^2)\right) - \frac{b^3}{a^2} \frac{1}{(a^2-b^2)} \frac{1}{d} \ln\left(\frac{-a \exp(I(d*x^n+2c)) - \exp(Ic) * b + (\exp(2Ic) * b^2 - a^2 \exp(2Ic))^{1/2}}{-\exp(Ic) * b + (\exp(2Ic) * b^2 - a^2 \exp(2Ic))^{1/2}}\right) / (\exp(2Ic) * b^2 - a^2 \exp(2Ic))^{1/2} * x^n / n / e * (e^n)^2 \exp\left(\frac{1}{2} I (-2\pi n \text{csgn}(Ie*x)^3 + 2\pi n \text{csgn}(Ie) \text{csgn}(Ie*x)^2 + 2\pi n \text{csgn}(Ix) \text{csgn}(Ie*x)^2 - 2\pi n \text{csgn}(Ie) \text{csgn}(Ix) \text{csgn}(Ie*x) + \pi \text{csgn}(Ie*x)^3 - \pi \text{csgn}(Ie) \text{csgn}(Ie*x)^2 - \pi \text{csgn}(Ix) \text{csgn}(Ie*x)^2 + \pi \text{csgn}(Ie*x) \text{csgn}(Ie) \text{csgn}(Ix) + 2c)\right) - 2 \frac{b}{a^2} \frac{1}{(a^2-b^2)} \frac{1}{d} \ln\left(\frac{a \exp(I(d*x^n+2c)) + \exp(Ic) * b + (\exp(2Ic) * b^2 - a^2 \exp(2Ic))^{1/2}}{\exp(Ic) * b + (\exp(2Ic) * b^2 - a^2 \exp(2Ic))^{1/2}}\right) / (\exp(2Ic) * b^2 - a^2 \exp(2Ic))^{1/2} * x^n / n / e * (e^n)^2 \exp\left(\frac{1}{2} I (-2\pi n \text{csgn}(Ie*x)^3 + 2\pi n \text{csgn}(Ie) \text{csgn}(Ie*x)^2 + 2\pi n \text{csgn}(Ix) \text{csgn}(Ie*x)^2 - 2\pi n \text{csgn}(Ie) \text{csgn}(Ix) \text{csgn}(Ie*x) + \pi \text{csgn}(Ie*x)^3 - \pi \text{csgn}(Ie) \text{csgn}(Ie*x)^2 - \pi \text{csgn}(Ix) \text{csgn}(Ie*x)^2 + \pi \text{csgn}(Ie*x) \text{csgn}(Ie) \text{csgn}(Ix) + 2c)\right) + \frac{b^3}{a^2} \frac{1}{(a^2-b^2)} \frac{1}{d} \ln\left(\frac{a \exp(I(d*x^n+2c)) + \exp(Ic) * b + (\exp(2Ic) * b^2 - a^2 \exp(2Ic))^{1/2}}{\exp(Ic) * b + (\exp(2Ic) * b^2 - a^2 \exp(2Ic))^{1/2}}\right) / (\exp(2Ic) * b^2 - a^2 \exp(2Ic))^{1/2} * x^n / n / e * (e^n)^2 \exp\left(\frac{1}{2} I (-2\pi n \text{csgn}(Ie*x)^3 + 2\pi n \text{csgn}(Ie) \text{csgn}(Ie*x)^2 + 2\pi n \text{csgn}(Ix) \text{csgn}(Ie*x)^2 - 2\pi n \text{csgn}(Ie) \text{csgn}(Ix) \text{csgn}(Ie*x) + \pi \text{csgn}(Ie*x)^3 - \pi \text{csgn}(Ie) \text{csgn}(Ie*x)^2 - \pi \text{csgn}(Ix) \text{csgn}(Ie*x)^2 + \pi \text{csgn}(Ie*x) \text{csgn}(Ie) \text{csgn}(Ix) + 2c)\right) + 2 \frac{b}{a^2} \frac{1}{(a^2-b^2)} \frac{1}{d} \ln\left(\frac{-a \exp(I(d*x^n+2c)) - \exp(Ic) * b + (\exp(2Ic) * b^2 - a^2 \exp(2Ic))^{1/2}}{-\exp(Ic) * b + (\exp(2Ic) * b^2 - a^2 \exp(2Ic))^{1/2}}\right) / (\exp(2Ic) * b^2 - a^2 \exp(2Ic))^{1/2} * x^n / n / e * (e^n)^2 \exp\left(\frac{1}{2} I (-2\pi n \text{csgn}(Ie*x)^3 + 2\pi n \text{csgn}(Ie) \text{csgn}(Ie*x)^2 + 2\pi n \text{csgn}(Ix) \text{csgn}(Ie*x)^2 - 2\pi n \text{csgn}(Ie) \text{csgn}(Ix) \text{csgn}(Ie*x) + \pi \text{csgn}(Ie*x)^3 - \pi \text{csgn}(Ie) \text{csgn}(Ie*x)^2 - \pi \text{csgn}(Ix) \text{csgn}(Ie*x)^2 + \pi \text{csgn}(Ie*x) \text{csgn}(Ie) \text{csgn}(Ix) + 2c)\right) + 2 \frac{b^2}{a^2} \frac{1}{(a^2-b^2)} \frac{1}{d^2} \ln(\exp(Ix^n d)) / n / e * (e^n)^2 \exp\left(\frac{1}{2} I \text{csgn}(Ie*x) \pi (-1+2n) (\text{csgn}(Ie*x) - \text{csgn}(Ix)) * (-\text{csgn}(Ie*x) + \text{csgn}(Ie))\right) + \frac{b^2}{a^2} \frac{1}{(a^2-b^2)} \frac{1}{d^2} \ln(\exp(2I(c+d*x^n)) + a + 2b \exp(I(c+d*x^n))) / n / e * (e^n)^2 \exp\left(\frac{1}{2} I \text{csgn}(Ie*x) \pi (-1+2n) (\text{csgn}(Ie*x) - \text{csgn}(Ix)) * (-\text{csgn}(Ie*x) + \text{csgn}(Ie))\right) + \frac{b^3}{a^2} \frac{1}{(a^2-b^2)} \frac{1}{d^2} \text{dilog}\left(\frac{a \exp(I(d*x^n+2c)) + \exp(Ic) * b + (\exp(2Ic) * b^2 - a^2 \exp(2Ic))^{1/2}}{\exp(Ic) * b + (\exp(2Ic) * b^2 - a^2 \exp(2Ic))^{1/2}}\right) / (\exp(2Ic) * b^2 - a^2 \exp(2Ic))^{1/2} / n / e * (e^n)^2 \exp\left(\frac{1}{2} I (-2\pi n \text{csgn}(Ie*x)^3 + 2\pi n \text{csgn}(Ie) \text{csgn}(Ie*x)^2 + 2\pi n \text{csgn}(Ix) \text{csgn}(Ie*x)^2 - 2\pi n \text{csgn}(Ie) \text{csgn}(Ix) \text{csgn}(Ie*x) + \pi \text{csgn}(Ie*x)^3 - \pi \text{csgn}(Ie) \text{csgn}(Ie*x)^2 - \pi \text{csgn}(Ix) \text{csgn}(Ie*x)^2 + \pi \text{csgn}(Ie*x) \text{csgn}(Ie) \text{csgn}(Ix) + 2c)\right)$

$x) * \operatorname{csgn}(I * e * x)^{2-2 * \operatorname{Pi} * n} * \operatorname{csgn}(I * e) * \operatorname{csgn}(I * x) * \operatorname{csgn}(I * e * x) + \operatorname{Pi} * \operatorname{csgn}(I * e * x)^{3-\operatorname{Pi}} * \operatorname{csgn}(I * e) * \operatorname{csgn}(I * e * x)^{2-\operatorname{Pi}} * \operatorname{csgn}(I * x) * \operatorname{csgn}(I * e * x)^{2+\operatorname{Pi}} * \operatorname{csgn}(I * e * x) * \operatorname{csgn}(I * e) * \operatorname{csgn}(I * x + 2 * c) + 2 * b / (a^2 - b^2) / d^2 * \operatorname{dilog}((-a * \exp(I * (d * x^n + 2 * c)) - \exp(I * c) * b + (\exp(2 * I * c) * b^2 - a^2 * \exp(2 * I * c))^{1/2}) / (-\exp(I * c) * b + (\exp(2 * I * c) * b^2 - a^2 * \exp(2 * I * c))^{1/2})) / (\exp(2 * I * c) * b^2 - a^2 * \exp(2 * I * c))^{1/2} / n / e * (e^n)^2 * \exp(1/2 * I * (-2 * \operatorname{Pi} * n * \operatorname{csgn}(I * e * x)^3 + 2 * \operatorname{Pi} * n * \operatorname{csgn}(I * e) * \operatorname{csgn}(I * e * x)^2 + 2 * \operatorname{Pi} * n * \operatorname{csgn}(I * x) * \operatorname{csgn}(I * e * x)^2 - 2 * \operatorname{Pi} * n * \operatorname{csgn}(I * e) * \operatorname{csgn}(I * x) * \operatorname{csgn}(I * e * x) + \operatorname{Pi} * \operatorname{csgn}(I * e * x)^3 - \operatorname{Pi} * \operatorname{csgn}(I * e) * \operatorname{csgn}(I * e * x)^2 - \operatorname{Pi} * \operatorname{csgn}(I * x) * \operatorname{csgn}(I * e * x)^2 + \operatorname{Pi} * \operatorname{csgn}(I * e * x) * \operatorname{csgn}(I * e) * \operatorname{csgn}(I * x + 2 * c)) - b^3 / a^2 / (a^2 - b^2) / d^2 * \operatorname{dilog}((-a * \exp(I * (d * x^n + 2 * c)) - \exp(I * c) * b + (\exp(2 * I * c) * b^2 - a^2 * \exp(2 * I * c))^{1/2}) / (-\exp(I * c) * b + (\exp(2 * I * c) * b^2 - a^2 * \exp(2 * I * c))^{1/2})) / (\exp(2 * I * c) * b^2 - a^2 * \exp(2 * I * c))^{1/2} / n / e * (e^n)^2 * \exp(1/2 * I * (-2 * \operatorname{Pi} * n * \operatorname{csgn}(I * e * x)^3 + 2 * \operatorname{Pi} * n * \operatorname{csgn}(I * e) * \operatorname{csgn}(I * e * x)^2 + 2 * \operatorname{Pi} * n * \operatorname{csgn}(I * x) * \operatorname{csgn}(I * e * x)^2 - 2 * \operatorname{Pi} * n * \operatorname{csgn}(I * e) * \operatorname{csgn}(I * x) * \operatorname{csgn}(I * e * x) + \operatorname{Pi} * \operatorname{csgn}(I * e * x)^3 - \operatorname{Pi} * \operatorname{csgn}(I * e) * \operatorname{csgn}(I * e * x)^2 - \operatorname{Pi} * \operatorname{csgn}(I * x) * \operatorname{csgn}(I * e * x)^2 + \operatorname{Pi} * \operatorname{csgn}(I * e * x) * \operatorname{csgn}(I * e) * \operatorname{csgn}(I * x + 2 * c))$

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^(-1+2\*n)/(a+b\*sec(c+d\*x^n))^2,x, algorithm="maxima")

[Out] Timed out

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(ex)^{2n-1}}{\left(a + \frac{b}{\cos(c+dx^n)}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x)^(2\*n - 1)/(a + b/cos(c + d\*x^n))^2,x)

[Out] int((e\*x)^(2\*n - 1)/(a + b/cos(c + d\*x^n))^2, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^{2n-1}}{(a + b \sec(c + dx^n))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(-1+2*n)/(a+b*sec(c+d*x**n))**2,x)
```

```
[Out] Integral((e*x)**(2*n - 1)/(a + b*sec(c + d*x**n))**2, x)
```

$$3.83 \quad \int \frac{(ex)^{-1+3n}}{(a+b \sec(c+dx^n))^2} dx$$

**Optimal.** Leaf size=1384

$$\frac{2ib^2(ex)^{3n} \operatorname{Li}_2\left(-\frac{ae^{i(dx^n+c)}}{b-i\sqrt{a^2-b^2}}\right) x^{-3n}}{a^2(a^2-b^2)d^3en} - \frac{2ib^2(ex)^{3n} \operatorname{Li}_2\left(-\frac{ae^{i(dx^n+c)}}{b+i\sqrt{a^2-b^2}}\right) x^{-3n}}{a^2(a^2-b^2)d^3en} + \frac{4ib(ex)^{3n} \operatorname{Li}_3\left(-\frac{ae^{i(dx^n+c)}}{b-\sqrt{b^2-a^2}}\right) x^{-3n}}{a^2\sqrt{b^2-a^2}d^3en} - \frac{2ib^3(ex)^{3n} \operatorname{Li}_3\left(-\frac{ae^{i(dx^n+c)}}{b+\sqrt{b^2-a^2}}\right) x^{-3n}}{a^2\sqrt{b^2-a^2}d^3en}$$

[Out]  $\frac{1}{3} \frac{(ex)^{3n}}{a^2} \frac{\ln(1+a \exp(I(c+dx^n)))}{b+(-a^2+b^2)^{1/2}} - \frac{1}{3} \frac{(ex)^{3n}}{a^2} \frac{\ln(1+a \exp(I(c+dx^n)))}{b-(-a^2+b^2)^{1/2}} + \frac{2}{3} \frac{(ex)^{3n}}{a^2} \frac{\operatorname{Li}_2\left(-\frac{ae^{i(dx^n+c)}}{b-i\sqrt{a^2-b^2}}\right)}{d^3en} - \frac{2}{3} \frac{(ex)^{3n}}{a^2} \frac{\operatorname{Li}_2\left(-\frac{ae^{i(dx^n+c)}}{b+i\sqrt{a^2-b^2}}\right)}{d^3en} + \frac{4}{3} \frac{(ex)^{3n}}{a^2} \frac{\operatorname{Li}_3\left(-\frac{ae^{i(dx^n+c)}}{b-\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}d^3en} - \frac{4}{3} \frac{(ex)^{3n}}{a^2} \frac{\operatorname{Li}_3\left(-\frac{ae^{i(dx^n+c)}}{b+\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}d^3en}$

**Rubi [A]** time = 2.37, antiderivative size = 1384, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 13, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.542$ , Rules used = {4208, 4204, 4191, 3324, 3321, 2264, 2190, 2531, 2282, 6589, 4522, 2279, 2391}

$$\frac{2ib^2(ex)^{3n} \operatorname{PolyLog}\left(2, -\frac{ae^{i(dx^n+c)}}{b-i\sqrt{a^2-b^2}}\right) x^{-3n}}{a^2(a^2-b^2)d^3en} - \frac{2ib^2(ex)^{3n} \operatorname{PolyLog}\left(2, -\frac{ae^{i(dx^n+c)}}{b+i\sqrt{a^2-b^2}}\right) x^{-3n}}{a^2(a^2-b^2)d^3en} + \frac{4ib(ex)^{3n} \operatorname{PolyLog}\left(3, -\frac{ae^{i(dx^n+c)}}{b-\sqrt{b^2-a^2}}\right) x^{-3n}}{a^2\sqrt{b^2-a^2}d^3en} - \frac{4ib^3(ex)^{3n} \operatorname{PolyLog}\left(3, -\frac{ae^{i(dx^n+c)}}{b+\sqrt{b^2-a^2}}\right) x^{-3n}}{a^2\sqrt{b^2-a^2}d^3en}$$

Antiderivative was successfully verified.

[In] Int[(e\*x)^(-1 + 3\*n)/(a + b\*Sec[c + d\*x^n])^2,x]

[Out] (e\*x)^(3\*n)/(3\*a^2\*e\*n) - (I\*b^2\*(e\*x)^(3\*n))/(a^2\*(a^2 - b^2)\*d\*e\*n\*x^n) + (2\*b^2\*(e\*x)^(3\*n)\*Log[1 + (a\*E^(I\*(c + d\*x^n)))/(b - I\*Sqrt[a^2 - b^2])])/(a^2\*(a^2 - b^2)\*d^2\*e\*n\*x^(2\*n)) + (2\*b^2\*(e\*x)^(3\*n)\*Log[1 + (a\*E^(I\*(c + d\*x^n)))/(b + I\*Sqrt[a^2 - b^2])])/(a^2\*(a^2 - b^2)\*d^2\*e\*n\*x^(2\*n)) - (I\*b^3\*(e\*x)^(3\*n)\*Log[1 + (a\*E^(I\*(c + d\*x^n)))/(b - Sqrt[-a^2 + b^2])])/(a^2\*(-a^2 + b^2)^(3/2)\*d\*e\*n\*x^n) + ((2\*I)\*b\*(e\*x)^(3\*n)\*Log[1 + (a\*E^(I\*(c + d\*x^n)))/(b - Sqrt[-a^2 + b^2])])/(a^2\*Sqrt[-a^2 + b^2]\*d\*e\*n\*x^n) + (I\*b^3\*(e\*x)^(3\*n)\*Log[1 + (a\*E^(I\*(c + d\*x^n)))/(b + Sqrt[-a^2 + b^2])])/(a^2\*(-a^2 + b^2)^(3/2)\*d\*e\*n\*x^n) - ((2\*I)\*b\*(e\*x)^(3\*n)\*Log[1 + (a\*E^(I\*(c + d\*x^n)))/(b + Sqrt[-a^2 + b^2])])/(a^2\*Sqrt[-a^2 + b^2]\*d\*e\*n\*x^n) - ((2\*I)\*b^2\*(e\*x)^(3\*n)\*PolyLog[2, -((a\*E^(I\*(c + d\*x^n)))/(b - I\*Sqrt[a^2 - b^2])])]/(a^2\*(a^2 - b^2)\*d^3\*e\*n\*x^(3\*n)) - ((2\*I)\*b^2\*(e\*x)^(3\*n)\*PolyLog[2, -((a\*E^(I\*(c + d\*x^n)))/(b + I\*Sqrt[a^2 - b^2])])]/(a^2\*(a^2 - b^2)\*d^3\*e\*n\*x^(3\*n)) - (2\*b^3\*(e\*x)^(3\*n)\*PolyLog[2, -((a\*E^(I\*(c + d\*x^n)))/(b - Sqrt[-a^2 + b^2])])]/(a^2\*(-a^2 + b^2)^(3/2)\*d^2\*e\*n\*x^(2\*n)) + (4\*b\*(e\*x)^(3\*n)\*PolyLog[2, -((a\*E^(I\*(c + d\*x^n)))/(b - Sqrt[-a^2 + b^2])])]/(a^2\*Sqrt[-a^2 + b^2]\*d^2\*e\*n\*x^(2\*n)) + (2\*b^3\*(e\*x)^(3\*n)\*PolyLog[2, -((a\*E^(I\*(c + d\*x^n)))/(b + Sqrt[-a^2 + b^2])])]/(a^2\*(-a^2 + b^2)^(3/2)\*d^2\*e\*n\*x^(2\*n)) - (4\*b\*(e\*x)^(3\*n)\*PolyLog[2, -((a\*E^(I\*(c + d\*x^n)))/(b + Sqrt[-a^2 + b^2])])]/(a^2\*Sqrt[-a^2 + b^2]\*d^2\*e\*n\*x^(2\*n)) - ((2\*I)\*b^3\*(e\*x)^(3\*n)\*PolyLog[3, -((a\*E^(I\*(c + d\*x^n)))/(b - Sqrt[-a^2 + b^2])])]/(a^2\*(-a^2 + b^2)^(3/2)\*d^3\*e\*n\*x^(3\*n)) + ((4\*I)\*b\*(e\*x)^(3\*n)\*PolyLog[3, -((a\*E^(I\*(c + d\*x^n)))/(b - Sqrt[-a^2 + b^2])])]/(a^2\*Sqrt[-a^2 + b^2]\*d^3\*e\*n\*x^(3\*n)) + ((2\*I)\*b^3\*(e\*x)^(3\*n)\*PolyLog[3, -((a\*E^(I\*(c + d\*x^n)))/(b + Sqrt[-a^2 + b^2])])]/(a^2\*(-a^2 + b^2)^(3/2)\*d^3\*e\*n\*x^(3\*n)) - ((4\*I)\*b\*(e\*x)^(3\*n)\*PolyLog[3, -((a\*E^(I\*(c + d\*x^n)))/(b + Sqrt[-a^2 + b^2])])]/(a^2\*Sqrt[-a^2 + b^2]\*d^3\*e\*n\*x^(3\*n)) + (b^2\*(e\*x)^(3\*n)\*Sin[c + d\*x^n])/(a\*(a^2 - b^2)\*d\*e\*n\*x^n\*(b + a\*Cos[c + d\*x^n]))

### Rule 2190

Int[(((F\_)^(g\_)\*((e\_) + (f\_)\*(x\_)))^(n\_)\*((c\_) + (d\_)\*(x\_))^(m\_))/((a\_) + (b\_)\*((F\_)^(g\_)\*((e\_) + (f\_)\*(x\_)))^(n\_)), x\_Symbol] := Simp[(((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2264

Int[(((F\_)^(u\_)\*((f\_) + (g\_)\*(x\_))^(m\_))/((a\_) + (b\_)\*(F\_)^(u\_) + (c\_)\*(F\_)^(v\_)), x\_Symbol] := With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[(2\*c)/q, Int[(f + g\*x)^m\*F^u/(b - q + 2\*c\*F^u), x], x] - Dist[(2\*c)/q, Int[(f + g\*x)^m\*F^u/(b + q + 2\*c\*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,

2\*u] && LinearQ[u, x] && NeQ[b^2 - 4\*a\*c, 0] && IGtQ[m, 0]

### Rule 2279

Int[Log[(a\_) + (b\_.)\*((F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(n\_.)], x\_Symbol]  
 := Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

### Rule 2282

Int[u\_, x\_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_.)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_.)\*((a\_.) + (b\_.)\*x))\*(F\_)[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rule 2531

Int[Log[1 + (e\_.)\*((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(n\_.)]\*((f\_.) + (g\_.)\*(x\_)^(m\_.), x\_Symbol] := -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x))))^n]]/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m - 1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

### Rule 3321

Int[((c\_.) + (d\_.)\*(x\_)^(m\_.))/((a\_) + (b\_.)\*sin[(e\_.) + Pi\*(k\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Dist[2, Int[((c + d\*x)^m\*E^(I\*Pi\*(k - 1/2))\*E^(I\*(e + f\*x)))/(b + 2\*a\*E^(I\*Pi\*(k - 1/2))\*E^(I\*(e + f\*x)) - b\*E^(2\*I\*k\*Pi)\*E^(2\*I\*(e + f\*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2\*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]

### Rule 3324

Int[((c\_.) + (d\_.)\*(x\_)^(m\_.))/((a\_) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^2, x\_Symbol] := Simp[(b\*(c + d\*x)^m\*Cos[e + f\*x])/(f\*(a^2 - b^2)\*(a + b\*Sin[e + f\*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d\*x)^m/(a + b\*Sin[e + f\*x]), x], x] - Dist[(b\*d\*m)/(f\*(a^2 - b^2)), Int[((c + d\*x)^(m - 1)\*Cos[e + f\*x])/(a + b\*Sin[e + f\*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^

2, 0] && IGtQ[m, 0]

### Rule 4191

Int[(csc[(e\_.) + (f\_.)\*(x\_.)]\*(b\_.) + (a\_.))^(n\_.)\*((c\_.) + (d\_.)\*(x\_.))^(m\_.), x\_Symbol] :> Int[ExpandIntegrand[(c + d\*x)^m, 1/(Sin[e + f\*x]^n/(b + a\*Sine[e + f\*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGtQ[m, 0]

### Rule 4204

Int[(x\_)^(m\_.)\*((a\_.) + (b\_.)\*Sec[(c\_.) + (d\_.)\*(x\_)^(n\_.)])^(p\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*Sec[c + d\*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]

### Rule 4208

Int[((e\_.)\*(x\_.))^(m\_.)\*((a\_.) + (b\_.)\*Sec[(c\_.) + (d\_.)\*(x\_)^(n\_.)])^(p\_.), x\_Symbol] :> Dist[(e^IntPart[m]\*(e\*x)^FracPart[m])/x^FracPart[m], Int[x^m\*(a + b\*Sec[c + d\*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]

### Rule 4522

Int[(((e\_.) + (f\_.)\*(x\_.))^(m\_.)\*Sin[(c\_.) + (d\_.)\*(x\_.)]/(Cos[(c\_.) + (d\_.)\*(x\_.)]\*(b\_.) + (a\_.)), x\_Symbol] :> Simp[(I\*(e + f\*x)^(m + 1))/(b\*f\*(m + 1)), x] + (Int[((e + f\*x)^m\*E^(I\*(c + d\*x)))/(I\*a - Rt[-a^2 + b^2, 2] + I\*b\*E^(I\*(c + d\*x))), x] + Int[((e + f\*x)^m\*E^(I\*(c + d\*x)))/(I\*a + Rt[-a^2 + b^2, 2] + I\*b\*E^(I\*(c + d\*x))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NegQ[a^2 - b^2]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_.))^(p\_.)]/((d\_.) + (e\_.)\*(x\_.)), x\_Symbol] :> Simp[PolyLog[n + 1, c\*(a + b\*x)^p/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rubi steps

$$\begin{aligned}
\int \frac{(ex)^{-1+3n}}{(a+b \sec(c+dx^n))^2} dx &= \frac{(x^{-3n}(ex)^{3n}) \int \frac{x^{-1+3n}}{(a+b \sec(c+dx^n))^2} dx}{e} \\
&= \frac{(x^{-3n}(ex)^{3n}) \text{Subst} \left( \int \frac{x^2}{(a+b \sec(c+dx))^2} dx, x, x^n \right)}{en} \\
&= \frac{(x^{-3n}(ex)^{3n}) \text{Subst} \left( \int \left( \frac{x^2}{a^2} + \frac{b^2 x^2}{a^2(b+a \cos(c+dx))^2} - \frac{2bx^2}{a^2(b+a \cos(c+dx))} \right) dx, x, x^n \right)}{en} \\
&= \frac{(ex)^{3n}}{3a^2 en} - \frac{(2bx^{-3n}(ex)^{3n}) \text{Subst} \left( \int \frac{x^2}{b+a \cos(c+dx)} dx, x, x^n \right)}{a^2 en} + \frac{(b^2 x^{-3n}(ex)^{3n}) \text{Subst} \left( \int \frac{x^2}{b+a \cos(c+dx)} dx, x, x^n \right)}{a^2 en} \\
&= \frac{(ex)^{3n}}{3a^2 en} + \frac{b^2 x^{-n}(ex)^{3n} \sin(c+dx^n)}{a(a^2-b^2) den (b+a \cos(c+dx^n))} - \frac{(4bx^{-3n}(ex)^{3n}) \text{Subst} \left( \int \frac{e^{i(c+dx)}}{a+2be^{i(c+dx)}} dx, x, x^n \right)}{a^2 en} \\
&= \frac{(ex)^{3n}}{3a^2 en} - \frac{ib^2 x^{-n}(ex)^{3n}}{a^2(a^2-b^2) den} + \frac{b^2 x^{-n}(ex)^{3n} \sin(c+dx^n)}{a(a^2-b^2) den (b+a \cos(c+dx^n))} - \frac{(2b^3 x^{-3n}(ex)^{3n}) \text{Subst} \left( \int \frac{e^{i(c+dx)}}{a+2be^{i(c+dx)}} dx, x, x^n \right)}{a^2 en} \\
&= \frac{(ex)^{3n}}{3a^2 en} - \frac{ib^2 x^{-n}(ex)^{3n}}{a^2(a^2-b^2) den} + \frac{2b^2 x^{-2n}(ex)^{3n} \log \left( 1 + \frac{ae^{i(c+dx^n)}}{b-i\sqrt{a^2-b^2}} \right)}{a^2(a^2-b^2) d^2 en} + \frac{2b^2 x^{-2n}(ex)^{3n} \log \left( 1 + \frac{ae^{i(c+dx^n)}}{b-i\sqrt{a^2-b^2}} \right)}{a^2(a^2-b^2) d^2 en} \\
&= \frac{(ex)^{3n}}{3a^2 en} - \frac{ib^2 x^{-n}(ex)^{3n}}{a^2(a^2-b^2) den} + \frac{2b^2 x^{-2n}(ex)^{3n} \log \left( 1 + \frac{ae^{i(c+dx^n)}}{b-i\sqrt{a^2-b^2}} \right)}{a^2(a^2-b^2) d^2 en} + \frac{2b^2 x^{-2n}(ex)^{3n} \log \left( 1 + \frac{ae^{i(c+dx^n)}}{b-i\sqrt{a^2-b^2}} \right)}{a^2(a^2-b^2) d^2 en} \\
&= \frac{(ex)^{3n}}{3a^2 en} - \frac{ib^2 x^{-n}(ex)^{3n}}{a^2(a^2-b^2) den} + \frac{2b^2 x^{-2n}(ex)^{3n} \log \left( 1 + \frac{ae^{i(c+dx^n)}}{b-i\sqrt{a^2-b^2}} \right)}{a^2(a^2-b^2) d^2 en} + \frac{2b^2 x^{-2n}(ex)^{3n} \log \left( 1 + \frac{ae^{i(c+dx^n)}}{b-i\sqrt{a^2-b^2}} \right)}{a^2(a^2-b^2) d^2 en} \\
&= \frac{(ex)^{3n}}{3a^2 en} - \frac{ib^2 x^{-n}(ex)^{3n}}{a^2(a^2-b^2) den} + \frac{2b^2 x^{-2n}(ex)^{3n} \log \left( 1 + \frac{ae^{i(c+dx^n)}}{b-i\sqrt{a^2-b^2}} \right)}{a^2(a^2-b^2) d^2 en} + \frac{2b^2 x^{-2n}(ex)^{3n} \log \left( 1 + \frac{ae^{i(c+dx^n)}}{b-i\sqrt{a^2-b^2}} \right)}{a^2(a^2-b^2) d^2 en} \\
&= \frac{(ex)^{3n}}{3a^2 en} - \frac{ib^2 x^{-n}(ex)^{3n}}{a^2(a^2-b^2) den} + \frac{2b^2 x^{-2n}(ex)^{3n} \log \left( 1 + \frac{ae^{i(c+dx^n)}}{b-i\sqrt{a^2-b^2}} \right)}{a^2(a^2-b^2) d^2 en} + \frac{2b^2 x^{-2n}(ex)^{3n} \log \left( 1 + \frac{ae^{i(c+dx^n)}}{b-i\sqrt{a^2-b^2}} \right)}{a^2(a^2-b^2) d^2 en}
\end{aligned}$$



**Mathematica** [F] time = 11.75, size = 0, normalized size = 0.00

$$\int \frac{(ex)^{-1+3n}}{(a + b \sec(c + dx^n))^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(e\*x)^(-1 + 3\*n)/(a + b\*Sec[c + d\*x^n])^2, x]

[Out] Integrate[(e\*x)^(-1 + 3\*n)/(a + b\*Sec[c + d\*x^n])^2, x]

**fricas** [C] time = 1.31, size = 3855, normalized size = 2.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^(-1+3\*n)/(a+b\*sec(c+d\*x^n))^2,x, algorithm="fricas")

[Out] 
$$\begin{aligned} & 1/12*(4*(a^5 - 2*a^3*b^2 + a*b^4)*d^3*e^{(3*n - 1)*x^{(3*n)}}*\cos(d*x^n + c) + \\ & 4*(a^4*b - 2*a^2*b^3 + b^5)*d^3*e^{(3*n - 1)*x^{(3*n)}} + 12*(a^3*b^2 - a*b^4)* \\ & d^2*e^{(3*n - 1)*x^{(2*n)}}*\sin(d*x^n + c) - (12*(2*a^3*b^2 - a*b^4)*d*e^{(3*n - 1)*x^n}*\sqrt{-(a^2 - b^2)/a^2} - (-12*I*a^2*b^3 + 12*I*b^5)*e^{(3*n - 1)} + ( \\ & 12*(2*a^4*b - a^2*b^3)*d*e^{(3*n - 1)*x^n}*\sqrt{-(a^2 - b^2)/a^2} - (-12*I*a^3*b^2 + 12*I*a*b^4)*e^{(3*n - 1)})*\cos(d*x^n + c))*\operatorname{dilog}(-1/2*(2*(a*\sqrt{-(a^2 - b^2)/a^2} + b)*\cos(d*x^n + c) + (2*I*a*\sqrt{-(a^2 - b^2)/a^2} + 2*I*b)*\sin(d*x^n + c) + 2*a)/a + 1) - (12*(2*a^3*b^2 - a*b^4)*d*e^{(3*n - 1)*x^n}*\sqrt{-(a^2 - b^2)/a^2} - (12*I*a^2*b^3 - 12*I*b^5)*e^{(3*n - 1)} + (12*(2*a^4*b - a^2*b^3)*d*e^{(3*n - 1)*x^n}*\sqrt{-(a^2 - b^2)/a^2} - (12*I*a^3*b^2 - 12*I*a*b^4)*e^{(3*n - 1)})*\cos(d*x^n + c))*\operatorname{dilog}(-1/2*(2*(a*\sqrt{-(a^2 - b^2)/a^2} + b)*\cos(d*x^n + c) + (-2*I*a*\sqrt{-(a^2 - b^2)/a^2} - 2*I*b)*\sin(d*x^n + c) + 2*a)/a + 1) + (12*(2*a^3*b^2 - a*b^4)*d*e^{(3*n - 1)*x^n}*\sqrt{-(a^2 - b^2)/a^2} + (12*I*a^2*b^3 - 12*I*b^5)*e^{(3*n - 1)} + (12*(2*a^4*b - a^2*b^3)*d*e^{(3*n - 1)*x^n}*\sqrt{-(a^2 - b^2)/a^2} + (12*I*a^3*b^2 - 12*I*a*b^4)*e^{(3*n - 1)})*\cos(d*x^n + c))*\operatorname{dilog}(1/2*(2*(a*\sqrt{-(a^2 - b^2)/a^2} - b)*\cos(d*x^n + c) - (2*I*a*\sqrt{-(a^2 - b^2)/a^2} - 2*I*b)*\sin(d*x^n + c) - 2*a)/a + 1) + (12*(2*a^3*b^2 - a*b^4)*d*e^{(3*n - 1)*x^n}*\sqrt{-(a^2 - b^2)/a^2} + (-12*I*a^2*b^3 + 12*I*b^5)*e^{(3*n - 1)} + (12*(2*a^4*b - a^2*b^3)*d*e^{(3*n - 1)*x^n}*\sqrt{-(a^2 - b^2)/a^2} + (-12*I*a^3*b^2 + 12*I*a*b^4)*e^{(3*n - 1)})*\cos(d*x^n + c))*\operatorname{dilog}(1/2*(2*(a*\sqrt{-(a^2 - b^2)/a^2} - b)*\cos(d*x^n + c) - (-2*I*a*\sqrt{-(a^2 - b^2)/a^2} + 2*I*b)*\sin(d*x^n + c) - 2*a)/a + 1) + ((6*I*(2*a^4*b - a^2*b^3)*c^2*\sqrt{-(a^2 - b^2)/a^2} - 12*(a^3*b^2 - a*b^4)*c)*e^{(3*n - 1)}*\cos(d*x^n + c) + (6*I*(2*a^3*b^2 - a*b^4)*c^2*\sqrt{-(a^2 - b^2)/a^2} - 12*(a^2*b^3 - b^5)*c)*e^{(3*n - 1)}*\log(2*a*\cos(d*x^n + c) + 2*I*a*\sin(d*x^n + c) + 2*a*\sqrt{-(a^2 - b^2)/a^2} + 2*b) + ((-6*I*(2*a^4*b - a^2*b^3)*c^2*\sqrt{-(a^2 - b^2)/a^2} - 12*(a^3*b^2 - a*b^4)*c)*e^{(3*n - 1)}*\cos(d \end{aligned}$$

$$\begin{aligned}
& *x^n + c) + (-6*I*(2*a^3*b^2 - a*b^4)*c^2*\sqrt{-(a^2 - b^2)/a^2} - 12*(a^2* \\
& b^3 - b^5)*c)*e^{(3*n - 1)}*\log(2*a*\cos(d*x^n + c) - 2*I*a*\sin(d*x^n + c) + \\
& 2*a*\sqrt{-(a^2 - b^2)/a^2} + 2*b) + ((6*I*(2*a^4*b - a^2*b^3)*c^2*\sqrt{-(a^2 - b^2)/a^2} - 12*(a^3*b^2 - a*b^4)*c)*e^{(3*n - 1)}*\cos(d*x^n + c) + (6*I*( \\
& 2*a^3*b^2 - a*b^4)*c^2*\sqrt{-(a^2 - b^2)/a^2} - 12*(a^2*b^3 - b^5)*c)*e^{(3*n - 1)}*\log(-2*a*\cos(d*x^n + c) + 2*I*a*\sin(d*x^n + c) + 2*a*\sqrt{-(a^2 - b^2)/a^2} - 2*b) + ((-6*I*(2*a^4*b - a^2*b^3)*c^2*\sqrt{-(a^2 - b^2)/a^2} - 1 \\
& 2*(a^3*b^2 - a*b^4)*c)*e^{(3*n - 1)}*\cos(d*x^n + c) + (-6*I*(2*a^3*b^2 - a*b^4)*c^2*\sqrt{-(a^2 - b^2)/a^2} - 12*(a^2*b^3 - b^5)*c)*e^{(3*n - 1)}*\log(-2*a \\
& *\cos(d*x^n + c) - 2*I*a*\sin(d*x^n + c) + 2*a*\sqrt{-(a^2 - b^2)/a^2} - 2*b) \\
& + (-6*I*(2*a^3*b^2 - a*b^4)*d^2*e^{(3*n - 1)}*x^{(2*n)}*\sqrt{-(a^2 - b^2)/a^2} \\
& + 12*(a^2*b^3 - b^5)*d*e^{(3*n - 1)}*x^n + (6*I*(2*a^3*b^2 - a*b^4)*c^2*\sqrt{-(a^2 - b^2)/a^2} + 12*(a^2*b^3 - b^5)*c)*e^{(3*n - 1)} + (-6*I*(2*a^4*b - a^2*b^3)*d^2*e^{(3*n - 1)}*x^{(2*n)}*\sqrt{-(a^2 - b^2)/a^2} + 12*(a^3*b^2 - a*b^4) \\
& )*d*e^{(3*n - 1)}*x^n + (6*I*(2*a^4*b - a^2*b^3)*c^2*\sqrt{-(a^2 - b^2)/a^2} + \\
& 12*(a^3*b^2 - a*b^4)*c)*e^{(3*n - 1)}*\cos(d*x^n + c))*\log(1/2*(2*(a*\sqrt{-(a^2 - b^2)/a^2} + b)*\cos(d*x^n + c) + (2*I*a*\sqrt{-(a^2 - b^2)/a^2} + 2*I*b) \\
& )*\sin(d*x^n + c) + 2*a)/a) + (6*I*(2*a^3*b^2 - a*b^4)*d^2*e^{(3*n - 1)}*x^{(2*n)}*\sqrt{-(a^2 - b^2)/a^2} + 12*(a^2*b^3 - b^5)*d*e^{(3*n - 1)}*x^n + (-6*I*(2 \\
& *a^3*b^2 - a*b^4)*c^2*\sqrt{-(a^2 - b^2)/a^2} + 12*(a^2*b^3 - b^5)*c)*e^{(3*n - 1)} + (6*I*(2*a^4*b - a^2*b^3)*d^2*e^{(3*n - 1)}*x^{(2*n)}*\sqrt{-(a^2 - b^2)/ \\
& a^2} + 12*(a^3*b^2 - a*b^4)*d*e^{(3*n - 1)}*x^n + (-6*I*(2*a^4*b - a^2*b^3)*c \\
& ^2*\sqrt{-(a^2 - b^2)/a^2} + 12*(a^3*b^2 - a*b^4)*c)*e^{(3*n - 1)}*\cos(d*x^n \\
& + c))*\log(1/2*(2*(a*\sqrt{-(a^2 - b^2)/a^2} + b)*\cos(d*x^n + c) + (-2*I*a*\sqrt{-(a^2 - b^2)/a^2} - 2*I*b)*\sin(d*x^n + c) + 2*a)/a) + (-6*I*(2*a^3*b^2 - \\
& a*b^4)*d^2*e^{(3*n - 1)}*x^{(2*n)}*\sqrt{-(a^2 - b^2)/a^2} + 12*(a^2*b^3 - b^5) \\
& )*d*e^{(3*n - 1)}*x^n + (6*I*(2*a^3*b^2 - a*b^4)*c^2*\sqrt{-(a^2 - b^2)/a^2} + \\
& 12*(a^2*b^3 - b^5)*c)*e^{(3*n - 1)} + (-6*I*(2*a^4*b - a^2*b^3)*d^2*e^{(3*n - 1)}*x^{(2*n)}*\sqrt{-(a^2 - b^2)/a^2} + 12*(a^3*b^2 - a*b^4)*d*e^{(3*n - 1)}*x^n \\
& + (6*I*(2*a^4*b - a^2*b^3)*c^2*\sqrt{-(a^2 - b^2)/a^2} + 12*(a^3*b^2 - a*b^4) \\
& )*c)*e^{(3*n - 1)}*\cos(d*x^n + c))*\log(-1/2*(2*(a*\sqrt{-(a^2 - b^2)/a^2} - b) \\
& )*\cos(d*x^n + c) - (2*I*a*\sqrt{-(a^2 - b^2)/a^2} - 2*I*b)*\sin(d*x^n + c) - \\
& 2*a)/a) + (6*I*(2*a^3*b^2 - a*b^4)*d^2*e^{(3*n - 1)}*x^{(2*n)}*\sqrt{-(a^2 - b^2) \\
& )/a^2} + 12*(a^2*b^3 - b^5)*d*e^{(3*n - 1)}*x^n + (-6*I*(2*a^3*b^2 - a*b^4)*c \\
& ^2*\sqrt{-(a^2 - b^2)/a^2} + 12*(a^2*b^3 - b^5)*c)*e^{(3*n - 1)} + (6*I*(2*a^4 \\
& *b - a^2*b^3)*d^2*e^{(3*n - 1)}*x^{(2*n)}*\sqrt{-(a^2 - b^2)/a^2} + 12*(a^3*b^2 \\
& - a*b^4)*d*e^{(3*n - 1)}*x^n + (-6*I*(2*a^4*b - a^2*b^3)*c^2*\sqrt{-(a^2 - b^2) \\
& )/a^2} + 12*(a^3*b^2 - a*b^4)*c)*e^{(3*n - 1)}*\cos(d*x^n + c))*\log(-1/2*(2*( \\
& a*\sqrt{-(a^2 - b^2)/a^2} - b)*\cos(d*x^n + c) - (-2*I*a*\sqrt{-(a^2 - b^2)/a^2} \\
& + 2*I*b)*\sin(d*x^n + c) - 2*a)/a) + 2*((12*I*a^4*b - 6*I*a^2*b^3)*e^{(3*n - 1)}*\sqrt{-(a^2 - b^2)/a^2}*\cos(d*x^n + c) + (12*I*a^3*b^2 - 6*I*a*b^4)*e^{(3*n - 1)}*\sqrt{-(a^2 - b^2)/a^2})*\text{polylog}(3, -1/2*(2*(a*\sqrt{-(a^2 - b^2)/a^2} + b)*\cos(d*x^n + c) - (2*I*a*\sqrt{-(a^2 - b^2)/a^2} + 2*I*b)*\sin(d*x^n + c))/a) + 2*((-12*I*a^4*b + 6*I*a^2*b^3)*e^{(3*n - 1)}*\sqrt{-(a^2 - b^2)/a^2})*\cos(d*x^n + c) + (-12*I*a^3*b^2 + 6*I*a*b^4)*e^{(3*n - 1)}*\sqrt{-(a^2 - b^2)
\end{aligned}$$

)/a^2))\*polylog(3, -1/2\*(2\*(a\*sqrt(-(a^2 - b^2)/a^2) + b)\*cos(d\*x^n + c) - (-2\*I\*a\*sqrt(-(a^2 - b^2)/a^2) - 2\*I\*b)\*sin(d\*x^n + c))/a) + 2\*((12\*I\*a^4\*b - 6\*I\*a^2\*b^3)\*e^(3\*n - 1)\*sqrt(-(a^2 - b^2)/a^2)\*cos(d\*x^n + c) + (12\*I\*a^3\*b^2 - 6\*I\*a\*b^4)\*e^(3\*n - 1)\*sqrt(-(a^2 - b^2)/a^2))\*polylog(3, 1/2\*(2\*(a\*sqrt(-(a^2 - b^2)/a^2) - b)\*cos(d\*x^n + c) + (2\*I\*a\*sqrt(-(a^2 - b^2)/a^2) - 2\*I\*b)\*sin(d\*x^n + c))/a) + 2\*((-12\*I\*a^4\*b + 6\*I\*a^2\*b^3)\*e^(3\*n - 1)\*sqrt(-(a^2 - b^2)/a^2)\*cos(d\*x^n + c) + (-12\*I\*a^3\*b^2 + 6\*I\*a\*b^4)\*e^(3\*n - 1)\*sqrt(-(a^2 - b^2)/a^2))\*polylog(3, 1/2\*(2\*(a\*sqrt(-(a^2 - b^2)/a^2) - b)\*cos(d\*x^n + c) + (-2\*I\*a\*sqrt(-(a^2 - b^2)/a^2) + 2\*I\*b)\*sin(d\*x^n + c))/a))/((a^7 - 2\*a^5\*b^2 + a^3\*b^4)\*d^3\*n\*cos(d\*x^n + c) + (a^6\*b - 2\*a^4\*b^3 + a^2\*b^5)\*d^3\*n)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^{3n-1}}{(b \sec(dx^n + c) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^(-1+3\*n)/(a+b\*sec(c+d\*x^n))^2,x, algorithm="giac")

[Out] integrate((e\*x)^(3\*n - 1)/(b\*sec(d\*x^n + c) + a)^2, x)

**maple** [F] time = 2.93, size = 0, normalized size = 0.00

$$\int \frac{(ex)^{-1+3n}}{(a + b \sec(c + dx^n))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x)^(-1+3\*n)/(a+b\*sec(c+d\*x^n))^2,x)

[Out] int((e\*x)^(-1+3\*n)/(a+b\*sec(c+d\*x^n))^2,x)

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^(-1+3\*n)/(a+b\*sec(c+d\*x^n))^2,x, algorithm="maxima")

[Out] Timed out

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(ex)^{3n-1}}{\left(a + \frac{b}{\cos(c+dx^n)}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x)^(3*n - 1)/(a + b/cos(c + d*x^n))^2, x)`

[Out] `int((e*x)^(3*n - 1)/(a + b/cos(c + d*x^n))^2, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^{3n-1}}{(a + b \sec(c + dx^n))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)**(-1+3*n)/(a+b*sec(c+d*x**n))**2, x)`

[Out] `Integral((e*x)**(3*n - 1)/(a + b*sec(c + d*x**n))**2, x)`

# Chapter 4

## Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

### 4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*      GradeAntiderivative[result,optimal] returns*)
(* "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(* "C" if result involves higher level functions than necessary*)
(* "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(* "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
```

```

If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
  If[LeafCount[result]<=2*LeafCount[optimal],
    "A",
    "B"],
  "C"],
If[FreeQ[result,Integrate] && FreeQ[result,Int],
  "C",
"F"]]

```

```
(* ::Text:: *)
```

```
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
```

```
(*1 = rational function*)
```

```
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
```

```
(*4 = special function*)
```

```
(*5 = hyperpergeometric function*)
```

```
(*6 = appell function*)
```

```
(*7 = rootsum function*)
```

```
(*8 = integrate function*)
```

```
(*9 = unknown function*)
```

```
ExpnType[expn_] :=
```

```
  If[AtomQ[expn],
```

```
    1,
```

```
  If[ListQ[expn],
```

```
    Max[Map[ExpnType,expn]],
```

```
  If[Head[expn]===Power,
```

```
    If[IntegerQ[expn[[2]],
```

```
      ExpnType[expn[[1]],
```

```
    If[Head[expn[[2]]===Rational,
```

```
      If[IntegerQ[expn[[1]] || Head[expn[[1]]===Rational,
```

```
        1,
```

```
        Max[ExpnType[expn[[1]],2]],
```

```
      Max[ExpnType[expn[[1]],ExpnType[expn[[2]],3]],
```

```
    If[Head[expn]===Plus || Head[expn]===Times,
```

```
      Max[ExpnType[First[expn],ExpnType[Rest[expn]]],
```

```
    If[ElementaryFunctionQ[Head[expn]],
```

```
      Max[3,ExpnType[expn[[1]]],
```

```
    If[SpecialFunctionQ[Head[expn]],
```

```
      Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
```

```
    If[HypergeometricFunctionQ[Head[expn]],
```

```
      Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
```

```
    If[AppellFunctionQ[Head[expn]],
```

```
      Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
```

```

If[Head[expn]===RootSum,
  Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
If[Head[expn]===Integrate || Head[expn]===Int,
  Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
9]]]]]]]]]]

ElementaryFunctionQ[func_] :=
MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]

SpecialFunctionQ[func_] :=
MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]

HypergeometricFunctionQ[func_] :=
MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]

AppellFunctionQ[func_] :=
MemberQ[{AppellF1},func]

```

## 4.0.2 Maple grading function

```

# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017 Use Maple leaf count instead since buildin
#Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017 corrected the check for complex result
#Nasser 10/27/2017 check for leafsize and do not call ExpnType()
#
# if leaf size is "too large". Set at 500,000

```

```

#Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
# see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

    leaf_count_result:=leafcount(result);
    #do NOT call ExpnType() if leaf size is too large. Recursion problem
    if leaf_count_result > 500000 then
        return "B";
    fi;

    leaf_count_optimal:=leafcount(optimal);

    ExpnType_result:=ExpnType(result);
    ExpnType_optimal:=ExpnType(optimal);

    if debug then
        print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
ExpnType_optimal);
    fi;

# If result and optimal are mathematical expressions,
# GradeAntiderivative[result,optimal] returns
# "F" if the result fails to integrate an expression that
# is integrable
# "C" if result involves higher level functions than necessary
# "B" if result is more than twice the size of the optimal
# antiderivative
# "A" if result can be considered optimal

#This check below actually is not needed, since I only
#call this grading only for passed integrals. i.e. I check
#for "F" before calling this. But no harm of keeping it here.
#just in case.

if not type(result,freeof('int')) then
    return "F";
end if;

if ExpnType_result<=ExpnType_optimal then
    if debug then
        print("ExpnType_result<=ExpnType_optimal");
    fi;

```



```

if is_contains_complex(result) then
  if is_contains_complex(optimal) then
    if debug then
      print("both result and optimal complex");
    fi;
    #both result and optimal complex
    if leaf_count_result<=2*leaf_count_optimal then
      return "A";
    else
      return "B";
    end if
  else #result contains complex but optimal is not
    if debug then
      print("result contains complex but optimal is not");
    fi;
    return "C";
  end if
else # result do not contain complex
  # this assumes optimal do not as well
  if debug then
    print("result do not contain complex, this assumes optimal do not
as well");
  fi;
  if leaf_count_result<=2*leaf_count_optimal then
    if debug then
      print("leaf_count_result<=2*leaf_count_optimal");
    fi;
    return "A";
  else
    if debug then
      print("leaf_count_result>2*leaf_count_optimal");
    fi;
    return "B";
  end if
end if
else #ExpnType(result) > ExpnType(optimal)
  if debug then
    print("ExpnType(result) > ExpnType(optimal)");
  fi;
  return "C";
end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false

```

```

#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'^^') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
      max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  end if
end proc:

```

```

elif HypergeometricFunctionQ(op(0,expn)) then
  max(5,apply(max,map(ExpnType,[op(expn)])))
elif AppellFunctionQ(op(0,expn)) then
  max(6,apply(max,map(ExpnType,[op(expn)])))
elif op(0,expn)='int' then
  max(8,apply(max,map(ExpnType,[op(expn)]))) else
9
end if
end proc:

ElementaryFunctionQ := proc(func)
  member(func,[
    exp,log,ln,
    sin,cos,tan,cot,sec,csc,
    arcsin,arccos,arctan,arccot,arcsec,arccsc,
    sinh,cosh,tanh,coth,sech,csch,
    arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
    erf,erfc,erfi,
    FresnelS,FresnelC,
    Ei,Ei,Li,Si,Ci,Shi,Chi,
    GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
    EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product. rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u)
  else
    apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

```

```
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
    MmaTranslator[Mma][LeafCount](u);
end proc:
```

### 4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#           Port of original Maple grading function by
#           Albert Rich to use with Sympy/Python
#Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
#           added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
        asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
        asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
        fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
        gamma,loggamma,digamma,zeta,polylog,LambertW,
        elliptic_f,elliptic_e,elliptic_pi,exp_polar
        ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow): #type(expn,'`^`')
        if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
            return expnType(expn.args[0]) #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)
    ))
    else:
        return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
    ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type
    (expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))

```

```

elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
expn))
    m1 = max(map(expnType, list(expn.args)))
    return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
elif is_appell_function(expn.func):
    m1 = max(map(expnType, list(expn.args)))
    return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
elif isinstance(expn,RootSum):
    m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
Apply[List,expn]],7]],
    return max(7,m1)
elif str(expn).find("Integral") != -1:
    m1 = max(map(expnType, list(expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"

```

## 4.0.4 SageMath grading function

```

#Dec 24, 2019. Nasser: Ported original Maple grading function by
#           Albert Rich to use with Sagemath. This is used to
#           grade Fracas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#           'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow: #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
        'sin','cos','tan','cot','sec','csc',
        'arcsin','arccos','arctan','arccot','arcsec','arccsc',
        'sinh','cosh','tanh','coth','sech','csch',
        'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
    ]
    if debug:
        if m:

```

```

        print ("func ", func , " is elementary_function")
    else:
        print ("func ", func , " is NOT elementary_function")

    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
sinh_integral'
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e',
        'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")

    return m

def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in
sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:

```



```

        return expn.operator() is None
    if expn.parent() in (ZZ, QQ, AA, QQbar):
        return expn in expn.parent() # Should always return True
    if hasattr(expn.parent(), "base_ring") and hasattr(expn.parent(), "gens"):
        return expn in expn.parent().base_ring() or expn in expn.parent().
gens()
    return False

except AttributeError as error:
    return False

def expnType(expn):

    if debug:
        print(">>>>Enter expnType, expn=", expn)
        print(">>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list: #instance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(instance(expn.args[0],
Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
    elif expn.operator() == operator.pow: #instance(expn,Pow)
        if type(expn.operands()[1])==Integer: #instance(expn.args[1],Integer)
            return expnType(expn.operands()[0]) #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational: #instance(expn.args[1],
Rational)
            if type(expn.operands()[0])==Rational: #instance(expn.args[0],
Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
instance(expn,Add) or instance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))

```

```

    return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()): #is_elementary_function(expn.
func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
    return max(4,m1) #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
    return max(5,m1) #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
    return max(6,m1) #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
#is checked before calling the grading function that is passed.
#but kept it here.
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

```

```
#main function
```

```
def grade_antiderivative(result,optimal):
```

```

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex

```

```
        if leaf_count_result <= 2*leaf_count_optimal:
            return "A"
        else:
            return "B"
    else: #result contains complex but optimal is not
        return "C"
else: # result do not contain complex, this assumes optimal do not as
well
    if leaf_count_result <= 2*leaf_count_optimal:
        return "A"
    else:
        return "B"
else:
    return "C"
```